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Preface

How to Use This Book

"I understand the material but I just can't do the problems."
"I can do the homework problems but the problems on the test are just too different."
"If I only knew the right formula, I would have gotten that problem right."
"I recognized that problem but I just couldn't remember the solution."

If you have ever felt that way about physics problems, this booklet is for you. It is intended to provide guidance for students wishing to develop a technique for solving physics problems similar to that used by experts. The technique is based on research describing how experts in many fields solve real problems. You will not find clever procedures to shorten the solution of certain types of problems in this booklet. No mathematical tricks are presented. Instead we try to explain how to use a general strategy that works for physics and all other fields that use systematic problem solving.

No matter what your field of interest, problem solving consists of approximately the same steps: recognize the problem (what's going on here?), describe the problem in the terms used by your field (what does this have to do with what I know?), plan a solution (where do I go from here?), execute the plan (what's the answer?), and evaluate the solution (can my answer be true?). Although an expert might accomplish some of these steps mentally, a non-expert does not yet have the years of practice needed to develop the necessary memory structure. The non-expert can implement the process using writing as an extension of memory. A logical, well organized written analysis of the problem is the most important tool in problem solving. As most students know, the most difficult part of solving a problem is getting started on the right track. The problem solving technique in this booklet is designed to help you do just that.

The booklet is not really designed to be read from beginning to end. It is more like a reference book which can be entered anywhere. We suggest reading Chapter 1 to get the overall picture of the strategy. Then try working out some of the problems for which complete solutions are given. We have given solutions to typical textbook problems which occur at the beginning of an introductory physics course. Once you catch on, you should be able to apply this problem solving technique to any situation on your own. If you have trouble applying any of the problem solving steps, read the appropriate section in Chapter 2. If you have trouble with the specific approaches of kinematics, dynamics, or conservation, read the appropriate parts of Chapter 3, 4, or 5.

Problem solving is a skill and, as with any skill, the best technique often seems "unnatural." Just think about your favorite sport. With practice you will get better and more efficient. After a while, this technique will become "second nature" to you. Practice is the key. But practice is only useful if you are always practicing the same thing. Pick plenty of your own exercises from your textbook. Practice writing down <u>all</u> of the steps of the general problem solving strategy even if you know an "easier" way of solving the problem. After practicing on the "easy" textbook exercises try some of the "realistic" problems in this booklet. Only a few solutions are given but, if you are good at using the problem solving strategy, you will know if you got them right.

Many people have contributed a great deal to the evolution of this booklet. First we thank the numerous researchers in expert-novice problem solving upon whose work this booklet is based. Pure research is always the basis for sound practice. Many graduate students in both physics and science education at the University of Minnesota have made very important contributions. We especially wish to thank Ron Keith who was responsible for much of the original draft of the booklet. Bruce Palmquist, Scott Anderson, Doug Huffman, Jennifer Blue and James Flaten then extended and refined the work. Many physics teaching assistants have contributed to the development and testing of the general problem solving strategy and the evolution of this booklet. We are also grateful to the many undergraduates who have participated in the courses through which this booklet was developed. Finally we thank Professors Charles Campbell, Clayton Giese, Walter Johnson, Roger Jones, and Konrad Mauersberger at the University of Minnesota for their comments and contributions in the development of this booklet.

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University of Minnesota, 1995

Chapter 1

A Logical Problem Solving Strategy

Introduction

At one level, problem solving is just that, solving problems. Presented with a problem you try to solve it. If you have seen the problem before and you already know its solution, you can solve the problem by recall. Much of the time, however, you have never experienced this situation before (if you had, you would not call it a problem). Solving real problems involves making a logical chain of decisions which lead from an unclear situation to a solution. Solving physics problems is not very different from solving any kind of problem. In your professional life, you will encounter new and complex problems (after all if they knew how to solve them, why would they pay you?). The skillful problem solver is able to invent good solutions for these new problem situations. But how does the skillful problem solver create a solution to a new problem? And how do you learn to be a more skillful problem solver?

The purpose of this booklet is to provide you with some guidance for solving physics problems. The technique given in this booklet is based on research done in a variety of disciplines such as physics, medical diagnosis, engineering, project design and computer programming. There are many similarities in the way experts in these disciples solve problems. The most important result is that experts follow a *general strategy* for solving all complex problems. The following sections in this chapter describe the characteristics of this general problem-solving strategy. In Chapter 2 this general strategy is elaborated in a way that makes it particularly useful for solving the physics problems you will encounter in this course. Chapters 3, 4, and 5 describe how to use the problem solving strategy to solve problems using kinematics, dynamics, and conservation ideas. These chapters also include example textbook problems with solutions and some additional problems for you to practice solving using the problem solving strategy. If you learn this strategy you will be successful in this course. In addition, you will become familiar with a general strategy for solving problems that will be useful in your chosen profession.

A Logical Problem-Solving Strategy

Experts solve real problems in several steps. Getting started is the most difficult step. In the first and most important step, you must accurately *visualize the situation*, *identify the actual problem*, and *identify information relevant to the problem.* At first you must deal primarily with the qualitative aspects of the situation. You must interpret the problem in light of your own knowledge and experience. This enables you to decide what information is important, what information can be ignored, and what additional information may be needed, even though it was not explicitly provided. In this step drawing a useful picture of the problem situation is crucial to getting started correctly. A picture is worth a thousand words (if it is the right picture). In the second step, you must *represent the problem* in terms of formal concepts and principles, whether these are concepts of engineering design, concepts of medicine, or concepts of physics. These formal concepts and principles use the accumulated knowledge of your field and thus enable you to simplify a complex problem to its essential parts. Frequently, your field has developed a formalized way to diagram the situation which helps show how the concepts are usually applied to a problem. Third, you must use your representation of the problem to *plan a solution*. Planning results in an outline of the logical steps required to obtain a solution. In many cases the logical steps are conveniently expressed as mathematics. Fourth, you must determine a solution by actually *executing* the logical steps outlined in your plan. Finally, you must *evaluate* how well the solution resolves the original problem.

The general strategy can be summarized in terms of five steps.:

(1) Comprehend the problem.
(2) Represent the problem in formal terms.
(3) Plan a solution.
(4) Execute the plan.
(5) Interpret and evaluate the solution.

The strategy begins with the qualitative aspects of a problem and progresses toward the quantitative aspects of a problem. Each step uses information gathered in the previous step to translate the problem into more quantitative terms and to clarify the decisions which you must make. These steps should make sense to you. You have probably used a similar strategy, without thinking about it, when you have solved problems before.

The Importance of Writing

Solving a problem requires that you constantly make decisions. This is very difficult to do if you must also remember many pieces of information and the relationships between those pieces of information. Soon you overload your brain which has only a small number of short term memory locations. You could forget important parts of the problem or the steps in a mathematical procedure. The chain of decisions you construct may even have logical flaws. Drawing pictures and diagrams and writing your procedures using words, symbols, and mathematics makes the paper a part of your extended memory. Your brain is then free to deal with the decision-making process. The single biggest mistake of novice problem solvers is not writing down enough in a form which is organized to be a useful aid to their memory. If you have had the experience of understanding how to solve a problem when someone shows you how but "getting lost" when you try to do a similar problem yourself, the effective use of writing could be your primary trouble.

A Physics-Specific Strategy

Each profession has its own specialized knowledge and patterns of thought. The knowledge and thought processes that you use in each of the steps will depend on the discipline in which you operate. Taking into account the specific nature of physics, we choose to label and interpret the five steps of the general problem solving strategy as follows:

1. Focus the Problem: In this step you develop a qualitative description of the problem. First, visualize the events described in the problem using a sketch. Write down a simple statement of what you want to find out. Write down the physics ideas which might be useful in the problem and describe the approach you will use. When you finish this step, you should never have to refer to the problem statement again.

2. Describe the Physics: In this step you use your qualitative understanding of the problem to prepare for a quantitative solution. First, simplify the problem situation by describing it with a diagram in terms of simple physical objects and essential physical quantities. Restate what you want to find by naming specific mathematical quantity(ies). Using the physics ideas assembled in step 1, write down equations which specify how these physical quantities are related according to the principles of physics or mathematics. The results of this step contains all of the relevant information so you should not need to refer to step 1 again.

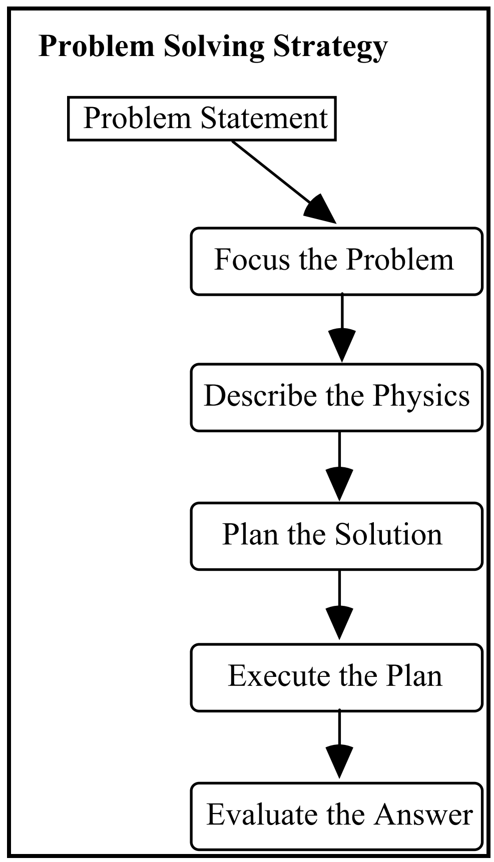
3. <u>Plan the Solution</u>: In this step you translate the physics description into a set of equations which represent the problem mathematically by using the equations assembled in step 2. Each equation should have a specific goal to find a single unknown quantity in the problem. An equation thus used may involve a new unknown quantity which must be determined using another equation. In other words, solving the original problem usually involves creating and solving sub-problems. As you do the mathematical operations to isolate your unknown quantities, you create an outline of how to arrive at a solution. You will find that most of your effort will go into deciding how to construct this logical chain of equations with less effort spent on mathematical operations.

4. <u>Execute the Plan</u>: In this step you actually execute the solution you have planned. Plug in all of the known quantities into the algebraic solution, which is the result of step 3, to determine a numerical value for the desired unknown quantity(ies).

5. <u>Evaluate the Answer</u>: Finally, check your work to see that it is properly stated, not unreasonable, and actually answers the question asked.

Consider each step as a translation of the previous step into a slightly different language. You begin with the full complexity of real objects interacting in the real world and through a series of decisions arrive at a simple and precise mathematical expression.

The solution to the following problem illustrates each step. On the right side of the page is the actual solution, as you might construct it. On the left side of the page are brief descriptions of each step of the solution. We have used a familiar situation so that you can concentrate on understanding how the strategy is applied. In later chapters, we will consider each of the steps individually and in more detail.



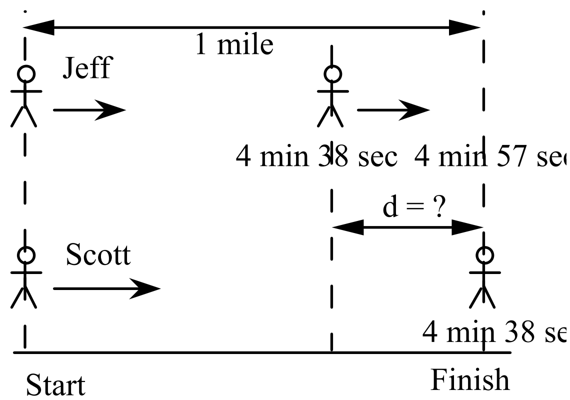
Example 1: Jeff and Scott are first rate runners. Jeff's best time in the mile is 4 min, 57 sec. Scott's best time in the mile is 4 min, 38 sec. If Scott and Jeff raced each other at their best in a mile run, by how far would Scott beat Jeff?

(1) <u>Focus the Problem</u>:

Visualize the events described in the problem and draw a sketch.

Write down what you want to find.

Write down the physics ideas that help you understand the events and describe the approach you will use.



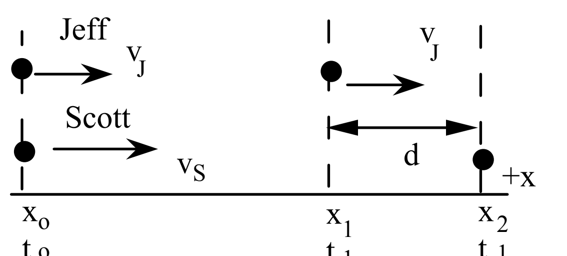
Question: What is the distance between the runners when Scott crosses the finish line?

Approach: Use the definition of average velocity to relate the runner's displacement to the elapsed time.

Assume that Jeff's average velocity is the same for this race as for the entire mile.

(2) <u>Describe the Physics</u>:

Simplify the problem by describing it in terms of simple physical objects and physical quantities. Usually this requires defining a coordinate system. Restate what you want to find in terms of a target quantity. Write down how these physical quantities are related by principles and definitions of physics.



$x_o = 0$ $\quad x_1 = ?$ $\quad x_2 = 1$ mile

$t_o = 0$ $\quad t_1 = 4$ min 38 s $= 278$ s

$$v_{J,Ave} = \frac{1 \text{ mile}}{297 \text{ sec}}, \quad v_{s,Ave} = \frac{1 \text{ mile}}{278 \text{ sec}}$$

Target Quantity: d

Quantitative Relationships: $v_{Ave} = \frac{\Delta x}{\Delta t}$

$$d = x_2 - x_1$$

(3) Plan the Solution:

Translate the physics description into a set of equations which relate specific physics quantities. Determine if the equations will allow you to find the answer you want. Check to see that there are no left over unknowns. Check the units.

unknowns

Find d: *d*

$$d = x_2 - x_1$$

Find x_1 *x1*

$$v_{J,Ave} = \frac{x_1 - x_o}{t_1 - t_o} = \frac{x_1}{t_1}$$

$$x_1 = v_{J,Ave}\, t_1$$

$$d = x_2 - v_{J,Ave}\, t_1$$

Check Units: $[\text{mi}] - \frac{[\text{mi}]}{[\text{sec}]}[\text{sec}] = [\text{mi}]$ *OK*

(4) Execute the Plan:

Put in known values of quantities to determine the numerical solution.

$$d = 1\ \text{mi} - \frac{1\ \text{mile}}{297\ \text{sec}}(278\ \text{sec}) = 0.064\ \text{mi}$$

(5) Evaluate the Answer:

Check your work to see that it is properly stated, not unreasonable and that you have actually answered the question asked.

The unit of distance is miles.

The answer is reasonable since the distance between them is a small fraction of the total distance.

The answer is the distance between the two runners at the finish.

In Example 1, you can clearly recognize the logical flow of the problem solving steps. In the first step, you focus your mental processes by expressing the problem in everyday terms in the form of a sketch, extracting a question and stating your approach. In the next step, you translate this information into important physical quantities and the relationships which characterize the physics of the problem. Then using this information, you construct a specific set of equations that relate the unknown physics quantities to those that you know and combine them to solve the problem. Before going to the next step you check your units to make sure you haven't made a math mistake. Next, you follow your plan by plugging in numbers to obtain a numerical solution. In the final step, you check your work, and consider how well the numerical solution answers the original problem. **The solution is complete when you are convinced that you have an answer, and that the answer is a good one.**

Example 1 serves to illustrate the steps of the strategy but not its value. Although you can understand the logical steps, you might question its usefulness. Perhaps after reading the problem, you knew just what equations you would use to solve the problem. You didn't need to reason through each step of the strategy. That is because this situation is simple enough that it is not a real problem for you. It does show a technique which will help you solve real problems, those you don't already know how to solve. This is the method that experts use to confront problems that they don't know how to solve. Expert problem solvers employ this strategy because it is the most effective and efficient way to solve realistic problems. Of course, those problems require many more steps and many more decisions. To prepare for the real world, you will need to master this powerful problem solving technique.

The five-step strategy represents an effective way to organize your thinking to produce a solution based on your best understanding of physics. The quality of the solution depends on the knowledge that you use in obtaining the solution. Your use of the strategy also makes it easier to look back through your solution to check for incorrect knowledge or assumptions. That makes it an important tool for learning physics. **If you learn to use the strategy effectively, you will find it a valuable tool to use for solving new and complex problems and for learning physics.**

Problem Difficulty

Throughout this course you will encounter problems whose difficulty ranges from the simple to the quite complex. Some straightforward, but not necessarily easy exercises (called problems) are given at the end of each chapter in your textbook. These exercises allow you to practice using the physics principles presented in the chapter. Exercises are usually (but not always) characterized by certain features:

- They may involve only a single application of one major principle, so that deciding on an approach to the problem is simple.
- The question is clearly stated as the need to find some specified physics quantity, e.g. velocity, energy, force, so that the relevant physics description is often suggested by the problem statement itself.
- Just enough information has been provided for you to determine a numerical value for the desired quantity, so that describing the situation and problem approach are simplified.
- All quantitative information is given in a simple set of units, so that if the correct principle is applied, the numerical solution will be correct. This simplifies evaluation of the solution.
- They often resemble other exercises which you have recently encountered.

Because the objects described in the exercise and their relationships are similar to other examples given, visualization of the problem is simpler.

It is **strongly recommended** that you use the five-step strategy when you are solving these textbook exercises. This will help make the strategy feel natural to you when you use it on real problems. Although the textbook exercises are straightforward, the physics principle and/or mathematical technique used may be new to you. Using the five-step strategy helps you learn the physics principle or mathematical technique because it provides a logical structure and organization to guide your thinking. Before you can solve the kinds of problems you might encounter in real life, you have to first practice your skills in simple situations, and then in increasingly complex situations as your skills improve.

Such well-focused exercises are not the only kind of problems that you will encounter in this course. On tests, you will be asked to solve more realistic types of problems that can be complex in several possible ways:

- The problems may require the application of multiple principles and/or multiple applications of the same principle.
- The question may not be stated as the need to find any particular quantity, much less a specified physics quantity; the problem may ask for a judgment, in which case you must decide what quantities you need to find in order to make a good judgment.
- The problem statement may include information which is not useful at all. On the other hand, some important information may not be expressly provided; you will have to provide that information from your own general knowledge.
- Quantitative information may be provided in unfamiliar or inconsistent units.
- The objects or interactions described in the problem situation may appear new to you; it may appear that you never have seen or solved a similar problem.

Problems with the above characteristics are similar to the problems that you will encounter in your chosen profession. It is important for you to practice on the simple textbook problems so that you develop the knowledge and skills required to solve realistic problems.

Next we give an example of a slightly more difficult problem. As before, on the right side is the actual solution, as you might construct it. On the left side are brief descriptions of what is being done at each step of the solution.

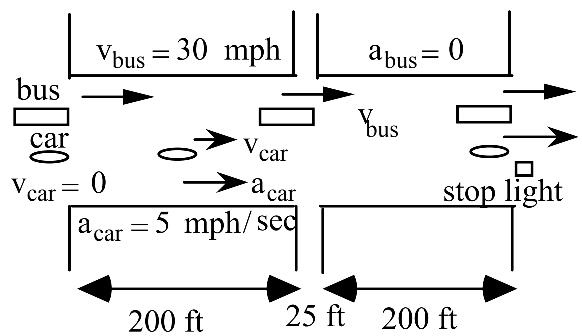
Example 2: Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30-mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200-ft long and the side streets are 25-ft wide, while the main avenue is 60-ft wide. If you increase your speed at a rate of 5-mph each second, will you make it?

(1) <u>Focus the Problem</u>: In this step of the problem solving strategy construct your initial qualitative understanding of the problem situation. Write down what you know, what you want to know, the physics you will use, and the assumptions you will make. This understanding can be usefully expressed as follows:

Picture & Given Information:

What's happening? Visualize the problem situation and make a sketch of the important objects and events.

Decide which given information may be useful and write it down on the sketch.



Question(s):

What is(are) the question(s)? Express it as some quantity to be found.

Find the distance the car travels to catch up to the bus. See if it is less than 425 feet.

Approach:

What approach shall I take? Outline the concepts which can relate the given information to the question.

Use the definition of average velocity for the bus since it travels at constant velocity.

Use the relationship between acceleration and position for the car since it travels at constant acceleration.

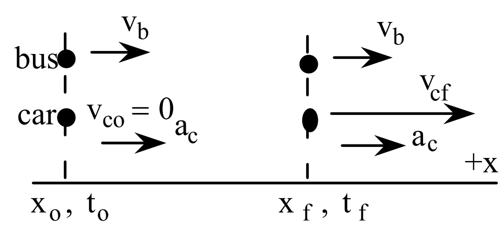
Initial time is when the bus and the car are first together. Final time is when the bus and the car are next together.

(2) **<u>Describe the Physics</u>**: In this step use your physics ideas to translate your initial understanding of the problem into a diagram of the actual problem. This diagram contains only idealized physical objects and representations of important physical quantities. Identify which of these physical quantities you need to find to answer the question. Write down the relationships between the quantities which will help you determine the unknowns. This information can be summarized with the following items:

Diagram & Define Quantities:

For kinematics problems, use a motion diagram. This diagram requires:
* Coordinate axes.
* Simplified representations (usually points) of objects.
* Indication of position, velocity and acceleration of objects at important times.

Identify known and unknown quantities.



$x_o = 0 \quad x_f = ?$

$t_o = 0 \quad t_f = ?$

$v_{co} = 0 \quad v_{cf} = ?$

$v_b = 30 \text{ mph} \quad a_c = 5 \text{ mph/ sec}$

Target Quantity(ies):

Decide which of your unknowns you will need to find in order answer the problem question.

$x_f = ?$

Quantitative Relationships:

Decide which physics principles or other mathematical relationships are applicable for the situation diagrammed above.

$$v_b = \frac{x_f - x_o}{t_f - t_o} = \frac{x_f}{t_f} \qquad v_b \text{ constant}$$

$$x_f = \frac{1}{2}a_c\left(t_f - t_o\right)^2 + v_{co}\left(t_f - t_o\right) + x_o$$

$$x_f = \frac{1}{2}a_c\left(t_f\right)^2 \qquad a_c \text{ constant}$$

(3) Plan the Solution: In this step translate your physics description of the problem into the particular equations, which will help you solve the problem. Always begin with an equation from your quantitative relationships containing the target quantity. If that equation contains additional unknowns, write down another equation from your quantitative relationships containing one of those unknowns. Continue until you have introduced a new equation for every unknown in your plan.

Construct Specific Equations:

Use your quantitative relationships to write specific equations relating unknown quantities to ones which are known.

unknowns

Find x_f $\qquad$ x_f

$$x_f = \frac{1}{2} a_c \left(t_f \right)^2$$

t_f

Find t_f

$$v_b = \frac{x_f}{t_f}$$

$$t_f = \frac{x_f}{v_b}$$

$$x_f = \frac{1}{2} a_c \left(\frac{x_f}{v_b} \right)^2$$

$$\frac{2v_b^2}{a_c} = x_f$$

Check Units:

Make sure the units on both sides of your equation are the same.

$$\frac{\left[\frac{mi}{hr}\right]^2}{\frac{mi}{hr^2}} = mi \qquad \textit{OK}$$

(4) <u>Execute the Plan</u>: In this step carry out the mathematics specified in your solution plan in order to determine a numerical value for your target quantity(ies).

Calculate Target Quantity(ies):

Put numerical values of known quantities into the equation for the target quantity. Convert units if necessary and calculate a value for the target quantity.

$$x_f = 2\frac{\left(30\frac{\text{mi}}{\text{hr}}\right)^2}{\left(\frac{5\frac{\text{mi}}{\text{hr}}}{\text{s}}\right)}$$

$$x_f = 360\left(\frac{\text{mi}}{\text{hr}}\right)\text{s}$$

$$x_f = 360\left(\frac{\text{mi}}{\text{hr}}\right)\text{s}\left(\frac{\text{hr}}{3600\,\text{s}}\right) = \frac{1}{10}\text{mi}$$

Since 0.1 miles is 528 feet, which is more than 425 feet, you do not make it.

(5) <u>Evaluate the Answer</u>: As a result of executing your plan, you have a numerical answer to the physics problem. In this final step, check that your answer is properly stated, not unreasonable, and complete.

Is Answer Properly Stated?:
Check that your answer has the appropriate units and sign.

Yes, miles are a correct unit for distance.

Is Answer Unreasonable?:
Check that the magnitude of your answer is not unexpectedly large or small.

The answer is only about 100 ft longer than the 2 block distance which is not unreasonable.

Is Answer Complete?:
Check that you have answered the original question.

The car does not make it. This answers the question.

The preceding example solutions introduced the essential features of each step of the physics problem solving strategy. In the table below these features are summarized. In the next chapter, these features are described more fully and are illustrated by example.

In the third chapter, we will introduce a format sheet which serves as a guide for solving problems. Using these format sheets, we will construct solutions to several exercises, ranging from intermediate difficulty to complex.

Finally, we will provide additional problems so that you can practice using the strategy. Practice is all important. Problem solving is a skill which can be developed, but as with all skills, your improvement depends upon the effort you invest.

Summary of the Physics Problem Solving Strategy

1. Focus the Problem
- **Picture & Given Information**
- **Question(s)**
- **Approach**

2. Describe the Physics
- **Diagrams & Define Physics Quantities**
- **Target Quantity(ies)**
- **Quantitative Relationships**

3. Plan the Solution
- **Start with equation which has target quantity(ies)**
- **Identify other unknowns in equation**
- **Solve a sub-problem for each unknown**
- **Check Units**

4. Execute the Plan
- **Calculate Target Quantity(ies)**

5. Evaluate the Answer
- **Is Answer Properly Stated?**
- **Is Answer Unreasonable?**
- **Is Answer Complete?**

Chapter 2

The Five-Step Physics Problem Solving Strategy

Introduction

The purpose of this chapter is to clarify the physics problem solving strategy. This will be accomplished by addressing each step of the strategy and illustrating that step by two examples. The main features of each step are first discussed, then that step is done for each example. The two examples are shown below.

Each step of the example solution consists of two parts: On the left-hand page you will find the statements, equations, diagrams, and other information that you might write down if you were to solve the example using the five-step strategy. On the right hand page is a commentary on the problem solving process. The commentary illustrates questions you ask yourself and decisions you must make when you are solving unfamiliar physics problems.

Example 3: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55-mph. Ahead you see a sign that says, "Curve Ahead 200 ft, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your brakes at the instant you pass the sign. You slow your car down at a rate of 7-mph each second. As you reach the curve, are you traveling within the posted speed limit?

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4-oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

1. Focus the Problem

The first and most important step to take in solving any problem is to understand just what the problem is. The goal of this step is to establish a qualitative understanding of the problem situation. To solve a problem about the real world, you want to have a clear mental image of the situation. This helps you to use all of your general knowledge and experience, not simply the physics formulas that you recall. The essential features of this step are:

Picture and Given Information

Begin by visualizing the events as portrayed in the problem statement. Identify the objects and the time sequence of events which are central to understanding the situation. Of particular importance are those times when an object experiences an abrupt change. A sketch of the situation is a useful way to focus your mind on the problem. All of the given information should be added to the sketch so you can see what is going on at a glance.

Question(s)

Every problem has a question. After you have reduced the problem situation to a sketch, you need to determine the question. For most textbook exercises this is not too difficult. Usually there is a direct statement which tells you what to find. For more realistic problems, the question may not be well formulated so that you can answer it with a calculation. For example, you may be asked to make a judgment. You must then restate the question in a form which can be answered by a physics calculation. Consider the following problem:

> You and your friend are racing to see who can be the first to make a purchase from Burger Bros. Starting at the same time from the McDonald's parking lot in Dinkytown, you head for the Roseville store and your friend heads for the Bloomington store. If you both agree to obey the speed limits and all traffic regulations, who will be the first to make a purchase?

This problem does not directly ask a question which can be answered using quantities determined using physics. However, the question really deals with time, a physics quantity. The question can be rephrased in terms of three different questions: "How much time does it take you to travel the distance to the Roseville store? How much time does it take your friend to travel the distance to the Bloomington store? Which of these time intervals is greater?" Time and distance are the basic physical quantities. The time can be calculated from other physical quantities such as velocities, positions and accelerations. It is clear you will have to make some assumptions about each person's average speed which depends on the speed limit. Rephrasing the question in terms of time enables you use physics to analyze the problem quantitatively. In many problems, success in solving the problem will depend critically upon your translation of the question(s) into physics terms.

Approach

Once you have a useful sketch of the situation and you have identified a question whose answer should resolve the problem, you can begin to think about how to approach the problem: What can physics tell you about the events? In your approach you gather your thoughts about how physics can help you solve the problem. It represents your first guess as to what facts and physics principles will be useful in the creation of a solution. Your approach helps you to make decisions regarding the following questions: What given information is potentially useful for answering the question? What additional information might I need, even if I have to estimate it? What information seems irrelevant and can be ignored?

In physics, there are a few basic patterns of explanation, fundamental principles, which have proved exceptionally valuable for thinking about natural events. The kinematics description of motion is one of these. Newtonian forces and energy conservation and momentum conservation are other fundamental ways of thinking about how the motion of objects is affected by interactions with other objects. Each fundamental principle typically relates several physical quantities through a mathematical relationship. Selecting one or a combination of fundamental principles determines your approach to the problem. Whether or not a physics principle will prove useful in a given situation depends upon characteristics of the problem. In the approach you write down your best guess of which of these principles will be most useful, and perhaps easiest for solving this particular problem.

Focus the Problem

Problem Statement

construct a mental image of the sequence of events described in the problem statement

- What's going on?
- What objects are involved?
- What are they doing?

sketch a picture which represents this mental image; include given information

- Are all the important objects shown?
- Are the spatial relations between the objects shown?
- Are the important times represented?
- Are the important motions represented?
- Are the important interactions represented?

determine the question

- Does the question ask about a specific measurable characteristic(s) about a particular object(s)? If not, reformulate it so it does.

select a qualitative approach which should lead to a solution to the problem

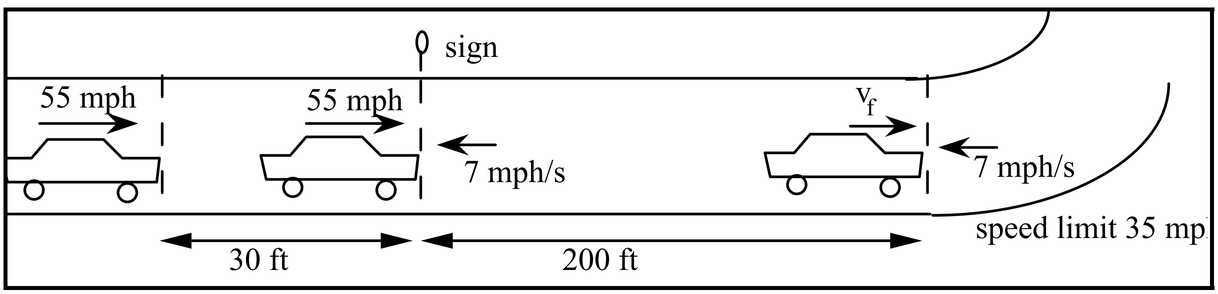
- What is the system of interest?
- Which physics principles could be used to solve the problem?
- What information is really needed?
- Are there only certain time intervals during which one approach is useful?
- Should we make any approximations?

Describe the Physics

Example 3: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55-mph. Ahead you see a sign that says, "Curve Ahead 200 ft, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your brakes at the instant you pass the sign. You slow your car down at a rate of 7-mph each second. As you reach the curve, are you traveling within the posted speed limit?

Focus the Problem

Picture and Given Information:



Question(s):

What is the speed of the car when it reaches the curve?
Is that speed less than 35 mph?

Approach:

Use kinematics.

Assume the car's acceleration is constant between the sign and the curve.

For constant acceleration, the average acceleration equals the instantaneous acceleration.

Use the definition of average acceleration to relate the change of velocity to the time interval.

Use the relationship between acceleration and position for constant acceleration.

<u>COMMENTARY</u>

<u>Picture and Given Information</u>:

First visualize the events described.

• ***What are the important objects and what do they do from beginning to end?***
The situation describes a car moving along a highway. The problem begins 30 ft from a highway sign and ends 200 ft beyond the sign, where a curve in the road begins.

• ***What is a good perspective from which to sketch the motion?***
Use a bird's eye view (view from above) to show the position of the car relative to the curve in the road.

• ***Does the motion of the car change between the beginning and the end of the described motion?***
Yes. The speed is constant up to the sign, but decreases thereafter. Put these points on the sketch as well.

<u>Question(s)</u>:

• ***What do I need to find out?***
The question is posed in the last sentence. I must calculate the speed of my car as I enter the curve and compare it to the speed limit of 35 mph.

<u>Approach</u>:

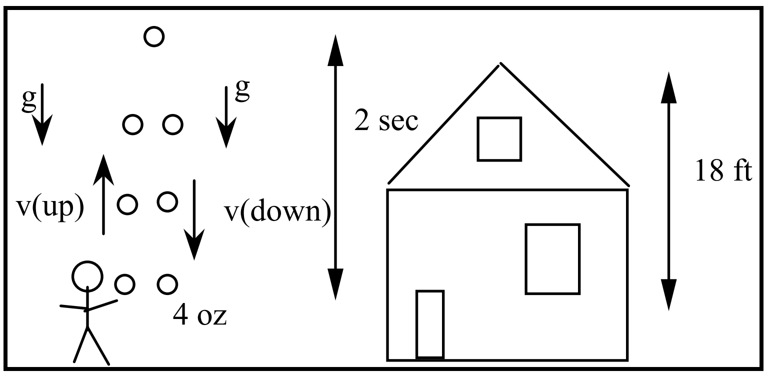
• ***Which general concepts and principles are useful for thinking about this problem? What kind of problem is it?***
Applying kinematics seems like a good approach. The final velocity of an object traveling from point to point depends on its initial velocity, its acceleration, and the distance between those points. Position, velocity, and acceleration are important quantities. But time is also an important quantity in kinematics. Since I don't know the time that the car takes to travel between the sign and the curve, I may need to use more than one kinematics relationship.

• ***Is any part of the problem not useful for answering the question?***
Yes. We can neglect the motion of the car before it reaches the sign because we know the speed of the car at this point and we know how far the car is from the curve.

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4-oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

Focus the Problem

Picture and Given Information:



Question(s):

What is the maximum height of the ball?

Is it less than the height of the house?

Approach:

Use kinematics.

The ball has a constant acceleration (down) throughout its flight so its average acceleration equals its instantaneous acceleration.

Use the relationship between acceleration and position for constant acceleration.

The velocity of the ball (but not its acceleration) is zero at its highest point.

Solve motion of ball for two different time intervals

1. *Hand to maximum height (know the final velocity)*
2. *Hand back to hand (know the time interval)*

Need to know height of house. Estimate it. Height from floor to ceiling is about 10 ft. Add 2 feet for floor above ground and space between first and second floor. About 22 ft .

Need to know height of hand when ball released. Estimate it. About 4 ft from ground.
Height of house above release point of ball is about 18 ft.

COMMENTARY

Picture and Given Information:

First visualize the events described.

- ***What are the important objects and what do they do from beginning to end?***
  Only the baseball is moving, but its motion is to be compared to the height of the house. Your brother tosses the ball straight up. After the ball leaves his hand, it travels up, but is slowing down. The ball reaches some maximum height, before it begins to fall back down to where he can catch it.

- ***What is a good perspective from which to sketch the motion?***
  A side view conveniently shows the position of the ball relative to the height of the house. Draw the path of the ball down a little offset from its path up.

- ***Does the motion of the ball change between the beginning and the end of the described motion?***
  Yes. From the instant your brother releases the ball upwards, until the instant just before he catches it again, the ball has a constant acceleration down. That acceleration causes the ball to slow down as it travels up and speed up as it travels down. At the top of its path, the instantaneous velocity of the ball is zero but its acceleration is still down. During the actual throwing and catching, the ball experiences different accelerations but these are not relevant.

Question(s):

- ***What do I need to find out?***
  The question is clearly stated in the last sentence. Find the baseball's highest point. Is that point above the top of the house?

Approach:

- ***Which general concepts and principles are useful for thinking about this problem? What kind of problem is it?***
  Kinematics is a good approach. How far the ball travels depends on its initial velocity and its acceleration and the time of travel. But the height that it reaches also depends on the height from which it is tossed. The acceleration is constant the entire time that it is in the air. The time that it is in the air depends on from what height it is thrown and at what height it is caught again. It may be useful to break the problem up into 2 time intervals even though the motion does not change since you are asked for the height up but are given the total time up and down.

- ***Do I need to assume any information which is not provided in the problem statement?***
  Yes. First I need to estimate the height of a two story house. The question asks to compare the ball's maximum height to the top of the house. Second, I have to make assumptions about the height from which the ball is thrown. How high the ball gets depends in part on the height at which he releases the ball upwards. Suppose he releases the ball at shoulder height, about 4 ft. It is also important to know where he catches the ball. An important piece of information may be the duration of time for which the ball is in the air. This is affected by the height at which he catches the ball. For convenience, assume that he catches the ball at shoulder height.

Summary of Focus the Problem

When you have brought the problem into focus in your mind and when you have made a sketch, you have decided upon the essential features of the problem. Moreover, you know what you want to find and how you might go about finding it from the given information. If this step is completed correctly, you have no need to refer to the original problem statement again. In the next step of the problem solving strategy, you use the ideas, sketch and information to construct a physics description of the problem, which in turn makes determination of a solution easier.

Summary of the Physics Problem Solving Strategy

1. **Focus the Problem**
   - **Picture & Given Information**
   - **Question(s)**
   - **Approach**

2. Describe the Physics
   - Diagrams & Define Physics Quantities
   - Target Quantity(ies)
   - Quantitative Relationships

3. Plan the Solution
   - Start with equation which has target quantity(ies)
   - Identify other unknowns in equation
   - Solve a sub-problem for each unknown
   - Check Units

4. Execute the Plan
   - Calculate Target Quantity(ies)

5. Evaluate the Answer
   - Is Answer Properly Stated?
   - Is Answer Unreasonable?
   - Is Answer Complete?

2. Describe the Physics

In the second step of solving a problem, uses the picture and approach from step 1 to reduce the problem situation to its essential physics concepts and principles. This step involves drawing a physics diagram, defining the target quantity(ies), and describing the quantitative relationships that apply to the problem. Here you specify the object or objects of interest (the system), the times of interest, and the positions of interest. You also specify a coordinate system which will allow you to define mathematically precise physics quantities.

Diagram & Define Quantities

The Physics Description reduces the stated problem to physics quantities which are related by principles that are expressed mathematically. Construct this ***physical representation*** of the problem by treating the real world objects as simplified objects which can be characterized by a few physics quantities. Assign unique names to represent the values of these important physics quantities at important times. A good diagram is the most useful tool in physics problem solving because it provides an easily understood summary of all the important information. By carefully examining your diagram, you can often recognize important relationships between quantities that might otherwise be missed.

Although there are different kinds of diagrams, they all share certain characteristics. The kind of diagram you use will depend on your basic approach to the problem. For example, motion diagrams conveniently summarize the important information concerning the motion (positions, times, velocity, and acceleration) of an object and are useful in a kinematics approach. Free body and force diagrams are used for expressing information about the interactions of objects and the vector nature of that interaction. They are useful in an interaction approach using Newtonian force laws. A good diagram always has a conveniently chosen coordinate axes, a simple representation of important objects, and unique representations of the values of the relevant physics quantities of the object at important times.

A good diagram requires coordinate axes. Coordinate axes provide the reference which allows you to express positions and vector directions mathematically. If you employ more than one axis, then the axes should be at right angles to each other. Other than that, the direction of coordinate axes is a matter of your convenience. You decide which directions will serve best as the axes, and you can define the origin of the coordinate axes as well as the positive direction. This freedom often enables you to simplify problems. For example, suppose there is a problem in which a car slides down a twenty foot long, icy driveway which is inclined at 10 degrees to the horizontal. You wish to find the speed of the car at the end of the driveway. If you choose axes which are horizontal and vertical, then the problem is difficult because the car moves along both directions. On the other hand, if you choose an axis which is parallel to the driveway then the problem is simpler because the motion of the car is entirely along that one direction.

On your diagram important objects are represented by simple objects, usually points. More elaborate drawings of the object provide no useful physics information and may confuse your mind with irrelevant information. All of the relevant information concerning an object and its behavior is expressed in terms of the interactions of that object with its environment, and its velocity and acceleration at each position and time. For example, in linear kinematics problems, any object can be considered to be a point. The size of the object, its shape, or its material composition are not important. For rotational motion, on the other hand, the object's size and shape may be crucial.

The diagram directly expresses all useful physics information. For example, a motion diagram shows the physics quantities which are relevant to a kinematics description of a problem. In that description, the motion of an object is completely determined by its position, velocity, and acceleration at each instant of time. In most cases of interest the motion of an object at one time (final time) can be predicted from a complete description of its motion (position, velocity, and acceleration) at another time (initial time). In the diagram, the position of an object at the different times is indicated by separate points. Next to each point, one arrow is drawn to indicate the velocity of the object at that instant. Another arrow is used to indicate the acceleration of the object at that instant.

Target Quantity(ies)

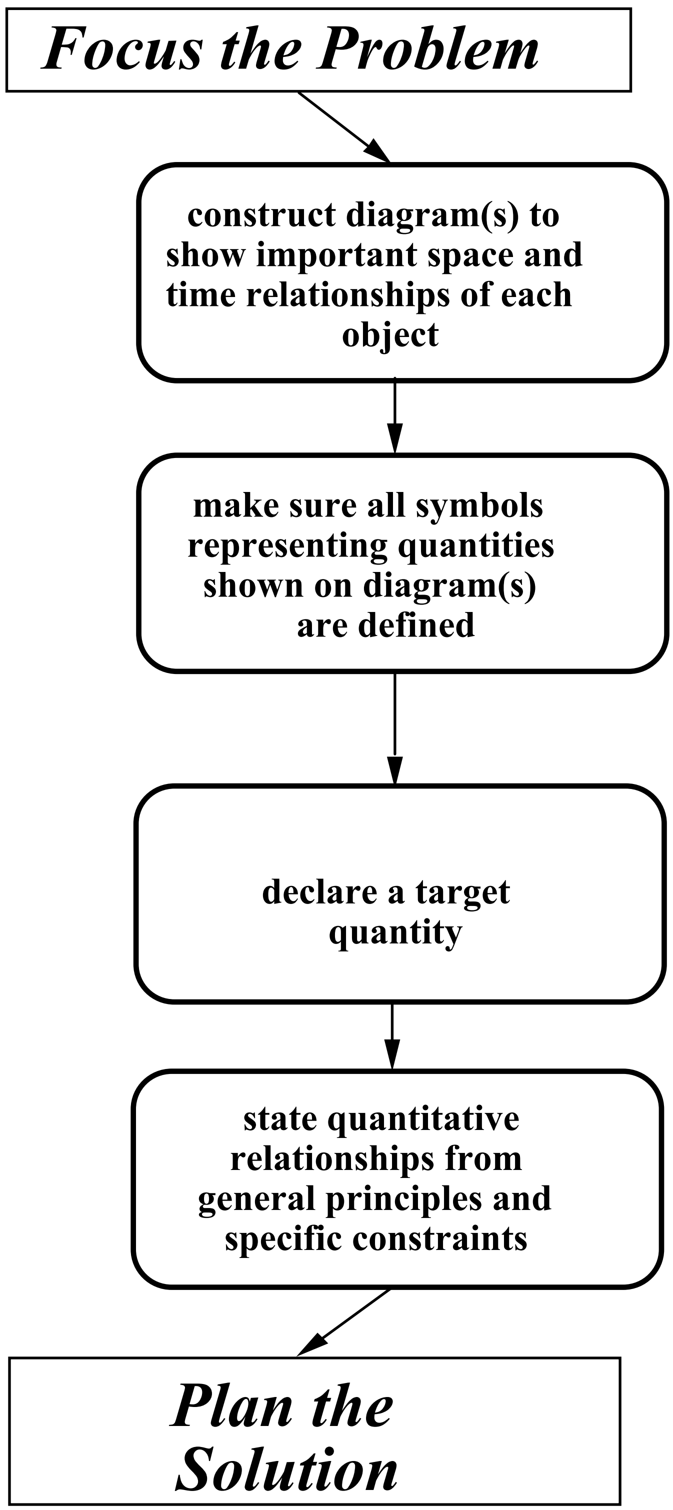
Once you have constructed a simple physics diagram, reformulate your original question in terms of the physics quantities you have defined. One (or more) of the unknown quantities in your description represents the information that you think will answer the question(s) posed in words in step 1. Identify that unknown by its symbol.

Quantitative Relationships

The final feature of the physics description is a list of the quantitative relationships between the physics quantities which are applicable to the problem. These relationships are of two kinds. First, there are **basic principles of physics or mathematics**, which always relate the quantities defined in your diagram to each other. For example, the average velocity is always the change in position of an object divided by the corresponding time interval to make that change: $v_{ave} = \Delta x/\Delta t$. After defining the appropriate coordinate axes and angles the mathematical expression such as $\sin\theta = V_y/V$ represents another such relationship. The other type of relationship which should be included in your physics description is often referred to as a **relationship of constraint**. This relationships is only true for the specific situation you are considering. For example, suppose in a problem it is stated that the distance traveled by a car is twice the distance traveled by a truck. This establishes a relationship of constraint between the quantity that represents the car's position and the one that represents the truck's positions at some specified time.

In your text you will come across many equations. Only a few of these, those corresponding to basic principles and definitions, belong in the physics description. Whenever you apply fundamental relationships to a specific type of problem, such as those dealing with freely falling objects, you can develop specialized equations which are valid only for that problem type. Since there are many different types of problems, the specialized equations are not very useful knowledge, unless you will encounter that specific situation over and over again in your work. It is much more practical to learn the few fundamental relationships and how to apply them to many types of problems. Developing the ability to apply fundamental knowledge to situations you have never encountered before is the aim of this course and the emphasis in this booklet.

Describe the Physics



- What coordinate axes are useful? Which direction should we call positive?
- Relative to the coordinate axes, where is (are) the object(s) for each important time?
- Relative to the coordinate axes, what is (are) the velocity and acceleration for each object at each important time?
- Are other diagrams necessary to represent the interactions of each object or the time evolution of its state?

- What quantities are needed to define the problem mathematically using the approach chosen?
- Which symbols represent known quantities?
  Which symbols represent unknown quantities?
- Are all quantities having different values labeled with unique symbols?
- Does the diagram(s) have all of the essential information from the sketch?

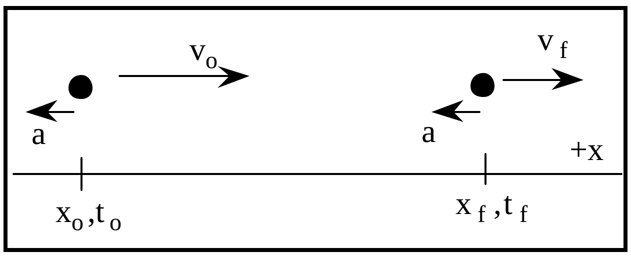
- Which of the unknowns defined on the diagram(s) answers the question?

- What equations represent the general principle(s) specified in our approach and relate the physics quantities defined in the diagram?
- During what time intervals are those relationships either true or useful?
- Are there any equations that represent special conditions that are true for some quantities in this problem?

Example 3: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55-mph. Ahead you see a sign that says, "Curve Ahead 200 ft, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your brakes at the instant you pass the sign. You slow your car down at a rate of 7-mph each second. As you reach the curve, are you traveling within the posted speed limit?

Describe the Physics

Diagram & Define Quantities:



$x_0 = 0$ $t_0 = 0$

$v_0 = 55\ mph$

$x_f = 200\ ft$ $t_f = ?$ $v_f = ?$

$a = \dfrac{-7mph}{s}$

Target Quantity(ies):

$v_f = ?$

Quantitative Relationships:

Since acceleration is constant,

$$a = \frac{v_f - v_0}{t_f - t_0} = \frac{v_f - v_0}{t_f}$$

instantaneous acceleration equals average acceleration

And

$$x_f = \frac{1}{2}(\ a)(t_f - t_0)^2 + v_0(t_f - t_0) + x_0 = \frac{1}{2}(\ a)(t_f)^2 + v_0(t_f)$$

COMMENTARY

Diagram & Define Physics Quantities:

For kinematics problems a motion diagram conveniently shows the velocity and acceleration of the object at every important position and time. Every diagram has coordinate axes, with the positive direction clearly indicated, as a reference for defining the positions of the object, indicated simply by a point (dot), and the orientation of its motion.

• ***What are convenient coordinate axes?***

In this problem, the velocity of the object is always along one direction, so a suitable coordinate system would be along the direction of the velocity (horizontal) with "+" chosen to be the direction the object is moving.

• ***Relative to the coordinate axis where is the object at important times?***

Important times and positions are where you know the values of important physical quantities (velocity and acceleration) of your chosen object, or where you think you may need to know their value. There are two important positions for the car. Call x_o the position of the car as it passes by the sign, and t_o the corresponding time. Call x_f the position of the car as it enters the curve, and t_f the corresponding time. For convenience take x_o to be the origin of the coordinate system and t_o to be the origin of time.

• ***Relative to the coordinate axis what is the magnitude and the orientation of the velocity of the object at each important point?***

The velocity of the object is parallel to the axis directed to the right. Because the object is slowing down, the velocity is larger at x_o than at x_f. Next to each point, add an arrow indicating the direction and relative magnitude of the velocity of the object at that point.

• ***Relative to the coordinate axis what is the acceleration of the object at each important point?***

The acceleration refers to the change in the velocity. Since the object is slowing down, the change in velocity is negative, $(v_f - v_o) < 0$. Therefore, the acceleration is parallel to the axis and points to the left.

• ***For which of the defined quantities do you have a numerical value?***

List known values for defined quantities. Also list which quantities are unknown.

Target :

• ***Of the unknowns defined above, for which do you want to determine a value in order to answer the question?***

Find the speed of the car as it enters the curve. That corresponds to v_f.

Quantitative Relationships:

• ***Which basic physics principles or other relationships of constraint can you use to relate known quantities to your target ?***

In every kinematics problem, the only basic principles are the definition of velocity and the definition of acceleration. For the special case of constant acceleration, we will use a relationship derived from those basic principles.

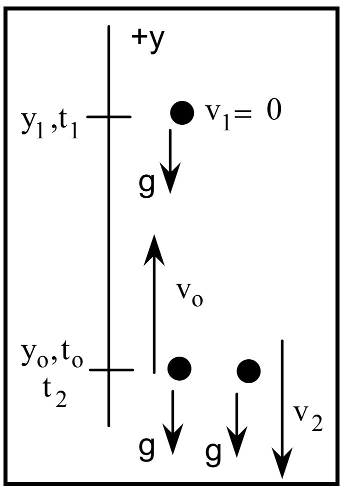
• ***Are there any conditions which must be satisfied in order for these relationships to be valid?***

Yes, the acceleration must be constant.

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4-oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

Describe the Physics

Diagram & Define Quantities:



$y_o = 0$	$t_o = 0$	$v_o = ?$
$y_1 = ?$	$t_1 = ?$	$v_1 = 0$
$y_2 = y_o = 0$	$t_2 = 3\ \text{s}$	$v_2 = ?$

$$g = -32\,\frac{ft}{s^2}$$

y_o is shoulder height

Target Quantity(ies):

$y_1 = ?$ *Is it greater than or equal to 18 ft?*

Quantitative Relationships:

For constant acceleration

$$g = \frac{v_f - v_o}{t_f - t_o} = \frac{v_f - v_o}{t_f}$$

And

$$y_f = \frac{1}{2}(\ g)\left(t_f - t_o\right)^2 + v_o\left(t_f - t_o\right) + y_o = \frac{1}{2}(\ a)\left(t_f\right)^2 + v_o\left(t_f\right)$$

COMMENTARY

Diagram & Define Physics Quantities:

For kinematics problems a motion diagram conveniently shows the velocity and acceleration of the object at every important position and time. Every diagram has coordinate axes as a reference for defining the positions of the object, indicated simply by a point (dot), and the orientation of its motion.

• ***What are convenient coordinate axes?***

If that the ball travels straight up and down, a suitable coordinate system would be a vertical axis with the upper side of the origin as chosen as the "+" direction.

• ***Relative to the coordinate axis where is the object at important times?***

Important times and positions are where you know the velocity and acceleration of your chosen object, or where you think you may need to know their value. Important positions for the ball are the positions where it is released, where it is caught, and its highest position.

• ***Relative to the coordinate axis what are the magnitude and the orientation of the velocity of the object at each important point?***

On its way up the velocity of the ball is parallel to the axis directed upwards. The magnitude of the velocity during this time decreases because the ball is slowing down. As it falls back down, the ball's velocity is parallel to the axis directed downwards. The magnitude of the velocity increases as it falls. At its highest point, the ball's velocity is zero at that instant.

• ***Relative to the coordinate axis what is the acceleration of the object at each important point?***

The acceleration refers to the change in the velocity. Since the baseball is slowing down on the way up, the change in velocity is down or in the negative direction in the chosen coordinate system. That is the direction of the acceleration. At its highest point, the ball has zero velocity at that instant but its acceleration is still down. As the ball moves downward, it speeds up so the direction of the acceleration is still down.

• ***For which of the defined quantities do you have a numerical value?***

List known values for defined quantities. Assume that the ball is released and caught at shoulder height, about 4 ft above the ground. This means the ball must go 18 ft to reach the top of the house.

Target :

• ***Of the unknowns defined above, which do you want to determine a value for in order to answer the question?***

Find the position of the baseball at its highest point. That corresponds to y_1 on the diagram.

Quantitative Relationships:

• ***Which basic physics principles or other relationships of constraint can you use to relate known quantities to your target?***

In every kinematics problem, the only basic principles are the definition of velocity and the definition of acceleration. For the special case of constant acceleration, we will use a relationship derived from those basic principles.

• ***Are there any conditions which must be satisfied for these relationships to be valid?***

Yes, the acceleration must be constant.

Summary of Describe the Physics

When you have completed the physics description, you have reduced the problem to its essentials. You have defined the quantity to be found and all of the quantities about which useful information is known, or can be reasonably assumed. You have also specified the quantitative relationships which link unknown quantities to known quantities. In the next step of the problem solving strategy, you translate this physics description into a set of mathematical equations with a prescription of how to use those equations to determine the target quantity.

Summary of the Physics Problem Solving Strategy

1. Focus the Problem
   - Picture & Given Information
   - Question(s)
   - Approach

2. **Describe the Physics**
   - **Diagrams & Define Physics Quantities**
   - **Target Quantity(ies)**
   - **Quantitative Relationships**

3. Plan the Solution
   - Start with equation which has target quantity(ies)
   - Identify other unknowns in equation
   - Solve a sub-problem for each unknown
   - Check Units

4. Execute the Plan
   - Calculate Target Quantity(ies)

5. Evaluate the Answer
   - Is Answer Properly Stated?
   - Is Answer Unreasonable?
   - Is Answer Complete?

3. <u>Plan the Solution</u>

The goal of planning the solution is to use the quantitative relationships between the quantities defined in the physics description to create a set of equations which is sufficient to determine the value(s) of the target quantity. In the process of creating this set of equations, you automatically create a logical chain of mathematical operations that will allow you to compute a solution to the problem.

Construct the Solution

Always begin with an equation, one of your quantitative relationships from the Physics Description, which contains the target quantity. This will ensure that your mathematical efforts will actually solve the problem you want. After writing this equation, examine it to see which quantities are known and which are unknown. It is useful to keep a list of the unknown quantities as they occur in the equations. If you only have one unknown, the target, then you're done.

Usually there will be more than one unknown in this equation. If you have more than one unknown, return to your quantitative relationships to write down a <u>different</u> equation which contains this new unknown. You can consider this the beginning of a new problem, a sub-problem. To solve your original problem, you must solve the sub-problem first. Solving a sub-problem does not require getting a numerical answer. You only need to solve for the sub-problem target unknown in terms of unknown quantities which have already been addressed earlier in the problem plan.
Check to see if a new equation introduces any new unknowns which were not already in previous equations. If so add them to your list of unknowns as they occur and solve for each unknown as a further sub-problem. If not then combine the equations to determine a value for the target unknown. If other unknowns have been introduced, return to your quantitative relationships and construct different equations containing the new unknowns. And so it goes, until your only unknown is the problem's target or you run out of different equations you can construct. If you run out of equations, either the problem has no solution, one of your unknowns does not actually determine the behavior of the object and will algebraically cancel out of the solution, or your Physics Description is incomplete. Go back and check your picture and approach to see if there is some information your left out of your Physics Description, or if your qualitative physic reasoning tells you that one of your unknowns is superfluous.

Remember, the rules of algebra are strict. You cannot change the equations so that the math works out. Usually a careless algebra error is due to not taking enough time with each step. It happens to everyone. One technique you can use to find such an error is to check the units of each term of each step of your algebra, starting from where you are and tracing back to the beginning.

Do not waste time plugging in numbers at intermediate stages. Sometimes many potential calculations will cancel out in the algebra. Of course it is always useful to put in the value of any quantities that are zero.

Check the Units

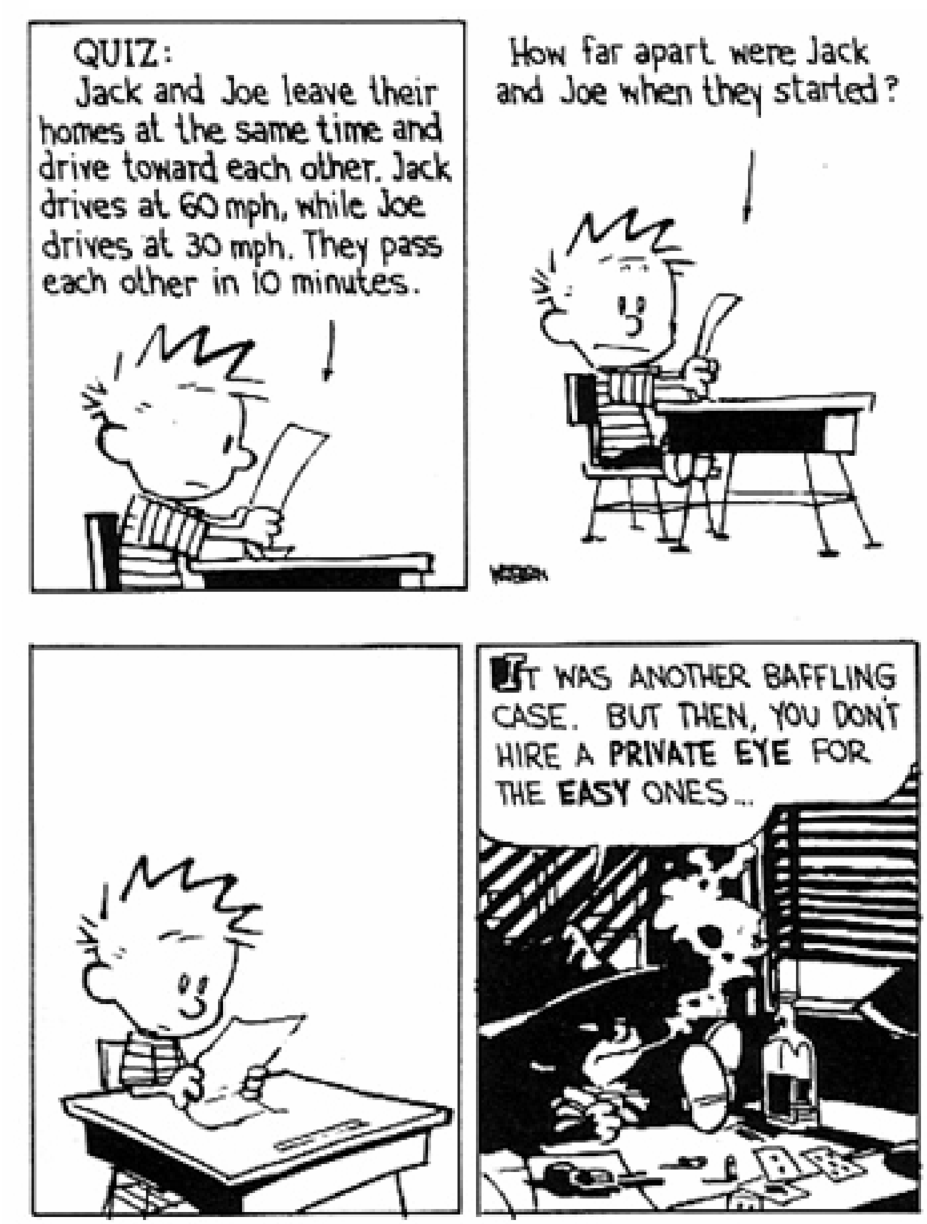
After you have an equation in which the only unknown is the target quantity, check your units to see if you have made a mathematical mistake. If you find your units don't balance, look back through your plan to find the mistake and correct it.

The old adage "You can't add apples and oranges." applies here. All terms which are added together (or subtracted) in your equation must have the same type of units. The earliest step in which this is not true is the step where you made your mistake. For example, in the equation:

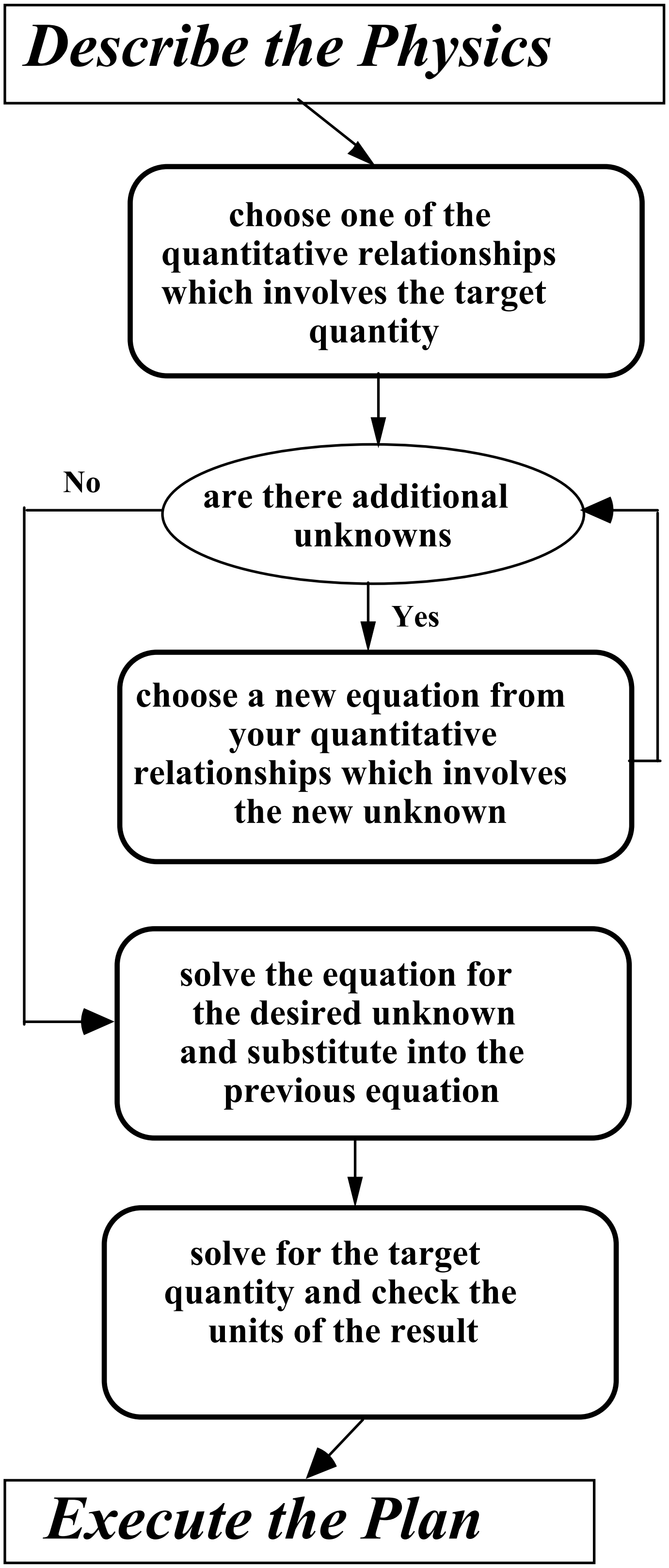
$$x = a + (bc) + \frac{de}{f}$$

x, a, (bc), and (de/f) must all have the same type of units. If x is a distance: a, (bc) and (de/f) must all have the units of distance (e.g. ft, meters, km, mm, inch).

Calvin and Hobbes / By Bill Watterson



Plan the Solution



- Which quantitative relationship includes the target quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?

- Are there any unknowns in the equation other than the target quantity?
- Are there any unknowns that cancel out in the algebra?

- Which quantitative relationship includes the unknown quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?
- Is this equation different from those already used in this problem?

- What unknown is the target of this specific equation?
- Which previous equations have that unknown?
- Are there any quantities that cancel out in the algebra?

- After all the substitution for unknowns, is the only unknown left the target quantity?
- Are the units the same on both sides of the equation?

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Plan the Solution

Construct the Solution:

unknowns

Find v_f — v_f

$$a = \frac{v_f - v_o}{t_f} \qquad [1]$$

t_f

Find t_f

$$x_f = \frac{1}{2}(\ a)(t_f)^2 + v_o(t_f) \qquad [2]$$

$$0 = \frac{1}{2}(\ a)(t_f)^2 + v_o(t_f) - x_f$$

$$t_f = \frac{-v_o \pm \sqrt{v_o^2 + 2ax_f}}{(\ a)}$$

$$a = \frac{v_f - v_o}{\dfrac{-v_o \pm \sqrt{v_o^2 + 2ax_f}}{(\ a)}}$$

$$a\frac{-v_o \pm \sqrt{v_o^2 + 2ax_f}}{(\ a)} = v_f - v_o$$

$$\pm\sqrt{v_o^2 + 2ax_f} = v_f$$ *velocity is + when car reaches the curve*

$$\sqrt{v_o^2 + 2ax_f} = v_f$$

Check Units

$$\sqrt{\left[\frac{m}{s}\right]^2 - \left[\frac{m}{s^2}\right][m]} = \sqrt{\left[\frac{m}{s}\right]^2} = \left[\frac{m}{s}\right] \qquad \mathit{OK}$$

COMMENTARY

Equation #1:

1) What's a specific equation from your quantitative relationships involving the target quantity?
   The definition of average acceleration includes the target quantity v_f.

2) Are the signs in front of the quantities in the equation consistent with the diagram?
   Yes. The acceleration is negative and the initial velocity and final velocity are positive.

3) Are there any additional unknowns in this equation?
   Yes. t_f is a new unknown. Finding it is a sub-problem.

Equation #2:

1) What's a specific equation from your quantitative relationships involving the new unknown?
   For constant acceleration the relationship between acceleration and position includes the unknown t_f.

2) Are there any unknowns in this equation which do not appear in the previous equation?
   No!

3) Since there are no additional unknowns, solve the sub-problem. Solve equation 2 for t_f and put it into equation 1.

4) Choose the sign which physically agrees with the solution you desire. The other sign is also a physical solution to the problem. Understand what it represents also. In this case if the car continued to move with a negative acceleration, its velocity would change direction and some time later it would return to the sign moving in the negative direction.

Check the Units:

- Do additive quantities have the same units?
  Yes.

- Are the units the same as those expected for the target quantity?
  Yes.

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4-oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

Plan the Solution

Construct the Solution:

			unknowns
Find y_1			y_1
$y_1 = \frac{1}{2}(\ \ g)(t_1)^2 + v_o(t_1)$	**[1]**	*motion of ball up*	v_o, t_1

Find t_1

$$g = \frac{0 - v_o}{t_1} \qquad \textbf{[2]}$$

$$t_1 = \frac{-v_o}{g}$$

$$y_1 = \frac{1}{2}(\ \ g)\left(\frac{v_o}{g}\right)^2 + v_o\left(\frac{-v_o}{g}\right)$$

$$y_1 = \ \frac{1}{2}\frac{v_o^2}{g} - \frac{v_o^2}{g}$$

$$y_1 = \frac{-v_o^2}{2g}$$

Find v_o

$$0 = \frac{1}{2}(\ \ g)(t_2)^2 + v_o(t_2) \qquad \textbf{[3]} \qquad \textit{motion of ball up and back down}$$

$$0 = \frac{1}{2}(\ \ g)t_2 + v_o$$

$$-\frac{1}{2}gt_2 = v_o$$

$$y_1 = \frac{-\left(\frac{-1}{2}gt_2\right)^2}{2g}$$

$$y_1 = \frac{-1}{8}gt_2^2$$

Check Units: $\left[\frac{m}{s^2}\right]\left[s^2\right] = [m]$ *OK*

COMMENTARY

Equation #1:

1) What's a specific equation from your quantitative relationships involving the target quantity ?

For constant acceleration the relationship between acceleration and position includes the target quantity y_1 if we consider only the motion of the ball from the instant after it leaves the hand until it reaches the top of its path.

2) Are there any additional unknowns in this equation?

Yes. t_1 and v_o are new unknowns. Finding them will give rise to sub-problems.

Equation #2:

1) What's a specific equation from your quantitative relationships involving one of the new unknowns ?

The definition of average acceleration includes the target quantity t_1. We could have chosen to solve for v_o first but either way is OK. Since we want t_1, we consider the motion of the ball from just after it leaves the hand until it reaches the top of its path.

2) Are there any unknowns in this equation which do not appear in the previous equation?

No!

3) Since there are no additional unknowns, solve the sub-problem. Solve equation 2 for t_1 and put it into equation 1. Some algebra can now simplify equation 1.

Equation #3:

1) What's a specific equation from your quantitative relationships involving the other of the new unknowns ?

For constant acceleration the relationship between acceleration and position includes the unknown v_o if we consider only the motion of the ball from the instant after it leaves the hand until just before it reaches the hand again. Because we are considering a different part of the ball's motion than before, this is a "new" equation.

2) Are there any unknowns in this equation which do not appear in the previous equation?

No!

3) Since there are no additional unknowns, solve the sub-problem. Solve equation 3 for v_o and put it into the results of equation 1 and equation 2. This determines the target quantity.

Check the Units:

- Do additive quantities have the same units?

  There are no additive quantities this time.
- Are the units the same as expected from the target quantity?

  Yes.

Summary of Plan the Solution

The purpose of this step of the problem solving strategy is to use the quantitative relationships from your physics description to generate a set of equations that can be used to solve for the target quantity. The result of this plan is the logical development of a solution which is easy to follow and check for mistakes. The plan requires decisions about which relationship to use when faced with an unknown quantity. As long as you always use new equations for each unknown, you will reach the solution. Unlike the world of consumption, in the world of physics, recycling a used equation is a bad idea. Reusing an equation adds no new information so it cannot give you the additional help you need to find a new unknown.

Checking units is important since it will save you from the algebraic mistakes that everyone makes. If your solution plan is neat and logical, you will usually find your mistake.

So far you have determined what is needed to solve the problem quantitatively and how to perform that solution, but no numerical values have been used. In the next step you will execute the plan in order to find a numerical answer to the problem.

Summary of the Physics Problem Solving Strategy

1. Focus the Problem
   - Picture & Given Information
   - Question(s)
   - Approach

2. Describe the Physics
   - Diagrams & Define Physics Quantities
   - Target Quantity(ies)
   - Quantitative Relationships

3. **Plan the Solution**
   - **Start with equation which has target quantity(ies)**
   - **Identify other unknowns in equation**
   - **Solve a sub-problem for each unknown**
   - **Check Units**

4. Execute the Plan
   - Calculate Target Quantity(ies)

5. Evaluate the Answer
   - Is Answer Properly Stated?
   - Is Answer Unreasonable?
   - Is Answer Complete?

4. Execute the Plan

By the time you reach this step, most of the work has been accomplished. You have gone from a qualitative understanding of the problem to an equation which represents the mathematical solution to the problem. The next step is to obtain a numerical value for your target quantity(ies).

Calculate the Target Quantity(ies)

When you finally have reduced the problem to a single equation you are ready to answer the question raised by that problem by calculating a value for the target quantity. Your equation consists of physics quantities which have units. Their value depends on their units. (12 inches is certainly different than 12 feet.) Your final calculation will only be meaningful if all of the physics quantities are expressed in a consistent set of units.

For example, suppose you want to calculate the value of the speed of a child as she runs a race with her older brother. The child is given a head start of 20 ft. Her older brother begins at the starting line of a 50 m track. She crosses the finish line in 15 seconds. How fast is she running on average? Using the definition of average velocity, $v_{ave} = (x_f - x_0)/\Delta t$. Δt is 15 sec, but what values do you put in for x_f and x_0? You would not subtract 20 from 50 because the "20" is a number of feet, while the "50" is a number of meters. You must first convert the number of feet into its equivalent number of meters (or vice versa).

Treat the units as if they were algebraic quantities. The same units in the numerator and the denominator give you 1 (i.e. the units cancel).

$$\frac{\text{feet}}{\text{feet}} = 1$$

But be careful, if you have

$$\frac{\text{feet} + \text{meters}}{\text{feet}}$$

You cannot "cancel" the feet. For the above example, your final equation is:

$$v_{ave} = \frac{(x_f - x_0)}{(t_f - t_0)}$$

$$v_{ave} = \frac{(50 \text{ meters} - 20 \text{ feet})}{15 \text{ s}}$$

This equation is completely correct, but you cannot extract a useful answer as it stands. If you want an answer in m/s, you must convert 20 feet to meters. On the other hand, if you want an answer in ft/s, convert 50 m to feet. Which expression you use depends on which set of units you want for your answer.

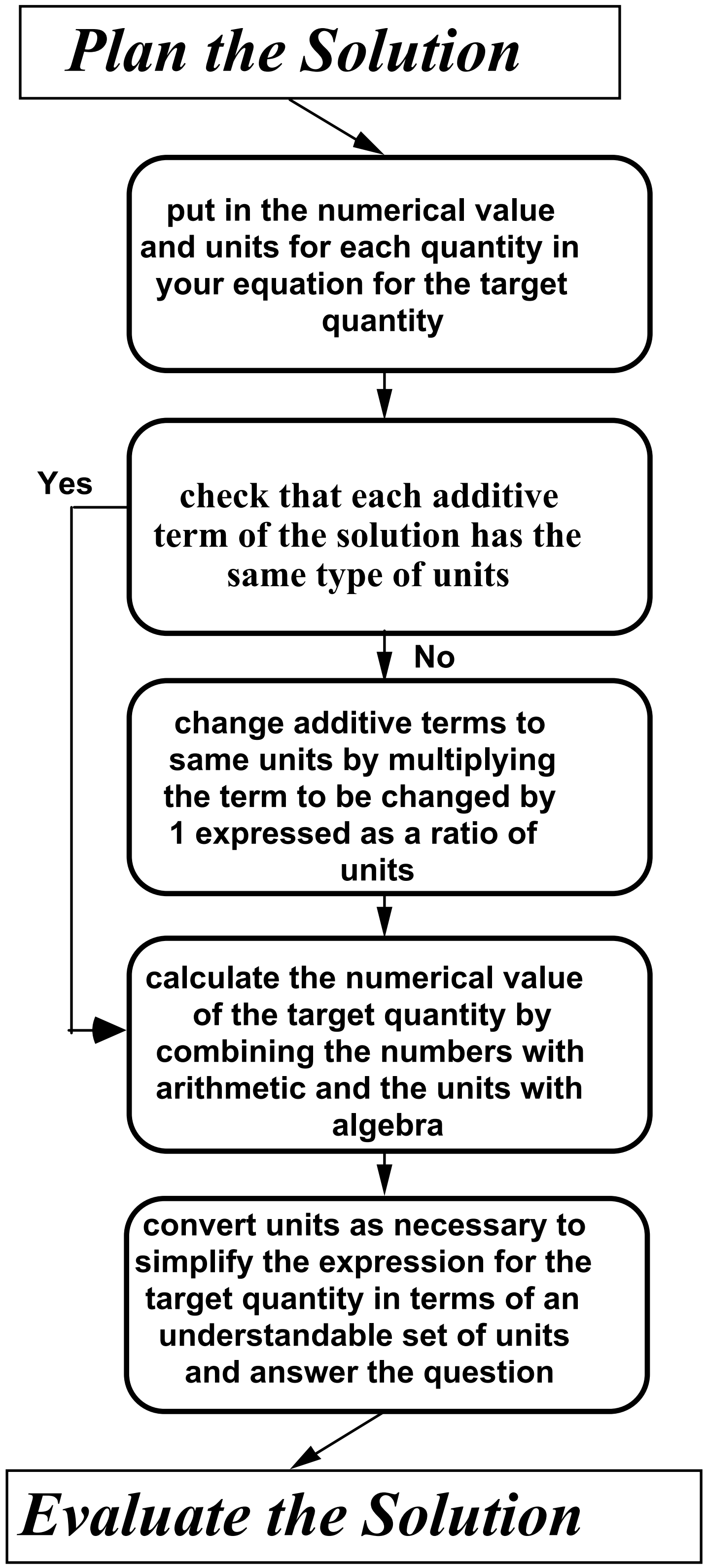
To change the units of a quantity without changing it value, you can only multiply that quantity by 1. You can express 1 as the ratio of two types of units whose denominator is the unit you wish to remove and whose numerator is the unit you wish to have. For example, to change a time in minutes to one in seconds multiply by 1 = (60 s/1 min). In the runner example, change ft to m by multiplying the quantity in ft by 1 = (1 m/3.3 ft):

$$v_{ave} = \frac{(50\text{m} - (20\text{ft})(\frac{1\text{m}}{3.3\text{ft}}))}{(15\text{s})}$$

$$v_{ave} = \frac{(50 \text{ m} - 6 \text{ m})}{15 \text{ s}}$$

$$v_{ave} = \frac{44\text{m}}{15\text{s}} = 2.9\frac{\text{m}}{\text{s}}$$

Execute the Plan



- Which values (numbers with units) from the physics description should be put into the equation for the target quantity?

- Do you need to convert units?

- What ratio of units equals 1?

- Use a calculator for the numbers and algebra for the units.
- Do any units cancel?

- Do we need to convert any units?
- What is the most reasonable set of consistent units for this problem?

Example 3: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55-mph. Ahead you see a sign that says, "Curve Ahead 200 ft, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your brakes at the instant you pass the sign. You slow your car down at a rate of 7-mph each second. As you reach the curve, are you traveling within the posted speed limit?

Execute the Plan

Calculate Target Quantity(ies):

$$\sqrt{v_o^2 + 2ax_f} = v_f$$

$$\sqrt{\left(55\frac{mi}{hr}\right)^2 + 2\left(-7\frac{\frac{mi}{hr}}{\sec}\right)(200ft)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 2800\left(\frac{mi}{hr\sec}\right)(ft)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 2800\left(\frac{mi}{hr\sec}\right)(ft)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 2800\left(\frac{mi}{hr\sec\frac{\min}{60\sec}\frac{hr}{60\min}}\right)(ft)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 2800(3600)\left(\frac{mi}{hr^2}\right)(ft)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 2800(3600)\left(\frac{mi}{hr^2}\right)(ft)\left(\frac{mi}{5280ft}\right)} = v_f$$

$$\sqrt{3025\left(\frac{mi}{hr}\right)^2 - 1909\left(\frac{mi^2}{hr^2}\right)} = v_f$$

$$\sqrt{3025 - 1909}\left(\frac{mi}{hr}\right) = v_f = 33\frac{mi}{hr}$$

You are under the speed limit of 35 mph.

COMMENTARY

Calculate Target Quantity(ies):

- **Are the additive quantities in the same units?**
  No. Need to change seconds to hours and feet to miles.
  $1 = (1 \text{ min})/(60 \text{ s})$
  $1 = (1 \text{ hr})/(60 \text{ min})$
  $1 = (5280 \text{ ft})/(1 \text{ mi})$

  Final units are in mi/hr to compare with problem question.

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4-oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

Execute the Plan

Calculate Target Quantity(ies):

$$y_1 = \frac{-1}{8} g t_2^2$$

$$y_1 = \frac{-1}{8}\left(-32\frac{\text{ft}}{\text{s}^2}\right)(2\,\text{s})^2$$

$$y_1 = 16\ \text{ft}$$

This is less than the 18 ft needed to reach the top of the house.

COMMENTARY

Calculate Target Quantity(ies):

- ***Are the known values for different quantities in the final equation expressed in units, consistent with the units you want for the target quantity?***
  Yes. You want value of the target quantity to have units of feet. The acceleration is given in ft/s^2 and the time is given in seconds.

Summary of Execute the Plan

The result of this step is the determination of those unknown quantities that you set out to find in the Describe the Physics. Now you have a solution to the physics problem. But is it a good solution? Addressing that issue is the aim of the last step of the problem solving strategy.

Summary of the Physics Problem Solving Strategy

1. Focus the Problem
   - Picture & Given Information
   - Question(s)
   - Approach

2. Describe the Physics
   - Diagrams & Define Physics Quantities
   - Target Quantity(ies)
   - Quantitative Relationships

3. Plan the Solution
   - Start with equation which has target quantity(ies)
   - Identify other unknowns in equation
   - Solve a sub-problem for each unknown
   - Check Units

4. **Execute the Plan**
   - **Calculate Target Quantity(ies)**

5. Evaluate the Answer
   - Is Answer Properly Stated?
   - Is Answer Unreasonable?
   - Is Answer Complete?

5. Evaluate the Answer

When you reach this step, you have calculated a quantitative solution and answered the question posed by the problem. You are not quite done since the **goal of problem solving is to get a correct solution**. In this final step, you evaluate your work by checking your answer and determining if that answer actually resolves the original problem. The important features of evaluation can be summarized in the following three questions:

Is Answer Properly Stated?

First, you must ***make sure that the value you write down is clearly and properly expressed.*** Remember that **most physics quantities have units.** Check that your answer has appropriate units. For example, a quantity which represents a distance or position should have units of distance (e.g. meters, feet, miles). Some quantities can be either positive or negative. Make sure that you have the proper sign for the value you obtained. **Vector quantities are described by both a magnitude and a direction.** If your answer is a vector, be sure you give both its magnitude and direction. Also, make sure that the direction is **defined with respect to the coordinate system used in your physics description**.

Is Answer Unreasonable?

Second, ***check if the answer you determined is unreasonable in either magnitude or direction***. If the numerical answer is very much larger or much smaller than the value you would have expected from how you know things work, then you have probably made an error in your solution. For example, cars do not travel at 1000 mph and atoms are much smaller than 1mm. If your plan is logical and clearly written, you can backtrack and fix the mistake. A common mistake would be algebraic or the use of inconsistent units. It is also possible that the mathematics given in your plan is correct and your execution of that plan is perfect but your application of physics is incorrect. This is actually the most common difficulty. An erroneous physics description usually leads to weird answers. If your answer is unreasonable, check your physics description. Are all your quantities given unique names? Do the signs of vector quantities agree with your coordinate system? Are all relevant interactions represented on your diagrams?

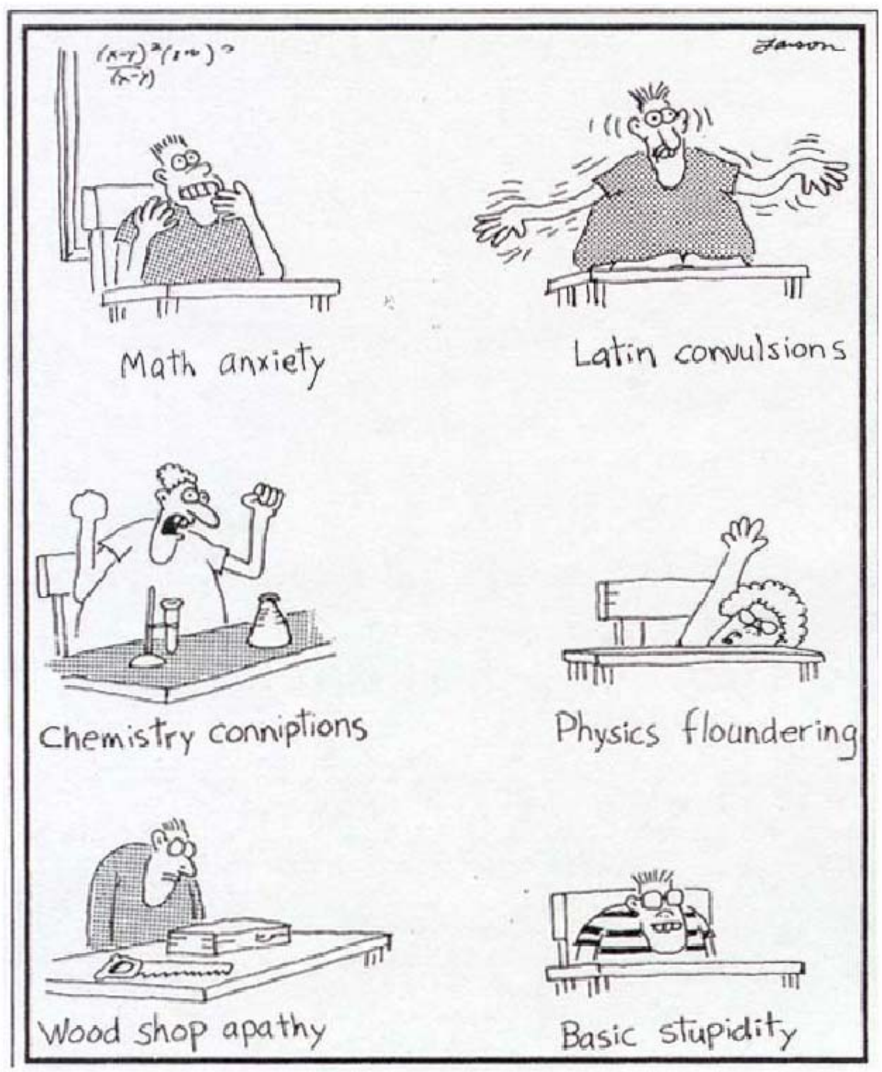
Another way to evaluate your answer is to estimate the value it should have using a simpler version of the problem. For example, if the question is to determine how far an object, accelerating in the same direction as the velocity, travels over a certain interval of time, you can ask the simpler question, "How far would the object travel during that time if it were moving at its original speed?" Because the object is accelerating, the answer to the original problem should be greater than the answer to the simpler problem. Sometimes a simpler problem results if you consider extreme values of some of the quantities in your problem. For example for an object moving down an ramp, it should not accelerate if the ramp were horizontal (ramp angle equals zero). On the other hand, the motion should be free fall if the ramp were vertical (ramp angle equals 90 degrees).

Is Answer Complete?

Third, having determined a numerical answer which is not unreasonable and is properly expressed, you need to ask the ultimate question, "Is your answer a solution to the problem?" Sometimes the answer to this question is trivial. For example, a problem might ask, "What is the frictional force on the car as it begins to move?" If the quantity you have solved for represents the frictional force,

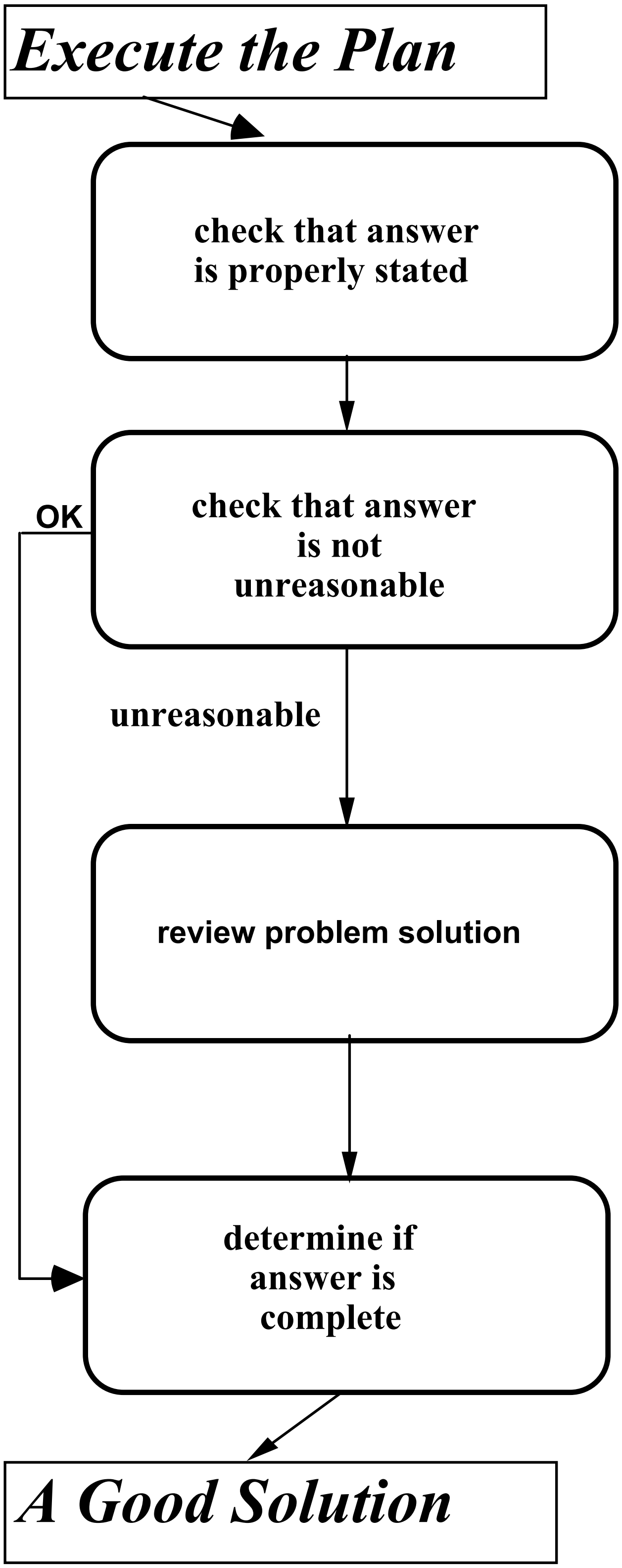
then you are done. However problems often require comparisons or judgments. In these cases the numerical answer simply provides the information on which you base your judgment.

Determine whether your numerical value resolves the original problem, by quickly read what you have written in that Focus the Problem step.



Classroom afflictions

Evaluate the Answer



- Do the units make sense?
- Do vector quantities have both magnitude and direction?
- If someone else read just your answer, would they know what it meant?

- Does the answer fit with your mental picture of the situation?
- Is the answer the magnitude you would expect in this situation?
- Do you have any knowledge of a similar situation that you can compare with to see if the answer is reasonable?
- Can you change the situation (and thus your equation for the target quantity) to describe a simpler problem to which you know the answer?

- Is your physics description complete?
- Are the definitions of your physics quantities unique?
- Do the signs of your physics quantities agree with your coordinate system?
- Can you justify all of the mathematical steps in your solution plan?
- Did you use units in a consistent manner in your execution?
- Is there a calculation mistake in the execution?

- Have you answered the question from the Focus the Problem step?
- Could someone else read and follow the solution plan?
- Are you sure you can justify each mathematical step in the plan?

Example 3: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55 mph. Ahead you see a sign that says, "Curve Ahead 200 ft, Slow to 35 mph." You are 30 ft from the sign when you first see it. You begin to apply your brakes at the instant you pass the sign. You slow your car down at a rate of 7 mph each second. As you reach the curve, are you traveling within the posted speed limit?

Evaluate the Answer:

Is Answer Properly Stated?

Yes. A speed has been calculated and the appropriate units are miles/hour. The positive sign has been chosen to correspond to the situation.

Is Answer Unreasonable?

No. 33 miles/hour is a typical speed for a car on a road. It is less than your initial speed which is reasonable because you are braking.

Is Answer Complete?

Yes. The car's speed was compared to the speed limit to answer the question. All steps in the plan are justified.

COMMENTARY

Is Answer Properly Stated?

- ***Does the value of the car's velocity include proper units?***

  Yes, miles/hour corresponds to the units of velocity.

- ***Does the sign indicate the direction for the car's velocity?***

  It is clear from the diagram that the velocity should be positive.

Is Answer Unreasonable?

- ***Is the car's velocity unreasonable in magnitude?***
  No. The car's final velocity is less than its initial velocity, and is close to 35 mph. You have direct experience that cars travel at that speed.

Is Answer Complete?

- ***Does this value of the car's velocity answer the original question?***

  No, but the question was answered by comparing that value to the speed limit.

- ***Could someone else read and follow the solution plan? Can you justify each mathematical step in the plan?***

  Yes, all steps are written down clearly and in a logical progression. The goal of each new equation introduced is given.

Example 4: Your younger brother is waiting outside for his friends to come over to play baseball. While he waits, he becomes restless and begins to play catch with himself with the 4 oz. baseball. He makes a vertical toss every 3 seconds. The ball returns to his hand two seconds after he releases it. Does the ball get as high as the top of your two story house?

<u>Evaluate the Answer</u>:

Is Answer Properly Stated?

Yes. The position of the ball is calculated in appropriate units, feet. The position is positive .

Is Answer Unreasonable?

No. A boy could throw a ball 16 feet straight up.

Is Answer Complete?

Yes. The position is compared to the height of the house.
All steps of the plan are justified.

COMMENTARY

Is Answer Properly Stated?

- ***Does the value of the ball's maximum height include proper units?***

  Yes. Feet correspond to units of distance.

- ***Does the sign indicate the position of the ball's maximum height?***

  It is clear from the diagram that the sign of y_1 should be positive.

Is Answer Unreasonable?

- ***Is the ball's maximum height unreasonable?***

  No. You have direct experience with people throwing balls.

Is Answer Complete?

- ***Does this value of the ball's height answer the original question?***

  No, but the comparison with the estimate of the height of a two story house is made.

- ***Could someone else read and follow the solution plan? Can you justify each mathematical step in the plan?***

  Yes, all steps are written down clearly and in a logical progression. The goal of each new equation introduced is given.

Summary of <u>Evaluate the Answer</u>

The evaluation performed in this step is a good way of preventing many mistakes by just taking a little bit of time to reflect on your answer. Passing these three checks doesn't guarantee that your answer is correct, but this step is a very efficient way of detecting difficulties.

Summary of the Physics Problem Solving Strategy

1. Focus the Problem
   - Picture & Given Information
   - Question(s)
   - Approach

2. Describe the Physics
   - Diagrams & Define Physics Quantities
   - Target Quantity(ies)
   - Quantitative Relationships

3. Plan the Solution
   - Start with equation which has target quantity(ies)
   - Identify other unknowns in equation
   - Solve a sub-problem for each unknown
   - Check Units

4. Execute the Plan
   - Calculate Target Quantity(ies)

5. **Evaluate the Answer**
   - **Is Answer Properly Stated?**
   - **Is Answer Unreasonable?**
   - **Is Answer Complete?**

FOCUS the PROBLEMPicture and Given Information

Question(s)

Approach

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

Target Quantity(ies)

Quantitative Relationships

PLAN the SOLUTION
Construct Specific Equations

Check Units

EXECUTE the PLAN
Calculate Target Quantity(ies)

EVALUATE the ANSWER
Is Answer Properly Stated?

Is Answer Unreasonable?

Is Answer Complete?

(extra space if needed)

Chapter 3

The Kinematics Approach

Introduction

Problem solving is a complex, cognitive skill. Learning to become a better problem solver is similar to learning to become a better musician, skier, or chess player. The most important factor in your improvement is practicing the right technique. It is also true that your progress is hindered the most by practicing using techniques which are not the best.

In any endeavor there are certain basic strategies or combination of actions that form a foundation for success. Mastering these basics is the prerequisite for developing a personal and creative style. It is the same for solving physics problems. When you "focus the problem," you decide which approach or basic technique you will use to solve the problem. As you gain more expertise you will come to use these techniques in combinations which fit your own personality and conceptual strengths. Luckily, there are only a few very powerful approaches to solving physics problems. Each centers around a basic principle of physics such as kinematics, force, or conservation principles. Your physics course is designed to introduce you to these few principles and show you some of their applications. That instruction will not be repeated here. Problem solving is the mechanism by which you practice applying those principles to the real world. Using these principles as approaches within the framework of the five-step problem solving strategy will make you a better problem solver and help solidify your understanding of the principles.

In order to use an approach effectively, you must be able to recognize it is useful to solve a particular problem. Much of this knowledge can only be gained through experience. Nevertheless there are some general guidelines which are helpful. This booklet briefly describes three approaches which you will encounter again and again.

This chapter discusses one of the three approaches to physics problems, kinematics. An outline of the approach is given in the first section of the chapter. The second section briefly describes how to construct a diagram which is particularly helpful in constructing a solution of problems using a kinematics approach. The third section includes practice exercises with sample solutions. The last section presents realistic practice problems taken from past exams.

As with any skill, it is important to begin by practicing using the full power of the five step strategy on simple exercises where its power is not really necessary. This will build up your ability to apply it to more complex problems and finally to real world problems.

1. The Kinematics Approach

The kinematics approach uses the concepts of position, time, velocity, and acceleration to determine the motion of objects. The relationship of velocity and acceleration to position and time is at the heart of this approach. In this approach, you need only determine how an object's position varies with time. The cause of that variation, due to the object's interaction with its environment, is not relevant. There are three fundamental kinematics equations that allow you to determine the object's position as a function of time:

I. $v_{ave} = \frac{x_f - x_i}{t_f - t_i}$ (definition of average velocity)

II. $a_{ave} = \frac{v_f - v_i}{t_f - t_i}$ (definition of average acceleration)

III. $v_{ave} = \frac{v_i + v_f}{2}$ (relationship between average velocity and instantaneous velocity when the acceleration is constant)

Although these three general relations are all you ever need to solve a kinematics problem, there is another general relationship which is made by combining them,

IV. $x_f = \frac{1}{2}a(t_f - t_i)^2 + v_i(t_f - t_i) + x_i$

(*only* when the acceleration is constant)

Relationships I, II, and III are fundamental. Relationship IV is not. This relationship is true for the motion of an object between points ONLY when the acceleration of the object is CONSTANT between the times you have chosen to be the initial and final times.

All problems that can be solved using the kinematics approach can always be solved using only equations I, II, and III. Equation IV can be used when it is applicable to shorten the amount of mathematics in a problem

2. The Motion Diagram

If you decide to use a kinematics approach to solve a problem, the motion diagram becomes your main tool for describing the physics. In this diagram you reduce every object to a point at a specified position and time. That point has a unique velocity and acceleration, which characterize the object, defined at that position and time. The position is specified with respect to a coordinate system which you choose as convenient for the problem. The object's position, time, velocity, and acceleration is drawn on the diagram at every instant of time that might be of interest in the context of the problem. Simple examples of motion diagrams are given in the four examples of the previous chapter.

For example, if you are given conditions at the beginning of an object's motion and want to find out something about its motion at some later time, the initial and final times are clearly of interest and should be drawn on the motion diagram. If, in addition, the object's acceleration changes between the initial and final time, the time when that change occurs is also of interest. The object's position, time, velocity, and acceleration should be drawn on your motion diagram at the instant of time of the change in acceleration. Your choice of a coordinate system can determine how difficult a problem is to solve mathematically. For example, if you have a single object in motion, it usually simplifies your mathematics to choose a coordinate axis in the direction of the object's velocity with the positive direction in the direction of that velocity.

3. Practice Textbook Exercises

The exercises listed below are taken from various textbooks. Practice applying the five-step problem solving strategy to them. To help using the strategy, we include solution format sheets at the end of chapter 2. These sheets mark off sections for each of the five problem-solving steps. Each section also includes brief prompts for the type of information to include in the space provided. Make copies of these sheets or sketch your own and use them to practice solving both simple exercises from your textbook and the realistic problems given at the end of this chapter. This will help the strategy become second nature to you.

Sample solutions to the exercises are worked out on the solution sheets in the next section of this chapter. Do not read a solution before you have tried to solve the exercises yourself. Your goals should be to understand (a) what kind of information belongs in each step, and (b) how one step follows naturally from the proceeding step and leads logically into the next step. *After* you have tried to solve an exercise, you can check your understanding by comparing your solution to the sample solution. When you have resolved any differences between the two solutions, go on and try to solve the next exercise.

When you are confident of your technique, apply that technique to the exercises (usually called problems) in your textbook. Repetition will help you be comfortable with basic problem solving skills so that you will not need to think about them. You will then be able to concentrate all of your thought on the decisions and physics necessary to solve a problem. Be careful that you do not practice weak problem solving skills. Practicing a bad technique which only works for exercises, will not help you solve real problems. In fact, such practice will only make bad habits harder to break and is worse than not practicing at all.

Problem #1: A burglar drops a bag of loot from a window in a hotel. The bag takes 0.15 seconds to pass the 1.6-m tall window of your room as it falls toward the ground. How far above the top of your window is the burglar who dropped the bag? (The bag's initial speed is zero.) (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 2.42)

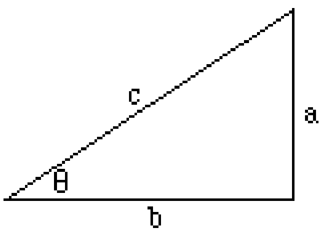
Exercise #2: A football player kicks off from the 40-yd line. How far will the ball travel before hitting the ground if its initial speed is 80-ft/s and the ball leaves the ground at an angle of 30˚? (Assume that air resistance can be ignored.) (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 3-7)

Exercise #3: A baseball leaves the bat of Henry Aaron with a speed of 34-m/s at an angle of 37˚ above the horizontal. The ball is 1.2-m off the ground when it leaves the bat. To be a home run, the ball must clear a fence that is 3.0-m high and 106-m from home plate. (a) At what times after being hit will it reach the height of the fence? (b) How far from home plate will the ball be at these times? (c) Will Henry have a home run? Explain. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 3.31)

Exercise #4: A car drives off a horizontal embankment and lands 11 m from the edge of the embankment in a field that is 3 m lower than the embankment. With what speed was the car traveling when it left the embankment? (Similar to Jones and Childers 1992, problem 3.34)

Below is information that may be helpful in solving these problems:

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi R^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Fundamental Concepts and Principles:

$$v_{ave} = \frac{\Delta x}{\Delta t} \qquad a_{average} = \frac{\Delta v}{\Delta t}$$

$$v_{instantaneous} = \lim(\Delta t \to 0)\frac{\Delta x}{\Delta t} \qquad a_{instantaneous} = \lim(\Delta t \to 0)\frac{\Delta v}{\Delta t}$$

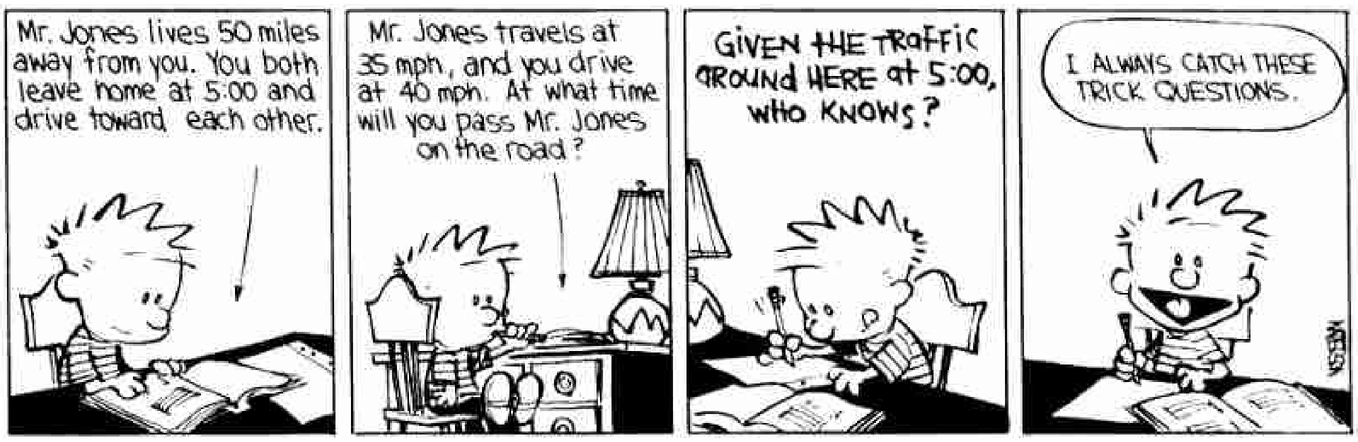
Under Certain Conditions:

$$x_f = \frac{1}{2}a(t_f - t_i)^2 + v_i(t_f - t_i) + x_i \qquad a = \frac{v^2}{r}$$

$$v_{ave} = \frac{v_i + v_f}{2}$$

Useful constants: 1 mile = 5280 ft, $g = 9.8\ m/s^2 = 32\ ft/s^2$

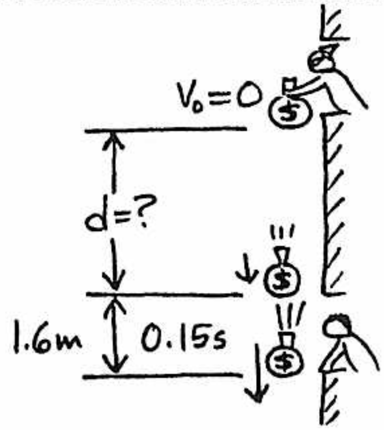
Calvin and Hobbes / By Bill Watterson



Problem #1: A burglar drops a bag of loot from a window in a hotel. The bag takes 0.15 seconds to pass the 1.6-m tall window of your room as it falls toward the ground. How far above the top of your window is the burglar who dropped the bag? (The bag's initial speed is zero.) (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 2.42)

FOCUS the PROBLEM

Picture and Given Information



Question(s) How far above the top of your window is the bag dropped?

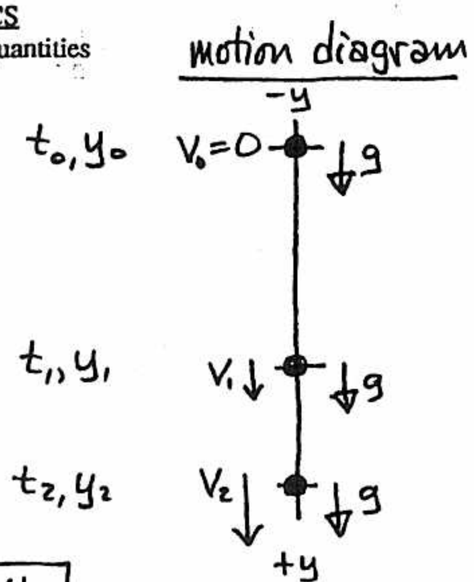
Approach Use kinematics -- this is constant acceleration motion.

Time: Initial time is the instant after bag is released.
Middle time is the instant bag reaches top of window.
Final time is the instant bag reaches bottom of window.

Assume air resistance can be neglected.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities



$y_0=0$ $V_0=0$ $t_0=0$

$y_1=?$ $V_1=?$ $t_1=?$

$y_2=?$ $V_2=?$ $t_2=?$

Target Quantity(ies) $\boxed{y_1}$

$a=g=9.8\,m/s^2$

$\Delta y = y_2 - y_1 = 1.6\,m$

$\Delta t = t_2 - t_1 = 0.15\,s$

Quantitative Relationships

Constant acceleration (g) in y-direction so use

$$y_f = \frac{1}{2} g (t_f - \cancelto{0}{t_0})^2 + \cancelto{0}{V_0} (t_f - \cancelto{0}{t_0}) + \cancelto{0}{y_0}$$

$$\Rightarrow y_f = \frac{1}{2} g t_f^2 \quad \therefore y_1 = \frac{1}{2} g t_1^2, \quad y_2 = \frac{1}{2} g t_2^2$$

PLAN the SOLUTION

Construct Specific Equations

Unknowns: $\boxed{y_1}$ $\boxed{t_1}$ $\boxed{t_2}$ $\boxed{y_2}$

<u>Find y_1</u>

$$y_1 = \frac{1}{2} g t_1^2 \quad (1)$$

<u>Find t_1</u>

$$\Delta t = t_2 - t_1 \quad (2)$$

<u>Find t_2</u>

$$y_2 = \frac{1}{2} g t_2^2 \quad (3)$$

<u>Find y_2</u>

$$\Delta y = y_2 - y_1 \quad (4)$$

$$y_2 = \Delta y + y_1$$

$$\Delta y + y_1 = \frac{1}{2} g t_2^2$$

$$t_2^2 = \frac{2}{g}(\Delta y + y_1)$$

$$t_2 = \sqrt{\frac{2}{g}(\Delta y + y_1)}$$

$$\Delta t = \sqrt{\frac{2}{g}(\Delta y + y_1)} - t_1$$

$$t_1 = \sqrt{\frac{2}{g}(\Delta y + y_1)} - \Delta t$$

$$y_1 = \frac{1}{2} g \left(\sqrt{\frac{2}{g}(\Delta y + y_1)} - \Delta t\right)^2$$

$$y_1 = \frac{1}{2} g \left(\frac{2}{g}(\Delta y + y_1) + \Delta t^2 - 2\Delta t \sqrt{\frac{2}{g}(\Delta y + y_1)}\right)$$

$$\cancel{y_1} = \Delta y + \cancel{y_1} + \frac{1}{2} g \Delta t^2 - g \Delta t \sqrt{\frac{2}{g}(\Delta y + y_1)}$$

$$g \Delta t \sqrt{\frac{2}{g}(\Delta y + y_1)} = \Delta y + \frac{1}{2} g \Delta t^2$$

$$g^{\cancel{2}} \Delta t^2 \frac{2}{\cancel{g}}(\Delta y + y_1) = \Delta y^2 + \frac{1}{4} g^2 \Delta t^4 + g \Delta t^2 \Delta y$$

$$2g\Delta t^2 \Delta y + 2g \Delta t^2 y_1 = \Delta y^2 + \frac{g^2 \Delta t^4}{4} + g \Delta t^2 \Delta y$$

continued

Check Units

$$\frac{[m]^2 + \left[\frac{m}{s^2}\right]^2 [s]^4 - \left[\frac{m}{s^2}\right][s]^2[m]}{\left[\frac{m}{s^2}\right][s]^2}$$

$$= \frac{[m]^2 + [m]^2 - [m]^2}{[m]} = [m] \quad \text{OK}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$y_1 = \frac{(1.6\,m)^2 + \frac{(9.8\,m/s^2)^2 (0.15\,s)^4}{4} - (9.8\,m/s^2)(0.15\,s)(1.6}{2(9.8\,m/s^2)(0.15\,s)^2}$$

$$y_1 = 5.0\,m$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected y_1 has units of length.

Is Answer Unreasonable?

No. 5.0m (just over 15 ft) sounds about right for spacing between windows.

Is Answer Complete?

Yes. 5.0m is the distance above the top of your window from which the bag was dropped which answers the question.

(extra space if needed)

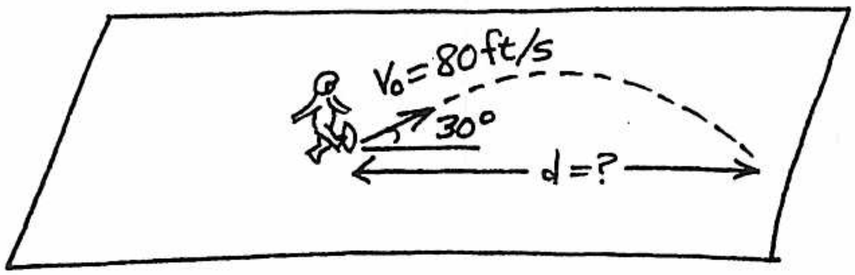
$$2g\Delta t^2 y_1 = \Delta y^2 + \frac{g^2 \Delta t^4}{4} - g \Delta t^2 \Delta y$$

$$\boxed{y_1 = \frac{\Delta y^2 + \frac{1}{4} g^2 \Delta t^4 - g \Delta t^2 \Delta y}{2g\Delta t^2}}$$

Problem #2: A football player kicks off from the 40-yd line. How far will the ball travel before hitting the ground if its initial speed is 80-ft/s and the ball leaves the ground at an angle of 30°? (Assume that air resistance can be ignored.) (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 3-7)

FOCUS the PROBLEM

Picture and Given Information



Question(s) How far will the ball travel before hitting the ground?

Approach Use kinematics; handle vertical and horizontal motion separately.

-- horizontal motion at constant velocity

-- vertical motion at constant acceleration

Time: Initial time is the instant after ball is kicked.

Final time is the instant ball lands.

Assume air resistance can be neglected.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

motion diagram

$x_o = 0$

$y_o = 0$

$V_o = 80\ ft/s$

$t_o = 0$

$\theta = 30°$

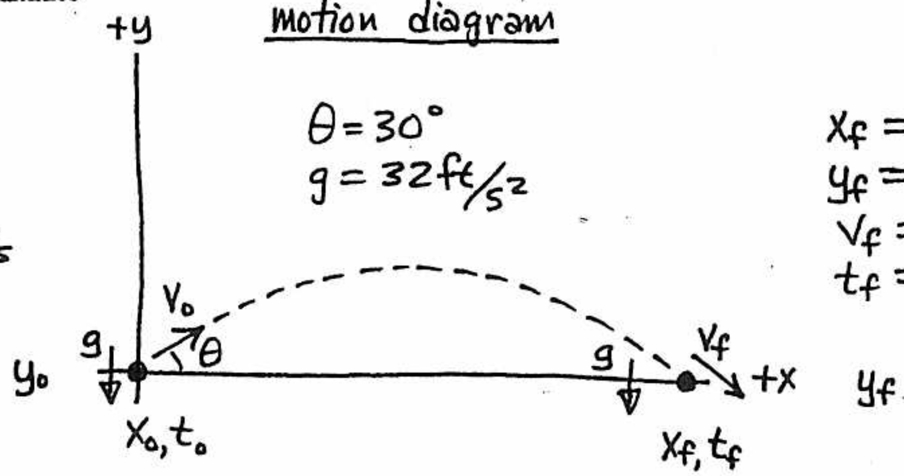
$g = 32\,ft/s^2$

$x_f = ?$

$y_f = 0$

$V_f = ?$

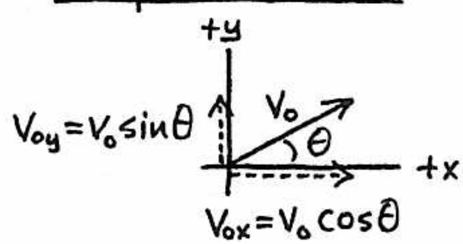
$t_f = ?$



Target Quantity(ies) $\boxed{x_f}$

Quantitative Relationships

components of V_o



Constant velocity in x-direction so use

$$V_{ox} = \overline{V}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - \cancelto{0}{x_o}}{t_f - \cancelto{0}{t_o}} \Rightarrow V_{ox} = x_f/t_f$$

Constant acceleration $(-g)$ in y-direction so use

$$\cancelto{0}{y_f} = \tfrac{1}{2}(-g)(t_f - \cancelto{0}{t_o})^2 + V_{oy}(t_f - \cancelto{0}{t_o}) + \cancelto{0}{y_o} \Rightarrow 0 = -\tfrac{1}{2} g t_f^2 + V_{oy} t_f$$

PLAN the SOLUTION

Construct Specific Equations

Unknowns

$\boxed{x_f}$

Find x_f

$V_{ox} = x_f / t_f$ ①

$\boxed{t_f, V_{ox}}$

Find V_{ox}

$V_{ox} = V_o \cos\theta$ ②

$V_o \cos\theta = x_f / t_f$

Find t_f

$0 = -\frac{1}{2} g t_f^2 + V_{oy} t_f$ ③

$\boxed{V_{oy}}$

Find V_{oy}

$V_{oy} = V_o \sin\theta$ ④

$0 = -\frac{1}{2} g t_f^2 + V_o \sin\theta\, t_f$

$\frac{1}{2} g t_f^{\cancel{2}} = V_o \sin\theta\, \cancel{t_f}$

$t_f = \dfrac{2 V_o \sin\theta}{g}$

$V_o \cos\theta = x_f \dfrac{g}{2 V_o \sin\theta}$

$$\boxed{x_f = \frac{2 V_o^2 \cos\theta \sin\theta}{g}}$$

Check Units

$$\frac{[ft/s]^2}{[ft/s^2]} = [ft] \quad \text{OK}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$x_f = \frac{2(80 ft/s)^2 \cos(30°) \sin(30°)}{32 ft/s^2}$$

$x_f = 173$ ft

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected x_f has units of length.

Is Answer Unreasonable?

No. 173 ft is nearly 58 yards -- a good kick off.

Is Answer Complete?

Yes. 173 ft is the distance down field the ball travels which answers the question.

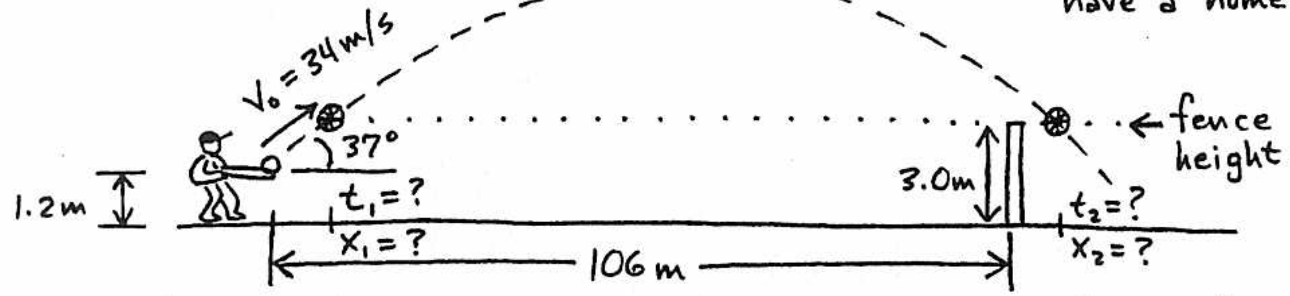
(extra space if needed)

Problem #3: A baseball leaves the bat of Henry Aaron with a speed of 34-m/s at an angle of 37° above the horizontal. The ball is 1.2-m off the ground when it leaves the bat. To be a home run, the ball must clear a fence that is 3.0-m high and 106-m from the home plate. (a) At what times after being hit will it reach the height of the fence? (b) How far from home plate will the ball be at these times? (c) Will Henry have a home run? Explain. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 3.31)

FOCUS the PROBLEM

Picture and Given Information

Picture drawn guessing he ~~does~~ have a home run.



Question(s)

a) At what times is the ball at the height of the fence?

b) How far from home plate is the ball at those times?

c) Will the ball make it over the fence for a home run?

Approach

Use kinematics; handle vertical and horizontal motion separately.

-- horizontal motion at constant (zero!) acceleration ⇒ const. velocity

-- vertical motion at constant (non-zero) acceleration

Time: Initial time is the instant ball leaves the bat.

Middle time is the instant ball reaches 3.0m (going up).

Final time is the instant ball reaches 3.0m (coming down).

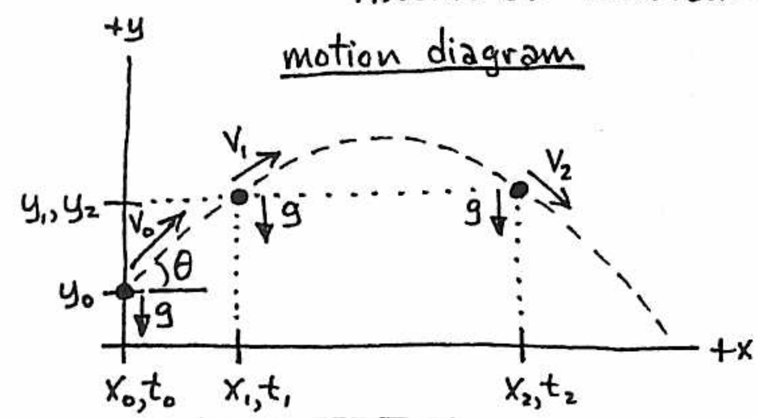
Assume "how far" means measured along the ground.

Assume air resistance can be neglected.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

motion diagram



$t_0 = 0$ $\quad x_0 = 0$ $\quad y_0 = 1.2m$

$t_1 = ?$ $\quad x_1 = ?$ $\quad y_1 = 3.0m$

$t_2 = ?$ $\quad x_2 = ?$ $\quad y_2 = 3.0m$

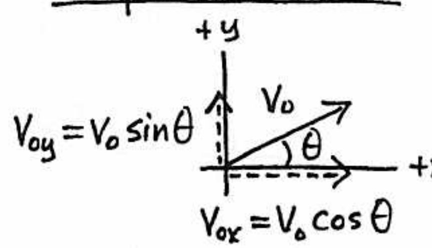
$V_0 = 34 m/s$ $\quad a_x = 0$

$\theta = 37°$ $\quad a_y = -g$

$g = 9.8 m/s^2$

Target Quantity(ies) $\boxed{t_1, t_2, x_1, x_2}$

components of V_0



Quantitative Relationships

Constant acceleration in both x-direction and y-direction so use

$$x_f = \tfrac{1}{2}(\cancelto{0}{a_x})(t_f - \cancelto{0}{t_0})^2 + V_{0x}(t_f - \cancelto{0}{t_0}) + \cancelto{0}{x_0}$$

$$\Rightarrow x_f = V_{0x} t_f \qquad \therefore x_1 = V_{0x} t_1, \quad x_2 = V_{0x} t_2$$

$$y_f = \tfrac{1}{2}(a_y)(t_f - \cancelto{0}{t_0})^2 + V_{0y}(t_f - \cancelto{0}{t_0}) + y_0$$

$$\Rightarrow y_f = -\tfrac{1}{2} g t_f^2 + V_{0y} t_f + y_0 \qquad \therefore y_1 = -\tfrac{1}{2} g t_1^2 + V_{0y} t_1 + y_0, \quad y_2 = -\tfrac{1}{2} g t_2^2 + V_{0y} t_2 + y_0$$

PLAN the SOLUTION
Construct Specific Equations

unknowns

$\boxed{t_1, t_2, x_1, x_2}$

Find t_1

$$y_1 = -\tfrac{1}{2} g t_1^2 + v_{oy} t_1 + y_o \quad (1)$$

$\boxed{v_{oy}}$

Find v_{oy}

$$v_{oy} = v_o \sin\theta \quad (2)$$

$$y_1 = -\tfrac{1}{2} g t_1^2 + v_o \sin\theta\, t_1 + y_o$$

$$-\tfrac{1}{2} g t_1^2 + v_o \sin\theta\, t_1 + y_o - y_1 = 0$$

$$t_1 = \frac{-v_o \sin\theta \pm \sqrt{v_o^2 \sin^2\theta - 4(-g/2)(y_o - y_1)}}{2(-g/2)}$$

$$\boxed{t_1 = \frac{-v_o \sin\theta \pm \sqrt{v_o^2 \sin^2\theta + 2g(y_o - y_1)}}{-g}}$$

Find t_2

$$y_2 = -\tfrac{1}{2} g t_2^2 + v_{oy} t_2 + y_o \quad (3)$$

$$-\tfrac{1}{2} g t_2^2 + v_o \sin\theta\, t_2 + y_o - y_2 = 0$$

$$\boxed{t_2 = \frac{-v_o \sin\theta \pm \sqrt{v_o^2 \sin^2\theta + 2g(y_o - y_2)}}{-g}}$$

Find x_1

$$x_1 = v_{ox} t_1 \quad (4)$$

$\boxed{v_{ox}}$

Find v_{ox}

$$v_{ox} = v_o \cos\theta \quad (5)$$

$$\boxed{x_1 = v_o \cos\theta\ t_1}$$

use result for t_1 from above

continued

Check Units

$$t_1, t_2 : \frac{[m/s] \pm \sqrt{[m/s]^2 + [m/s^2][m]}}{[m/s^2]}$$

$$= \frac{[m/s] \pm [m/s]}{[m/s^2]} = [s] \qquad \text{OK}$$

$$x_1, x_2 : [m/s][\ ][s] = [m] \qquad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$t_{1,2} = \frac{(-34\tfrac{m}{s})\sin(37^\circ) \pm \sqrt{(34\tfrac{m}{s})^2 \sin^2(37^\circ) + 2(9.8\tfrac{m}{s^2})(1.2m-3.(}}{-9.8\,m/s^2}$$

$$t_{1,2} = 2.09s \pm (-2.00s) \Rightarrow t_1 = 0.09s \quad t_2 = 4.09s$$

$$x_1 = (34\tfrac{m}{s})\cos(37^\circ)(0.09s) \qquad x_2 = (34\tfrac{m}{s})\cos(37^\circ)(4.09$$

$$x_1 = 2.4m \qquad x_2 = 111m$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected the t's came out in units of time and the x's in units of distance.

Is Answer Unreasonable?

No. t_1 and x_1 are both quite small. The values found for t_2 and x_2 seem plausible for a baseball.

Is Answer Complete?

Not quite. Since $x_2 = 111m > 106$ m, the ball should clear the fence for a home run. This answers the final question.

(extra space if needed)

Find x_2

$$x_2 = v_{ox} t_2 \quad (6)$$

$$\boxed{x_2 = v_o \cos\theta\ t_2}$$

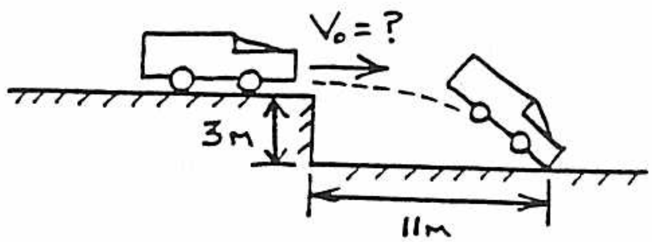
use result for t_2 from before

Notice that because $y_1 = y_2$ the expressions for t_1 and t_2 are the same! Thus use the plus sign for one time and the minus sign for the other. Choose so that $t_1 < t_2$.

Problem #4: A car drives off a horizontal embankment and lands 11-m from the edge of the embankment in a field that is 3-m lower than the embankment. With what speed was the car traveling when it left the embankment?
Based on Jones & Childers 1992, problem 3.34

FOCUS the PROBLEM

Picture and Given Information



Question(s) WHAT WAS THE INITIAL HORIZONTAL SPEED OF THE CAR?

Approach

USE DEFINITIONS OF VELOCITY AND ACCELERATION.

TREAT VERTICAL AND HORIZONTAL MOTIONS SEPARATELY.
ACCELERATION IS ONLY VERTICAL.
VELOCITY HAS BOTH HORIZONTAL AND VERTICAL COMPONENTS.

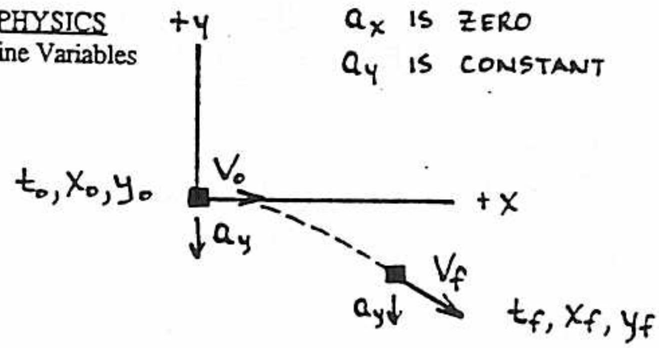
IGNORE AIR RESISTANCE.

DESCRIBE the PHYSICS

Diagram and Define Variables

a_x IS ZERO

a_y IS CONSTANT



$t_o = 0$	$t_f = ?$
$x_o = 0$	$x_f = 11\,m$
$y_o = 0$	$y_f = -3\,m$

$a_x = 0$

$a_y = -g$

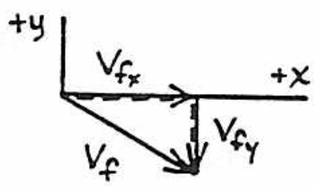
$= -9.8\ m/s^2$

$V_{ox} = V_o$

$V_{oy} = 0$

(BECAUSE INITIAL VELOCITY IS HORIZONTAL)

$V_{fy} = ?$



Target Variable(s) $\boxed{\text{FIND } V_o = ?}$

Quantitative Relationships

$\bar{V}_x = \dfrac{\Delta x}{\Delta t}$; DEFINITION

$\bar{V}_y = \dfrac{\Delta y}{\Delta t}$; DEFINITION

$\bar{a}_y = \dfrac{\Delta V_y}{\Delta t}$; DEFINITION

$\bar{V}_x = V_{ox}$; SINCE $a_x = 0$

$\bar{V}_y = \dfrac{V_{oy} + V_{fy}}{2}$; SINCE a_y = CONSTANT

$\bar{a}_y = a_y = -g$ SINCE a_y = CONSTANT

PLAN the SOLUTION

Construct specific equations

UNKNOWNS

FIND V_o:

(I) $V_o = V_{ox}$ — V_o, V_{ox}

FIND V_{ox}:

(II) $V_{ox} = \bar{V}_x$ — $\bar{V}_x$

FIND $\bar{V}_x$: $\bar{V}_x = \dfrac{x_f - x_o}{t_f - t_o}$

(III) $\bar{V}_x = \dfrac{x_f}{t_f}$ — t_f

FIND t_f: $\bar{a}_y = \dfrac{V_{fy} - V_{oy}}{t_f - t_o}$

$\bar{a}_y = \dfrac{V_{fy}}{t_f}$

(IV) $-g = \dfrac{V_{fy}}{t_f}$ — V_{fy}

FIND V_{fy}: $\bar{V}_y = \dfrac{V_{oy} + V_{fy}}{2}$

(V) $\bar{V}_y = \dfrac{V_{fy}}{2}$ — $\bar{V}_y$

FIND $\bar{V}_y$: $\bar{V}_y = \dfrac{y_f - y_o}{t_f - t_o}$

(VI) $\bar{V}_y = \dfrac{y_f}{t_f}$

Check for sufficiency

6 UNKNOWNS ($V_o, V_{ox}, \bar{V}_x, t_f, V_{fy}, \bar{V}_y$)

6 EQUATIONS (I, II, III, IV, V, VI)

Outline the Math Solution

SOLVE (VI) FOR $\bar{V}_y$ AND PUT INTO (V)

SOLVE (V) FOR V_{fy} AND PUT INTO (IV)

SOLVE (IV) FOR t_f AND PUT INTO (III)

SOLVE (III) FOR $\bar{V}_x$ AND PUT INTO (II)

SOLVE (II) FOR V_{ox} AND PUT INTO (I)

SOLVE (I) FOR V_o

EXECUTE the PLAN

Follow the Plan

SOLVE (VI) $\bar{V}_y = \dfrac{y_f}{t_f}$

PUT INTO (V) $\dfrac{y_f}{t_f} = \dfrac{V_{fy}}{2}$

SOLVE (V) $\dfrac{2y_f}{t_f} = V_{fy}$

PUT INTO (IV) $-g = \dfrac{\left(\frac{2y_f}{t_f}\right)}{t_f}$

SOLVE (IV) $-g t_f = \dfrac{2y_f}{t_f}$

$t_f^2 = \dfrac{2y_f}{-g}$

$t_f = \sqrt{\dfrac{2y_f}{-g}}$

PUT INTO (III) $\bar{V}_x = \dfrac{x_f}{\sqrt{\frac{2y_f}{-g}}}$

PUT INTO (II) $V_{ox} = \dfrac{x_f}{\sqrt{\frac{2y_f}{-g}}}$

PUT INTO (I) $V_o = \dfrac{x_f}{\sqrt{\frac{2y_f}{-g}}}$

SOLVE (I) $\boxed{V_o = x_f\sqrt{\dfrac{-g}{2y_f}}}$

O.K.

CHECK UNITS: $[M]\sqrt{\dfrac{[M]/[S]^2}{[M]}} = [M]\sqrt{\dfrac{1}{[S]^2}} = \dfrac{[M]}{[S]}$

Calculate Target Variable(s)

$$V_o = x_f\sqrt{\frac{-g}{2y_f}} = (11\,M)\sqrt{\frac{-9.8\,M/S^2}{2(-3\,M)}} = \boxed{14\,M/S}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. ALL STEPS FOLLOW RULES OF ALGEBRA. V_o IS IN THE UNITS OF SPEED (M/S).

Is Answer Reasonable?

CONVERT SPEED TO MI/HR FOR CHECK.

$$V_o = 14\,\frac{M}{S} = 14\,\frac{M}{S} * \frac{60\,S}{1\,MIN} * \frac{60\,MIN}{1\,HR} * \frac{100\,CM}{1\,M} * \frac{1\,IN}{2.54\,CM} * \frac{1\,FT}{12\,IN} * \frac{1\,MI}{5280\,FT} = 31\,\frac{MI}{HR}$$

SEEMS REASONABLE!

Is Answer Complete?

YES. THE INITIAL HORIZONTAL SPEED IS 14 M/S. THIS ANSWERS THE QUESTION.

4. Practice Exam Problems

The following realistic problems require you to sort through information and make judgments about what you need to find. The five-step problem solving strategy is the most effective way to work through such problems.

Problem #1: You are writing a short adventure story for your English class. In your story, two submarines on a secret mission need to arrive at a place in the middle of the Atlantic ocean at the same time. They start out at the same time from positions equally distant from the rendezvous point. They travel at different velocities but both go in a straight line. The first submarine travels at an average velocity of 20-km/hr for the first 500-km, 40-km/hr for the second 500-km, 30-km/hr for the next 500-km and 50-km/hr for the final 500-km. In the plot, the second submarine is required to travel at a constant velocity, so the captain needs to determine the magnitude of that velocity.

Problem #2: It's a sunny Sunday afternoon, about 65˚F, and you are walking around Lake Calhoun enjoying the last of the autumn color. The sidewalk is crowded with runners and walkers. You notice a runner approaching you wearing a tee-shirt with writing on it. You read the first two lines, but are unable to read the third and final line before he passes. You wonder, "Hmm, if he continues around the lake, I bet I'll see him again, but I anticipate the time when we'll pass again." You look at your watch and it is 3:07 p.m. You recall the lake is 3.4 miles in circumference. You estimate your walking speed at 3 miles per hour and the runner's speed to be twice your walking speed.

Problem #3 You are part of a citizen's group evaluating the safety of a high school athletic program. To help judge the diving program you would like to know how fast a diver hits the water in the most complicated dive. The coach has his best diver perform for your group. The diver, after jumping from the high board, moves through the air with a constant acceleration of 9.8-m/s^2. Later in the dive, she passes near a lower diving board which is 3.0-m above the water. With your trusty stop watch, you determine that it took 0.20 seconds to enter the water from the time the diver passed the lower board. How fast was she going when she hit the water?

Problem #4: Just for the fun of it, you and a friend decide to enter the famous Tour de Minnesota bicycle race from Rochester to Duluth and then to St. Paul. You are riding along at a comfortable speed of 20-mph when you see in your mirror that your friend is going to pass you at what you estimate to be a constant 30-mph. You will, of course, take up the challenge and accelerate just as she passes until you pass her. If you accelerate at a constant 0.25 miles per hour each second until you pass her, how long will she be ahead of you?

Problem #5: The University Skydiving Club has asked you to plan a stunt for an air show. In this stunt, two skydivers will step out of opposite sides of a stationary hot air balloon 5,000 feet above the ground. The second skydiver will leave the balloon 20 seconds after the first skydiver but you want them to both land on the ground at the same time. The show is planned for a day with no wind so assume that all motion is vertical. To get a rough idea of the situation, assume that the skydiver will fall with a constant acceleration of 32-ft/s^2 before the parachute opens. As soon as the parachute is opened, the skydiver falls with a constant velocity of 10-ft/s. If the first skydiver waits 3.0 seconds after stepping out of the balloon before opening his parachute, how long must the second skydiver wait after leaving the balloon before opening his parachute?

Problem #6: You are sitting around with a friend working physics problems while watching Wheel of Fortune on TV. You know that your TV set uses a beam of electrons to form the picture on the screen. You hope the presence of this unintentional product of basic physics research will inspire your physics thought (at least that's what you told your roommate). During a lull in the "action," you wonder how long it takes for the electron to get from the far end of the 2 1/2 foot long picture tube to the screen. Each electron starts essentially at rest from a hot filament at the rear of the picture tube and then undergoes a constant acceleration in a high voltage region of the picture tube. You guess that this high voltage region is the narrow straight section of the picture tube, about 2 inches in diameter, containing the filament at one end. This straight section is a 1 foot long cylinder before the picture tube flares out to match the 21 inch screen. When an electron leaves the high voltage region and travels straight to the screen, it no longer accelerates. It makes a flash of light when it finally hits the screen. Your friend remembers reading that the accelerating voltage is 5 kilovolts and that, just before it hits the screen, the electron is traveling at 1/10 the speed of light. You wish that you could remember the speed of light when your roommate comes in and tells you that light travels 1 foot per nanosecond and "everyone" knows that a nanosecond is 10^{-9} seconds. Now, before Vanna flips the next letter, what's the answer?

Problem 7: While on a vacation to Kenya, you visit the port city of Mombassa on the Indian Ocean. On the coast you find an old Portuguese fort probably built in the 16th century. Large stone walls rise vertically from the shore to protect the fort from cannon fire from pirate ships. Walking around on the ramparts, you find the fort's cannons mounted such that they fire horizontally out of holes near the top of the walls facing the ocean. Leaning out of one of these gun holes, you drop a rock which hits the ocean 3.0 seconds later. You wonder how close a pirate ship would have to sail to the fort to be in range of the fort's cannon? Of course you realize that the range depends on the velocity that the cannonball leaves the cannon. That muzzle velocity depends, in turn, on how much gunpowder was loaded into the cannon.

(a) Calculate the muzzle velocity necessary to hit a pirate ship 300 meters from the base of the fort.

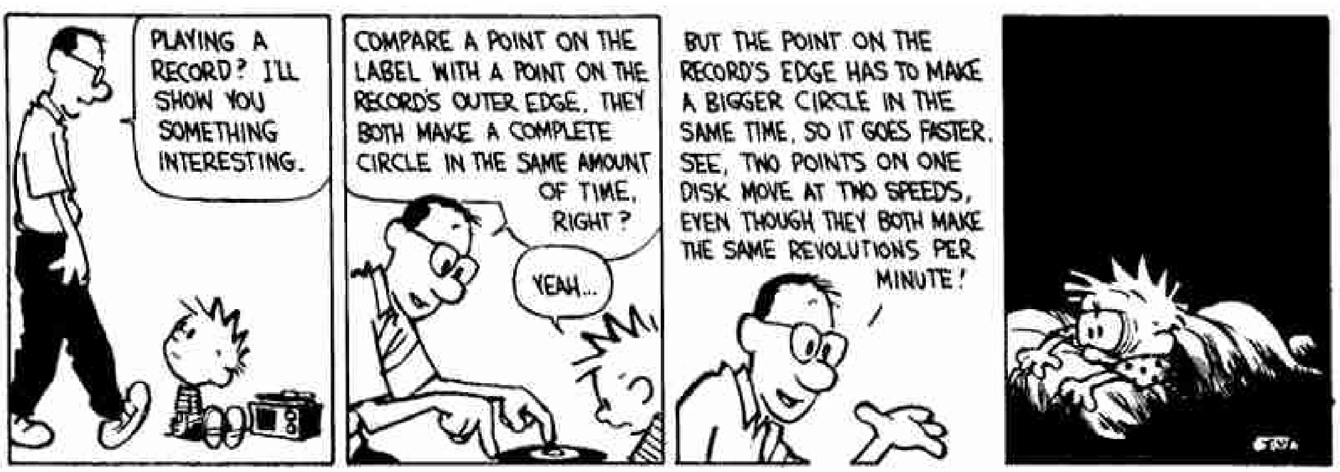
(b) (To determine how the muzzle velocity must change to hit ships at different positions, make a graph of horizontal distance traveled by the cannonball (range) before it hits the ocean as a function of muzzle velocity of the cannonball for this fort.

Problem #8: You are on the target range preparing to shoot a new rifle when it occurs to you that you would like to know how fast the bullet leaves the gun (the muzzle velocity). You bring the rifle up to shoulder level and aim it horizontally at the target center. Carefully you squeeze off the shot at the target which is 300 feet away. When you collect the target you find that your bullet hit 9.0 inches below where you aimed.

Problem #9: Tramping through the snow this morning, you were wishing that you were not here on your way to this test. Instead, you imagined yourself sitting in the Florida sun watching winter league softball. You have had baseball on the brain ever since the Twins actually won the World Series. One of the fielders seems very impressive. As you watch, the batter hits a low outside ball when it is barely off the ground. It looks like a home run over the left center field wall which is 200-ft from home plate. As soon as the left fielder sees the ball being hit, she runs to the wall, leaps high, and catches the ball just as it barely clears the top of the 10-ft high wall. You estimate that the ball left the bat at an angle of 30˚. How much time did the fielder have to react to the hit, run to the fence, and leap up to make the catch?

Problem #10: Your group has been selected to serve on a citizen's panel to evaluate a new proposal to search for life on Mars. On this unmanned mission, the lander will leave orbit around Mars falling through the atmosphere until it reaches 10,000 meters above the surface of the planet. At that time a parachute opens and takes the lander down to 500 meters. Because of the possibility of very strong winds near the surface, the parachute detaches from the lander at 500 meters and the lander falls freely through the thin Martian atmosphere with a constant acceleration of 0.40g for 1.0 second. Retrorockets then fire to bring the lander to a softly to the surface of Mars. A team of biologists has suggested that Martian life might be very fragile and decompose quickly in the heat from the lander. They suggest that any search for life should begin at least 9 meters from the base of the lander. This biology team has designed a probe which is shot from the lander by a spring mechanism in the lander 2.0 meters above the surface of Mars. To return the data, the probe cannot be more than 11 meters from the bottom of the lander. Combining the data acquisition requirements with the biological requirements the team designed the probe to enter the surface of Mars 10 meters from the base of the lander. For the probe to function properly it must impact the surface with a velocity of 8.0-m/s at an angle of 30 degrees from the vertical. Can this probe work as designed?

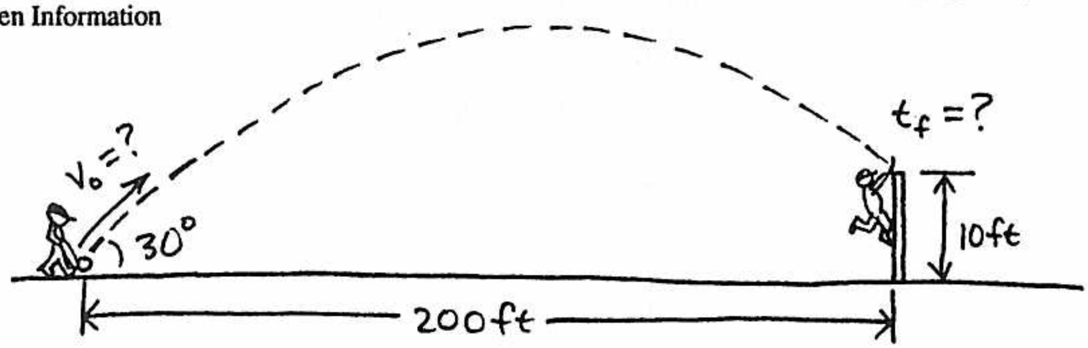
Calvin and Hobbes / By Bill Watterson



PRACTICE EXAM PROBLEM #9

FOCUS the PROBLEM

Picture and Given Information



Question(s) How long does the ball take to get to the fielder?

Approach Use kinematics; handle vertical and horizontal motion separately.

--horizontal motion at constant velocity

--vertical motion at constant acceleration

Time: Initial time is the instant ball leaves the bat.

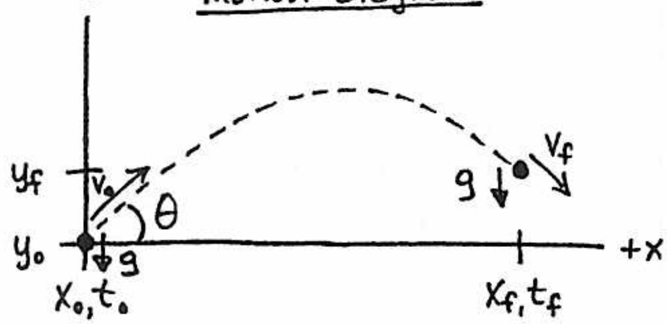
Final time is the instant fielder catches the ball.

Assume air resistance can be neglected.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

motion diagram



$y_f = 10\text{ft}$ $\quad V_f = ?$

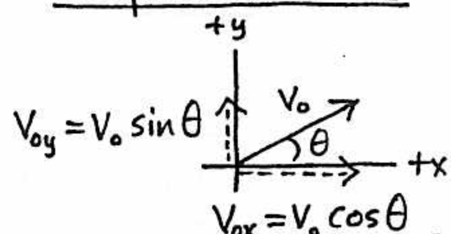
$y_o = 0$ $\quad V_o = ?$

$x_o = 0$ $\quad x_f = 200\text{ft}$ $\quad \theta = 30°$

$t_o = 0$ $\quad t_f = ?$ $\quad g = 32\text{ft}/\text{s}^2$

Target Quantity(ies) $\boxed{t_f}$

components of V_o



Also: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Quantitative Relationships

Constant velocity in x-direction so use

$$V_{ox} = \overline{V}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - \cancel{x_o}^{\,0}}{t_f - \cancel{t_o}^{\,0}} \Rightarrow V_{ox} = x_f/t_f$$

Constant acceleration (-g) in y-direction so use

$$y_f = \tfrac{1}{2}(-g)(t_f - \cancel{t_o}^{\,0})^2 + V_{oy}(t_f - \cancel{t_o}^{\,0}) + \cancel{y_o}^{\,0} \Rightarrow y_f = -\tfrac{1}{2}g t_f^2 + V_{oy} t_f$$

PLAN the SOLUTION
Construct Specific Equations

Unknowns: $\boxed{t_f}$, $\boxed{V_{ox}}$, $\boxed{V_o}$, $\boxed{V_{oy}}$

Find t_f

$$V_{ox} = x_f / t_f \quad (1)$$

Find V_{ox}

$$V_{ox} = V_o \cos\theta \quad (2)$$

Find V_o

$$V_{oy} = V_o \sin\theta \quad (3)$$

Find V_{oy}

$$y_f = -\tfrac{1}{2} g t_f^2 + V_{oy} t_f \quad (4)$$

$$V_{oy} t_f = y_f + \tfrac{1}{2} g t_f^2$$

$$V_{oy} = \frac{y_f + \frac{1}{2} g t_f^2}{t_f}$$

$$\frac{y_f + \frac{1}{2} g t_f^2}{t_f} = V_o \sin\theta$$

$$V_o = \frac{y_f + \frac{1}{2} g t_f^2}{t_f \sin\theta}$$

$$V_{ox} = \frac{(y_f + \frac{1}{2} g t_f^2)}{t_f \sin\theta} \cos\theta$$

$$\frac{(y_f + \frac{1}{2} g t_f^2)}{\cancel{t_f}} \frac{\cos\theta}{\sin\theta} = \frac{x_f}{\cancel{t_f}}$$

$$y_f + \tfrac{1}{2} g t_f^2 = x_f \tan\theta$$

$$\tfrac{1}{2} g t_f^2 + y_f - x_f \tan\theta = 0$$

$$t_f = \frac{0 \pm \sqrt{0 - 4(g/2)(y_f - x_f \tan\theta)}}{2(g/2)}$$

continued

Check Units

$$\frac{\sqrt{[ft/s^2]([ft][\,] - [ft])}}{[ft/s^2]}$$

$$= \frac{[ft/s]}{[ft/s^2]} = [s] \qquad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$t_f = \frac{\sqrt{2(32 ft/s^2)(200 ft \tan(30°) - 10 ft)}}{32 ft/s^2}$$

$$t_f = 2.6\,s$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected t_f is in units of time.

Is Answer Unreasonable?

No. A softball might reach the outfield wall in about $2\frac{1}{2}$ seconds.

Is Answer Complete?

Yes. t_f is the time the fielder has to make the catch. This answers the question.

(extra space if needed)

$$\boxed{t_f = \pm\frac{\sqrt{2g(x_f \tan\theta - y_f)}}{g}}$$

Since $t_f > t_o$ (ie. the ball is caught after it is hit) select the plus sign here.

Chapter 4

The Dynamics Approach

Introduction

Dynamics is an approach which describes how the motion of an object is changed by its interactions with other objects. The first section of this chapter discusses approaches to physics problems using concepts of dynamics. The second section describes how to use free-body and force diagrams to describe the physics of these problems. The third section includes practice exercises from textbooks, with sample solutions. Finally, the last section provides you with practice problems from past exams.

1. The Dynamics Approach

An interaction always involves two objects, an object which exerts the force and the object on which the force acts. In addition, there are only a few types of interactions which occur in nature, and they are usually easily identified. The most common types of interactions are either contact, gravitational, electric, or magnetic. Every interaction results in a force on an object in a specified direction which can be thought of as either a push or a pull. In summary, any force must be expressible in the following statement:

The ________ (*type of interaction*) (*push/pull*) exerted by the ________ (*object*) on the ________ (*different object*) .

For example, your weight is the gravitational pull exerted by the Earth on you. If you make sure that every force can be expressed by this general statement it will help you identify the forces on an object. Even more importantly, it will help you reject forces which do not exist.

The acceleration of an object is related to the sum of all of the forces which act on that object. The mass of the object is the factor which relates them. Mathematically speaking: $\sum \mathbf{F} = m\mathbf{a}$. The equation states that the acceleration ($\mathbf{a}$) of an object is directly proportional to the sum of the forces ($\sum \mathbf{F}$) which act on that object. The mass (m) of that object determines how much the acceleration changes as $\Sigma\mathbf{F}$ changes. Note that **bold** type designates vectors. When you have a problem which involves several objects and several interactions, remember that the motion of a given object is affected only by the forces exerted on that object.

Newton's Third Law gives an important relationship between forces acting on the two objects involved in some interaction. Newton's Third Law states that whenever one object (A) exerts a force on another object (B), the second object (B) exerts a force of equal magnitude on the first object (A), but in the opposite direction. Every force has a third law pair which acts on a different object. As an example, consider again the force which you call your weight. Your weight is the gravitational pull that the Earth exerts on you. The corresponding force, its Third Law pair, is the gravitational pull exerted by you on the Earth. Although they are equal in magnitude and opposite in direction, such Third Law pairs NEVER ADD UP TO ZERO. They cannot be added together because each force acts on a different object. The two gravitational forces in the example act on different objects. One force acts on you, the other force acts on the Earth.

Sometimes to solve a problem it can be useful to combine the approaches of dynamics and kinematics. For example, if you want to determine the subsequent motion of an object due to the forces exerted on it, you can use forces to determine the object's acceleration. Using that acceleration with kinematics, you could determine the object's subsequent position and velocity. Conversely, if you can describe an object's motion, you can use kinematics to determine its acceleration at important times. Using those accelerations and dynamics, you could determine the sum of the forces acting on that object.

2. Free-body and Force Diagrams

Suppose you have made a sketch of the problem situation and decided that the best approach to solve the problem is to apply $\sum \mathbf{F} = m\mathbf{a}$ to a particular system. The system is usually a single object (e.g., a box, a car). However, if two or more objects are attached and/or move together (e.g., a car and its driver), it may be more convenient to define the system as all of the objects that are moving together.

To solve the problem, you must determine what objects are interacting with the system object(s), the type of interaction, and the direction and relative magnitudes of the resulting forces acting on the system. Then you must represent the forces in a convenient way so they can be easily added. There are three drawings that are helpful in this process. (1) *Your sketch of the problem situation* helps you identify the most convenient system of interest and the objects in the environment that interact with that system. (2) The *free-body diagram* isolates the system of interest and helps you determine the qualitative behavior of that system in terms of the direction and relative magnitudes of the forces acting on it. (3) The *force diagram* represents the forces acting on the system of interest as mathematical vectors that can be conveniently added.

Using Your Sketch

To determine the forces acting on a system, you must first distinguish between the object(s) in your system and the objects in the system's environment. Then determine the objects in the environment that are actually interacting with the system object(s), and the type of interaction. Every force on a system requires that an identifiable object in the system's environment has an identifiable type of interaction with the system in an identifiable direction. If a force really exists, it can be described in the following form:

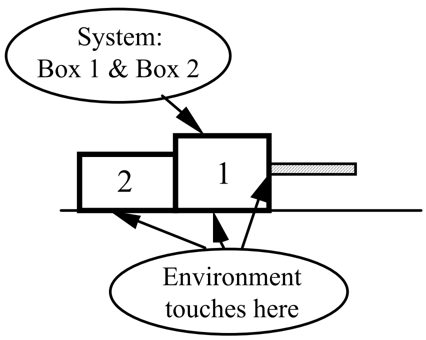
> Force **A** is the <u>(type of interaction)</u> pull or push exerted *by* <u>(an environmental object)</u> *on* the <u>(system object or objects)</u>.

For example, one force acting on a ball after is thrown up is the <u>gravitational</u> pull exerted by the <u>Earth</u> on the <u>ball</u>.

Use your sketch to choose the system you wish to consider to solve your problem and outline it. Look for two general categories of interactions with the object(s) in your system: (1) short-range interactions caused by the physical contact between the objects in the environment and the system object(s), and (2) long-range (action-at-a-distance) interactions between objects in the environment and the system object(s).

Look along the boundary of your system for objects in the environment that touch the system. Only environmental objects that actually touch the system can cause contact forces. For example, one type of contact interaction occurs when an environmental object such as a rope pulls on a system object such as a car. Long-range (action-at-a-distance) forces are caused by an object in the environment that does not have to be touching an object in the system, but nevertheless interacts with the system. A common example of a long-range interaction is the gravitational attraction between a system object (e.g., a ball) and an environmental object (e.g., the Earth). When the environmental object is a planet (such as the Earth), the gravitational force on the system object is called the "weight" of object.

As an example, consider a problem in which two attached boxes are accelerated across the floor by the pull of a rope. Since the boxes move together, we can define the system as the two boxes.



For this system, the floor touches the base of the boxes and exerts two types of contact forces on the boxes:

$\mathbf{F_{N1}}$ is the contact push exerted by the floor on Box 1 (normal force).

$\mathbf{F_{N2}}$ is the contact push exerted by the floor on Box 2 (normal force).

$\mathbf{F_{k1}}$ is the contact push exerted by the floor on the Box 1 (kinetic frictional force).

$\mathbf{F_{k2}}$ is the contact push exerted by the floor on the Box 2 (kinetic frictional force).

The rope also touches the system:

P is the contact pull exerted by the rope on the boxes (tension force).

There are also two long-range forces:

$\mathbf{W_1}$ is the gravitational pull exerted by the Earth on Box 1 (the weight of the Box 1).

$\mathbf{W_2}$ is the gravitational pull exerted by the Earth on Box 2 (the weight of the Box 2).

This description can be simplified by considering the system as a single "object." Then

$\mathbf{F_N}$ is the contact push exerted by the floor on the system (normal force: $\mathbf{F_N} = \mathbf{F_{N1}} + \mathbf{F_{N2}}$).

$\mathbf{F_k}$ is the contact push exerted by the floor on the system (kinetic frictional force: $\mathbf{F_k} = \mathbf{F_{k1}} + \mathbf{F_{k2}}$).

P is the contact pull exerted by the rope on the boxes (tension force).

W is the gravitational pull of the Earth on the system (weight of system: $\mathbf{W} = \mathbf{W_1} + \mathbf{W_2}$).

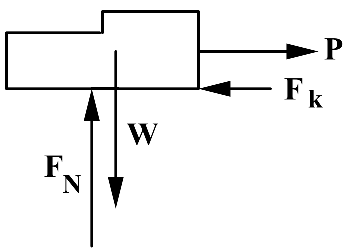
If you cannot identify the environmental object that is exerting the force on the system object(s) and determine the type of interaction, then the force does not exist. For example, a proposed "force of motion" acting on the accelerating boxes is not a real force because it is impossible to identify the specific environmental object exerting this proposed force, and motion is not a *type* of interaction.

Drawing a Free-body Diagram

The next step is to use a diagram to represent the qualitative behavior of the system. For force problems, this is achieved by representing both the directions and the relative magnitudes of the forces acting on a system by arrows. If you drew force vectors on your sketch, it would be easy to confuse the forces exerted by the environmental objects on the system with the forces exerted by the system on the environmental objects. To avoid this confusion, we will draw "free-body" diagrams. First, draw a separate picture of only the object(s) in the system. Now instead of drawing the environmental objects which are interacting with the system object(s), draw arrows representing the forces the environ-mental objects exert on the system. Label the force arrows with the same symbols you used to describe the forces.

An example of the free-body diagram for a system of two boxes pulled by a rope across the floor is shown on the next page. The direction of each force can be determined by considering the type of interaction:

Contact Forces: When a rope pulls on a system, the force is always directed along the rope. When the surface of an environmental object is pressed against the system, the normal force (push) is always perpendicular to the surface of contact. The kinetic frictional force on a system is always parallel to the surface in contact with the surface of the environmental object. Its direction is always opposite to the direction that the system moves relative to the environmental object.



Long-range Forces: A gravitational interaction between a system object and an environmental object always results in a force on the system object directed towards the center of mass of the environmental object.

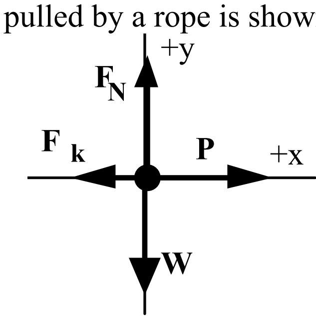
To make sure that all appropriate forces are included on your diagram with the correct qualitative behavior, draw the forces on the system where the interaction occurs. Look at the system boundary on your sketch to determine the contact interaction points or surfaces. For pushing (e.g., $\mathbf{F_N}$ and $\mathbf{F_k}$), place the *head* of an arrow representing the force at the approximate point or surface where the force acts on the system. For pulling forces (e.g., **P** and **W**), place the *tail* of an arrow representing the force at the approximate point where the force is acting. For long-range (action-at-a-distance) forces (e.g., **W**), place the tail of the arrow at the appropriate center of the system (e.g., the center of mass).

Finally, use your knowledge of how forces affect the motion of objects ($\sum \mathbf{F} = m\mathbf{a}$) to determine the relative magnitudes of the forces on your system. Look back at your sketch and given information of the problem situation (Focus the Problem) to check how your system is moving (or how you expect the system to move). You may have to draw a motion diagram to determine the direction of the acceleration. Then try to make the length of the force arrows representative of the relative magnitudes of the forces that would be necessary for the system to move as expected. For example, since the system of two boxes shown above is not accelerating in the vertical direction, the forces in the vertical direction ($\mathbf{F_N}$ and **W**) should be balanced -- equal in magnitude but opposite in direction (but remember these two forces are not a third law pair). The boxes are, however, accelerating to the right, so the forces in the horizontal direction are not balanced. The arrow representing the pull of the rope (**P**) is longer than the arrow representing the frictional push of the floor ($\mathbf{F_k}$) on the system.

Drawing a Force Diagram

The next step is to apply vector mathematics using a force diagram. Now only the forces are drawn as vectors originating at the origin of a coordinate system. First draw a set of coordinate axes. It is usually convenient to orient one axis in the direction of motion and the other axis perpendicular to the direction of motion. Draw the force vectors with the tails at the origin of the axes and the heads pointing in the appropriate directions. If more than one force acts in the same direction, draw the force vectors slightly offset from the origin so you can see them.

The force diagram for the system of two boxes pulled by a rope is shown below.



You can now apply Newton's Second Law in each coordinate direction:

$$\sum F_x = ma_x \text{ and } \sum F_y = ma_y$$

If one or more forces are at an angle to the coordinate axes, use trigonometry to determine the components of the forces in the x and y directions.

EXAMPLE 1: An explorer in Greenland wants to pull her sled up a hill. She estimates that the hill makes an angle of 15 degrees with the horizontal. Her loaded sled, which is at rest halfway up the hill, weighs 200-lbs. Before she left on the expedition, she determined that the coefficient of static friction between snow and her loaded sled is 0.30, and the coefficient of kinetic friction is 0.20. Her rope is rated for a maximum tension of 100-lbs. Will she be able to pull her sled up the hill?

Draw a sketch of the problem situation and a free-body and force diagram of the loaded sled. Cover the right side of the page and try each step before looking at the answer.

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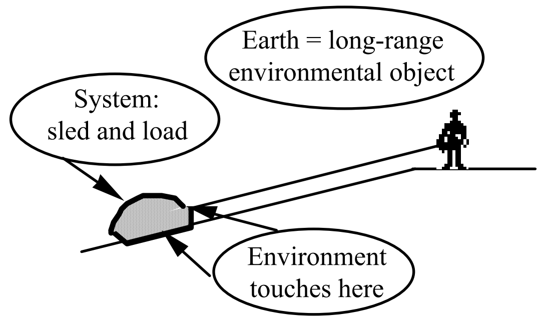
1. Make and Use a Sketch: Draw a sketch of the problem situation and then outline with a heavy line the system of interest. Examine the system boundary and identify the environmental objects that have contact interactions with the system. Identify the environmental objects that have long-range interactions with the system object(s).



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2. Identify and Describe the Forces: Choose a symbol for each of the forces acting on the system caused by either the contact or long-range interactions. Describe in words each force, using the following form:

> **X** is the <u>(type of interaction)</u> pull or push exerted *by* <u>(environmental object)</u> *on* the <u>(system object(s))</u>.

Contact Forces:

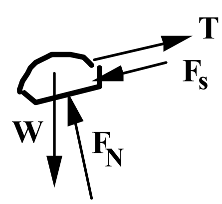
- $\mathbf{F_N}$ is the contact push exerted by the snow on the sled (normal force).
- **Fs** is the contact push exerted by the snow on the sled (static frictional force).
- **T** is the contact pull exerted by the rope on the sled.

Long-Range Forces:

- **W** is the gravitational pull exerted by the Earth on the sled (weight of sled and load).

---

3. Draw a Free-body Diagram: Make a separate picture of the system. Draw an arrow for each force acting on the system and label each arrow with the symbol you used in Step 2. Make the length of the force arrows representative of the relative magnitudes of the forces that would be necessary for the system to move as expected.



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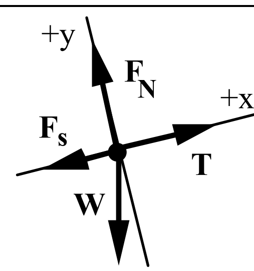
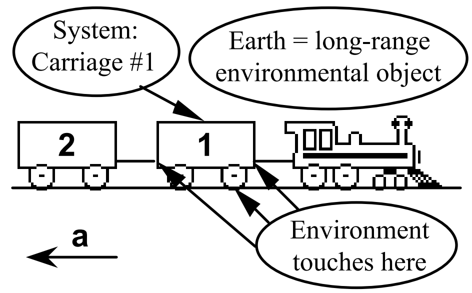
4. Draw a Force Diagram: Draw a coordinate system. It is usually convenient to orient one axis in the direction of motion and the other axis perpendicular to the direction of motion. Draw each force vector originating from the origin of the coordinate system.



EXAMPLE 2: You are investigating a train accident that occurred when two empty train carriages were being moved to another track. The engineer stated that the train was traveling at 45 miles per hour when he made an emergency stop in 30 seconds. Your boss suspects a faulty coupling between the engine and the first carriage. She knows you are taking a physics course, so she asks you to calculate the force exerted by the engine on the first carriage during the emergency stop. The train manufacturer claims that each carriage weighs 20,000-lbs, and the maximum coefficient of kinetic friction between the wheels and the track is 0.55.

Draw a sketch of the problem situation and a free-body and force diagram of the **first carriage**. Cover the right side of the page below and try each step before looking at the answer.

1. Use Your Sketch: Draw a sketch of the problem situation and then outline with a heavy line the system of interest. Examine the system boundary and identify the environmental objects that have contact interactions with the system. Identify the environmental objects that have long-range interactions with the system object(s).



2. Identify and Describe the Forces: Choose a symbol for each of the forces acting on the system caused by either the contact or long-range interactions. Describe in words each force, using the following form:

> **X** is the (type of interaction) pull or push exerted *by* (environmental object) *on* the (system object(s)).

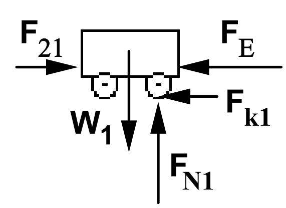
Contact Forces:

- $\mathbf{F_{N1}}$ is the contact push exerted by the track on carriage #1 (normal force).
- $\mathbf{F_{k2}}$ is the contact push exerted by the track on carriage #1 (kinetic frictional force).
- $\mathbf{F_E}$ is the contact push exerted by the engine on carriage #1.
- $\mathbf{F_{21}}$ is the contact push exerted by carriage #2 on carriage #1.

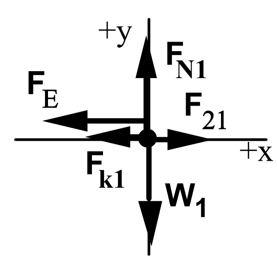
Long-Range Forces:

- $\mathbf{W_1}$ is the gravitational pull exerted by the Earth on Carriage #1 (weight of the carriage).

3. Draw a Free-body Diagram: Make a separate picture of the system. Draw an arrow for each force acting on the system and label each arrow with the symbol you used above. Make the length of the arrows representative of the relative magnitudes of the forces that would cause the system to move as expected.



4. Draw a Force Diagram: Draw a coordinate system. It is usually convenient to orient one axis in the direction of motion and the other axis perpendicular to the direction of motion. Draw each force vector originating from the origin of the coordinate system.



EXAMPLE 3: You are investigating a train accident that occurred when two empty train carriages were being moved to another track. The engineer stated that the train was traveling at 45 miles per hour when he made an emergency stop in 30 seconds. Your boss suspects a faulty coupling between the engine and the first carriage. She knows you are taking a physics course, so she asks you to calculate the force exerted by the engine on the first carriage during the emergency stop. The train manufacturer claims that each carriage weighs 20,000-lbs, and the maximum coefficient of kinetic friction between the wheels and the track is 0.55 when the brakes are applied.

Draw a sketch of the problem situation and a free-body and force diagram of the **second carriage**. Cover the right side of the page below and try each step before looking at the answer.

<table>
<tr>
<td>

1. Make and Use a Sketch: Draw a sketch of the problem situation and then outline with a heavy line the system of interest. Examine the system boundary and identify the environmental objects that have contact interactions with the system. Identify the environmental objects that have long-range interactions with the system object(s).

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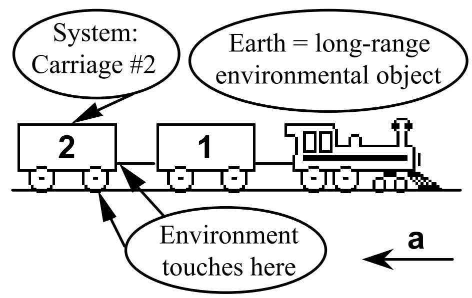

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2. Identify and Describe the Forces: Choose a symbol for each of the forces acting on the system caused by either the contact or long-range interactions. Describe in words each force, using the following form:

X is the <u>(type of interaction)</u> pull or push exerted *by* <u>(environmental object)</u> *on* the <u>(system object(s))</u>.

</td>
<td>

Contact Forces:

$\mathbf{F_{N2}}$ is the contact push exerted by the track on carriage #2 (normal force).

$\mathbf{F_{k2}}$ is the contact push exerted by the road on t carriage #2 (kinetic frictional force).

$\mathbf{F_{12}}$ is the contact push exerted by carriage #1 on carriage #2.

Long-Range Forces:

$\mathbf{W_2}$ is the gravitational pull exerted by the Earth on Carriage #2 (weight of the carriage).

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3. Draw a Free-body Diagram: Make a separate picture of the system. Next draw an arrow for each force acting on the system and label each arrow with the symbol used above. Make the length of the arrows representative of the relative magnitudes of the forces that would be necessary for the system to move as expected from the given problem situation.

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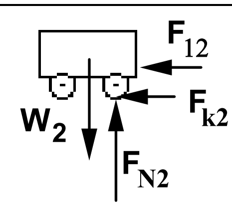

Note: $\mathbf{F_{12}}$ is the Third Law pair of $\mathbf{F_{21}}$ in Example 2, so its arrow is drawn the same length, but opposite direction

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4. Draw a Force Diagram: Draw a coordinate system. It is usually convenient to orient one axis in the direction of motion and the other axis perpendicular to the direction of motion. Draw each force vector originating from the origin of the coordinate system.

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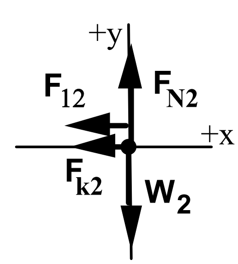

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</table>

EXAMPLE 4: (try this one on your own!) You are on your way to the University when your car breaks down. A tow truck weighing 4000-lbs comes along and agrees to tow your car, which weighs 2000-lbs, to the nearest service station. The driver of the truck attaches his cable to your car at an angle of 20 degrees to the horizontal. He tells you that his cable has a strength of 500-lbs and that he plans to take 15 seconds to tow your car at a constant acceleration from rest in a straight line along the flat stretch of freeway until he reaches the speed limit of 55 miles per hour. If rolling friction behaves like kinetic friction, and the coefficient of rolling friction between your tires and the road is 0.10, determine if the driver can carry out his plan.

3. Practice Textbook Exercises

The exercises listed below are taken from textbooks. Use the five-step problem solving strategy to solve them. It is the most effective way to work through new problems, and it will be a useful tool on exam days. To make it easier to practice using the strategy, we have included solution format sheets at the end of Chapter 2. These sheets mark off sections for each of the five problem-solving steps. Each section also includes brief prompts for the type of information to include in the space provided. Make copies of these sheets or sketch your own and use them to practice solving problems. This will help the strategy become second nature to you.

Example solutions to the problems are worked out on the solution sheets, using the problem solving strategy. Do not read the solution before you have tried to solve the exercise yourself. Your goals should be to understand (a) what kind of information belongs in each step, and (b) how one step leads logically into the next. *After* you have tried to solve an exercise, you can check your understanding by comparing your solution to the sample solution. When you have resolved any differences between the two solutions, go on and try to solve the next exercise.

Exercise #1: A car exerts a forward force on a trailer, and the trailer exerts an equal magnitude backward force on the car. Construct a force diagram for each vehicle and explain what force causes the car to accelerate when pulling the trailer. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.29)

Exercise #2: Consider two blocks connected by a horizontal rope. The pair of blocks are being accelerated across a horizontal frictionless surface by another rope which is attached to one of the blocks and slopes upward at some angle θ with respect to the horizontal surface. The blocks have equal mass. Construct force diagrams for each block. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.14)

Exercise #3: A rope connected to a 50-kg sled pulls it along a frictionless sheet of ice. The tension in the rope is 100-N and the rope is oriented at an angle of 30° above a line drawn parallel to the ice. Calculate the horizontal acceleration of the sled. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.14)

Exercise #4: Two blocks are connected by a cable strung over a pulley, which is mounted at the top of an incline. The 10-kg block hangs over the edge of the incline. The 20-kg block rests on the frictionless incline. The "toe" of the incline is at an angle of 37° with respect to the horizontal. Find the magnitude of the acceleration and the tension in the cable. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.14)

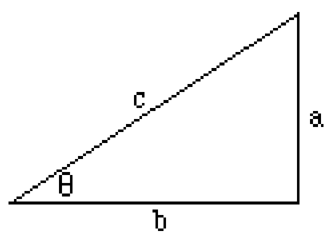
Exercise #5: A 3.0-kg box sits on a horizontal surface of your car seat as you drive at a speed of 20-m/s. The coefficient of friction between the box and the seat is 0.50. You apply the brakes to stop the car. Calculate the shortest possible stopping distance so that the box does not start to slide off the seat. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.30)

Exercise #6: A 20-kg crate sitting on a horizontal floor is attached to a rope that pulls 37˚ above the horizontal. The coefficient of static friction between the crate and the floor is 0.50. (a) Construct a force diagram for the crate. (b) Determine the least rope tension that will cause the crate to start sliding. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.29.)

Exercise #7: Four ions (Na^+, Cl^-, Na^+ and Cl^-) each separated from its neighbors by 3.0×10^{-10}-m are in a row. The charge of a sodium ion is +e and that of a chlorine ion is -e. Calculate the force on the chlorine ion at the end of the row due to the other three ions. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 22.29)

Below is information that may be useful in solving these problems

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi R^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Fundamental Concepts and Principles:

$v_{ave} = \dfrac{\Delta x}{\Delta t}$ $\qquad a_{average} = \dfrac{\Delta v}{\Delta t}$ $\qquad \Sigma F_r = ma_r$

$v_{instantaneous} = \lim(\Delta t \to 0)\dfrac{\Delta x}{\Delta t}$ $\qquad a_{instantaneous} = \lim(\Delta t \to 0)\dfrac{\Delta v}{\Delta t}$

Under Certain Conditions:

$x_f = \frac{1}{2}a(t_f - t_i)^2 + v_i(t_f - t_i) + x_i$ $\qquad F = \mu_k F_N$ $\qquad F = G\dfrac{m_1 m_2}{r^2}$

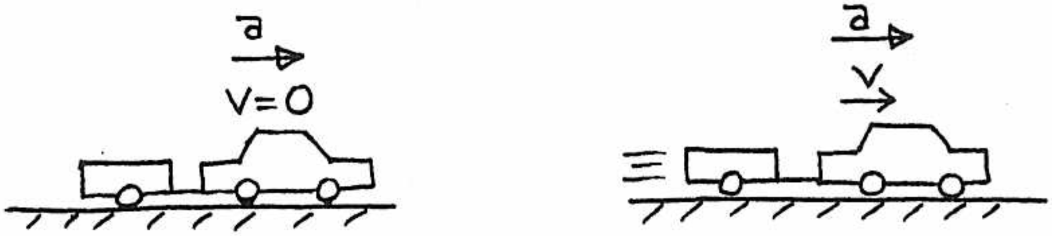
$a = \dfrac{v^2}{r}$ $\qquad v_{ave} = \dfrac{v_i + v_f}{2}$ $\qquad F \le \mu_s F_N$ $\qquad F = k_e\dfrac{q_1 q_2}{r^2}$

Useful constants: 1 mile = 5280 ft , $g = 9.8\ m/s^2 = 32\ ft/s^2$, $G = 6.7 \times 10^{-11}\ N\ m^2/kg^2$, $k_e = 9.0 \times 10^9\ N\ m^2/C^2$, $e = 1.6 \times 10^{-19}\ C$

Problem #1: A car exerts a forward force on a trailer, and the trailer exerts an equal magnitude backward force on the car. Construct a force diagram for each vehicle and explain what force causes the car to accelerate when pulling the trailer. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.29)

FOCUS the PROBLEM

Picture and Given Information



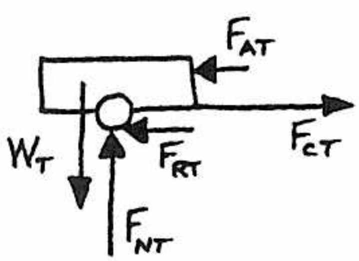
Question(s) Construct force diagrams for car and trailer.
What force causes the car to accelerate?

Approach Use dynamics. (Newton's 2nd and 3rd Laws)
Determine all forces acting on car and trailer separately.
Sum the forces acting on the car to explain its acceleration.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

free-body diagram (trailer)



F_{AT} : frictional force of air on trailer

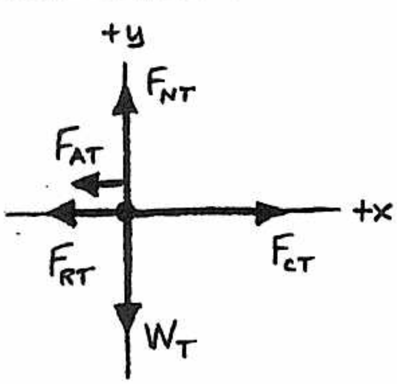
F_{RT} : frictional force of road on trailer

F_{CT} : contact (pulling) force of car on trailer

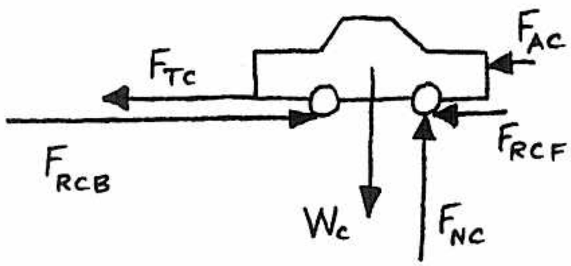
W_T : gravitational force of Earth on trailer

F_{NT} : normal (contact) force of road on trailer

force diagram (trailer)

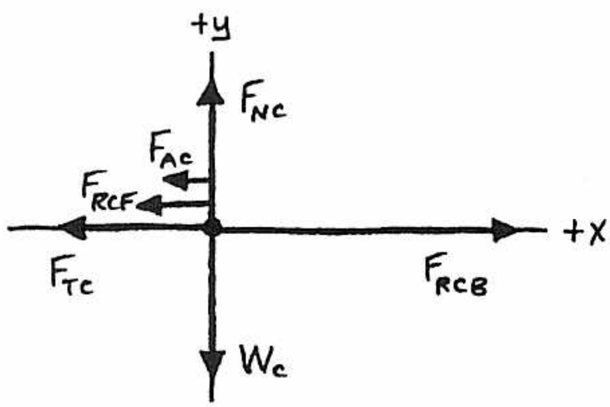


free-body diagram (car)



F_{AC}: frictional force of air on car
F_{RCF}: frictional force of road (backward) on front wheels of car
F_{RCB}: frictional force of road (forward!) on rear wheels of car (see discussion below)
F_{TC}: contact (pulling) force of trailer on car
W_c: gravitational force of Earth on car
F_{NC}: normal (contact) force of road on car

force diagram (car)



Discussion of forces on car and its acceleration.

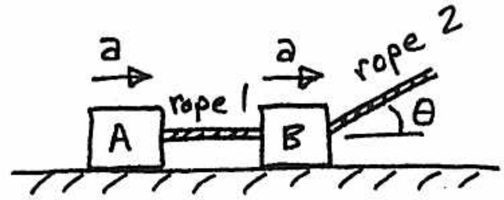
The vertical forces on the car are its weight (W_c) and the normal force (F_{NC}). These sum to zero so there is no net force and hence no acceleration vertically.

There are four horizontal forces on the car. In the -x-direction there is air friction (F_{AC}), friction with the road on the front wheels (F_{RCF}), and the force of the trailer (F_{TC}) which is the Newton's 3rd Law pair to F_{CT}. Assuming a rear-wheel drive car, the rear wheels make use of friction with the road to push backward on the road. The reaction to this force on the road is a frictional force forward on the car's rear wheels (F_{RCB}). The car will accelerate forward whenever F_{RCB} exceeds the sum of F_{AC}, F_{RCF}, and F_{CT}, resulting in a net force on the car in the +x-direction.

Problem #2: Consider two blocks connected by a horizontal rope. The pair of blocks are being accelerated across a horizontal frictionless surface by another rope which is attached to one of the blocks and slopes upward at some angle θ with respect to the horizontal surface. The blocks have equal mass. Construct force diagrams for each block. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.14)

FOCUS the PROBLEM

Picture and Given Information



frictionless surface

Question(s) What are the forces on each block?

Approach Use dynamics. Draw free-body diagrams and force diagrams for each block.

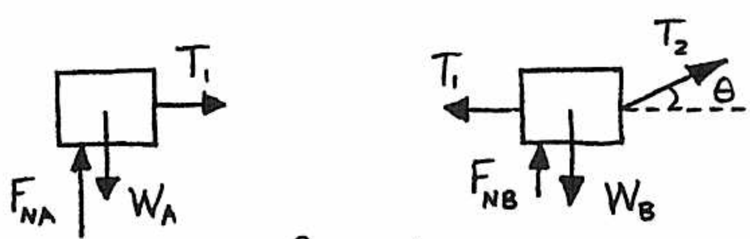
Assume massless ropes ⇒ tension is constant throughout each rope.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

free-body diagrams

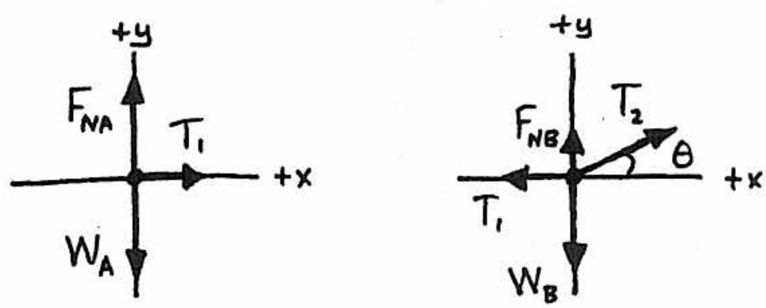
Block A



Block B

force diagrams

Block A



Block B

F_{NA} : normal (contact) force of surface on block A

W_A : gravitational force of Earth on block A

T_1 : contact force of rope 1 on block A

F_{NB} : normal (contact) force of surface on block B

W_B : gravitational force of Earth on block B

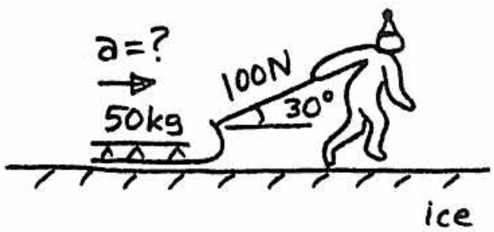
T_1, T_2 : contact force of rope 1, 2 on block B


SOMETIMES EVEN THE GREAT HAVE TO GET HIT ON THE HEAD TO LEARN THE TRUTH.

Problem #3: A rope connected to a 50-kg sled pulls it along a frictionless sheet of ice. The tension in the rope is 100-N and the rope is oriented at an angle of 30° above a line drawn parallel to the ice. Calculate the horizontal acceleration of the sled. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.14)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the horizontal acceleration of the sled?

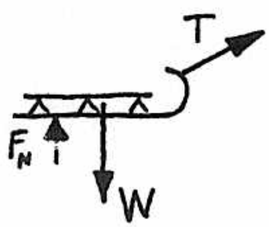
Approach Use dynamics. Find the horizontal and vertical forces acting on the sled. The net horizontal force will lead to a horizontal acceleration.

Since ice is slippery, assume friction can be neglected.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

free-body diagram

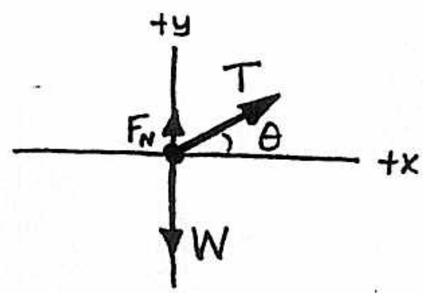


T: contact force of rope on sled

W: gravitational force of Earth on sled

F_N: normal force of ice on sled

force diagram



$T = 100N$ $m = 50kg$ $g = 9.8 m/s^2$

Target Quantity(ies) $\boxed{a_x}$

Quantitative Relationships

$$\sum F_x = T_x = ma_x$$

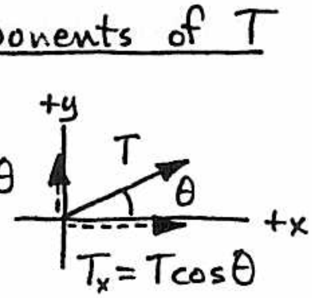
$$\sum F_y = T_y + F_N - W = m\cancel{a_y}^{\,0}$$

$$W = mg$$

$$\therefore T_y + F_N - mg = 0$$

components of T

$T_y = T\sin\theta$



PLAN the SOLUTION

Construct Specific Equations

unknowns

$\boxed{a_x}$

$\boxed{T_x}$

Find a_x

$T_x = ma_x$ (1)

Find T_x

$T_x = T\cos\theta$ (2)

$T\cos\theta = ma_x$

$$\boxed{a_x = \frac{T\cos\theta}{m}}$$

Check Units

$$\frac{[N]}{[kg]} = \frac{[kg\ m/s^2]}{[kg]} = [m/s^2] \quad \text{OK}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$a_x = \frac{100\ N \cos(30^\circ)}{50\ kg}$$

$$a_x = 1.7\ m/s^2$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected a_x has units of acceleration.

Is Answer Unreasonable?

No. This modest acceleration could be achieved by a sled.

Is Answer Complete?

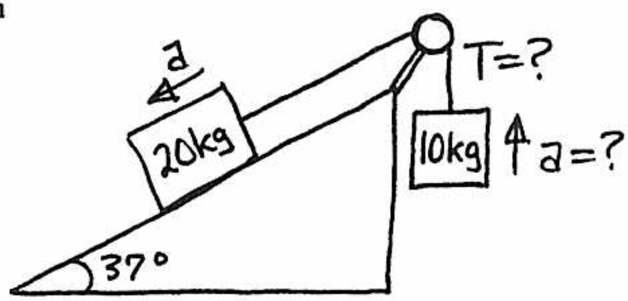
Yes. a_x is the horizontal acceleration of the sled which answers the question.

(extra space if needed)

Problem #4: Two blocks are connected by a cable strung over a pulley, which is mounted at the top of an incline. The 10-kg block hangs over the edge of the incline. The 20-kg block rests on the frictionless incline. The "toe" of the incline is at an angle of 37° with respect to the horizontal. Find the magnitude of the acceleration and the tension in the cable. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.14)

FOCUS the PROBLEM

Picture and Given Information



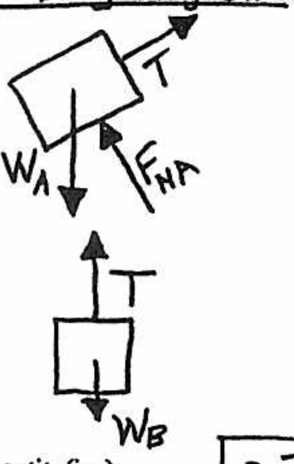
Question(s)
1) What is the acceleration of the blocks?
2) What is the tension in the cable?

Approach
Use dynamics -- sum the forces and find the acceleration of each block separately. The two accelerations must be the same because the blocks are connected by a taut cable.
Assume no friction in pulley and between 20kg block and incline.
Assume massless pulley. Assume massless cable ⇒ tension is the same throughout the cable.

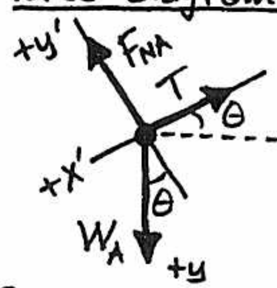
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

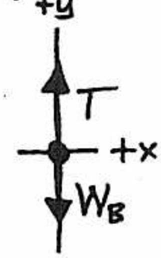
free-body diagrams



force diagrams



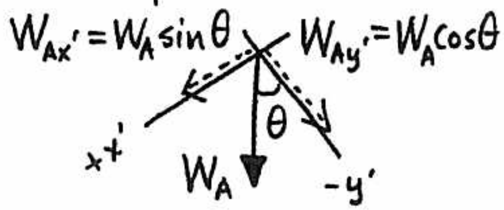
$m_A = 20kg$
$m_B = 10kg$
$\theta = 37°$
$g = 9.8 m/s^2$



F_{NA}: normal force of incline on block A
W_A, W_B: gravitational force of Earth on block A, B (also known as "weight")
T: contact force of cable on block A, B (also known as "tension")

Target Quantity(ies) $\boxed{a, T}$

components of W_A



Quantitative Relationships

$$\sum F_{Ax'} = W_{Ax'} - T = m_A a$$

$$\sum F_{Ay'} = F_{NA} - W_{Ay'} = 0 \qquad W_A = m_A g$$

$$\sum F_{Bx} = 0 \qquad W_B = m_B g$$

$$\sum F_{By} = T - W_B = m_B a \qquad \therefore T - m_B g = m_B a,\ W_{Ax'} = m_A g \sin\theta,\ W_{Ay'} = m_A g \cos\theta$$

PLAN the SOLUTION
Construct Specific Equations

Unknowns

$\boxed{a, T}$

Find a

$T - m_B g = m_B a$ (1)

Find T

$W_{Ax'} - T = m_A a$ (2)

$\boxed{W_{Ax'}}$

Find $W_{Ax'}$

$W_{Ax'} = m_A g \sin\theta$ (3)

$m_A g \sin\theta - T = m_A a$

$T = m_A g \sin\theta - m_A a$ (A)

$m_A g \sin\theta - m_A a - m_B g = m_B a$

$g(m_A \sin\theta - m_B) = (m_A + m_B) a$

$$\boxed{a = \frac{g(m_A \sin\theta - m_B)}{(m_A + m_B)}}$$

and $\boxed{T = m_A g \sin\theta - m_A a}$ ← result (A)

Check Units

$$a: \frac{[m/s^2]([kg]-[kg])}{[kg]} = [m/s^2] \quad \text{OK}$$

$$T: [kg][m/s^2] - [kg][m/s^2]$$
$$= [kg\ m/s^2] = [N] \quad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$a = \frac{(9.8 m/s^2)(20kg \sin(37°) - (10kg))}{(20kg + 10kg)}$$

$a = 0.66\ m/s^2$

$T = (20kg)(9.8 m/s^2)\sin(37°) - (20kg)(0.66 m/s^2)$

$T = 105\ N$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. Both the acceleration and the tension came out with the expected units.

Is Answer Unreasonable?

No. The acceleration is a small fraction of g and the tension exceeds the weight of block B just a little.

Is Answer Complete?

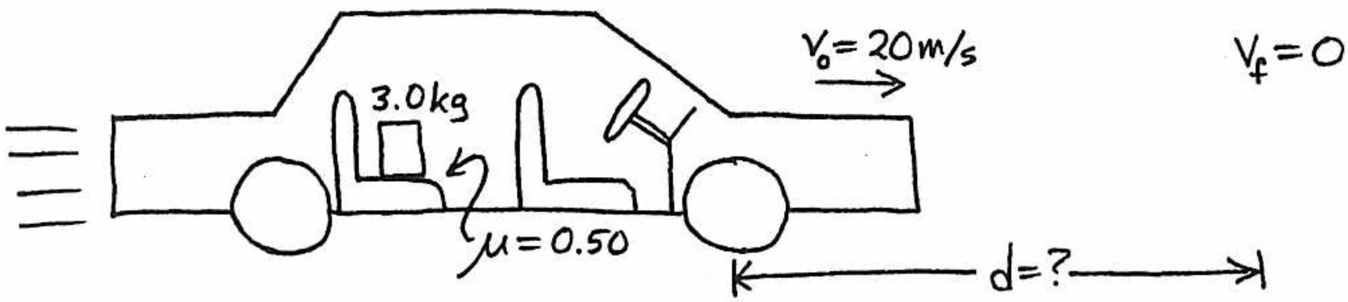
Yes. The acceleration of the blocks and the tension in the cable have been found. This answers the question.

(extra space if needed)

Problem #5: A 3.0-kg box sits on a horizontal surface of your car seat as you drive at a speed of 20-m/s. The coefficient of friction between the box and the seat is 0.50. You apply the brakes to stop the car. Calculate the shortest possible stopping distance so that the box does not start to slide off the seat. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.30)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the shortest distance in which the car can stop without allowing the box to slide?

Approach

Use dynamics to evaluate the forces on the box and the acceleration of the car/box -- to stay together they must accelerate the same. Then use kinematics to get the distance.

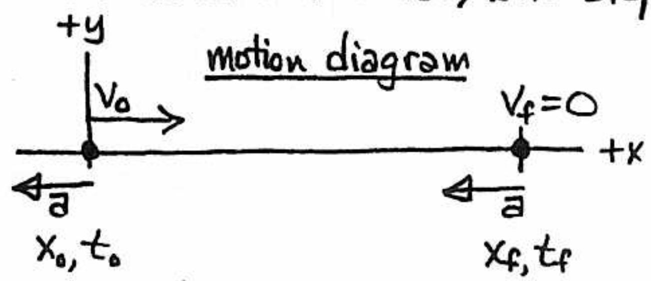
Time: Initial time is the instant you apply the brakes.
Final time is the instant the car/box stops.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

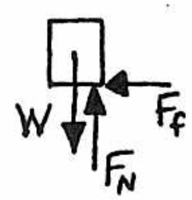
$X_o = 0$
$V_o = 20\,m/s$
$t_o = 0$
$a = ?$

motion diagram

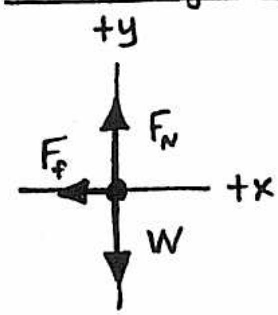


$X_f = ?$
$V_f = 0$
$t_f = ?$

free-body diagram



force diagram



W: gravitational force of Earth on box

F_N: normal force of seat on box

F_f: frictional force of seat on box

Target Quantity(ies) $\boxed{X_f}$

$m = 3.0\,kg$
$\mu = 0.50$
$g = 9.8\,m/s^2$

Quantitative Relationships

$$\sum F_y = F_N - W = 0 \qquad W = mg$$
$$\therefore F_N - mg = 0$$
$$\sum F_x = -F_f = m(-a) \qquad F_f = \mu F_N$$
$$\therefore \mu F_N = ma$$

Negative because both are in the -x-direction.

Constant acceleration (-a) in x-direction so use

$$x_f = \tfrac{1}{2}(-a)(t_f - \cancel{t_o})^2 + V_o(t_f - \cancel{t_o}) + \cancel{x_o}$$
$$\Rightarrow x_f = -\tfrac{1}{2} a t_f^2 + V_o t_f$$

Also: $-a = -\bar{a} = \dfrac{(\cancel{V_f} - V_o)}{(t_f - \cancel{t_o})} = -V_o/t_f \Rightarrow a = \dfrac{V_o}{t_f}$

PLAN the SOLUTION
Construct Specific Equations

unknowns

$\boxed{x_f}$

Find x_f

$x_f = -\frac{1}{2} a t_f^2 + v_0 t_f$ (1) $\boxed{a, t_f}$

Find a

$\mu F_N = ma$ (2) $\boxed{F_N}$

Find F_N

$F_N - mg = 0$ (3)

$F_N = mg$

$\mu \cancel{m} g = \cancel{m} a$

$a = \mu g$ (A)

$x_f = -\frac{1}{2} \mu g t_f^2 + v_0 t_f$

Find t_f

$a = v_0 / t_f$ (4)

$t_f = v_0 / a$

$t_f = v_0 / \mu g$ used result (A)

$x_f = -\frac{1}{2} \mu g \left(\frac{v_0}{\mu g}\right)^2 + v_0 \left(\frac{v_0}{\mu g}\right)$

$x_f = -\frac{1}{2} \frac{v_0^2}{\mu g} + \frac{v_0^2}{\mu g}$

$$\boxed{x_f = \frac{1}{2} \frac{v_0^2}{\mu g}}$$

Check Units

$$\frac{[m/s]^2}{[\,][m/s^2]} = [m] \qquad \text{OK}$$

μ is unitless

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$x_f = \frac{1}{2} \frac{(20\,m/s)^2}{(0.50)(9.8\,m/s^2)}$$

$x_f = 40.8\,m$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected x_f came out in units of length.

Is Answer Unreasonable?

No. While stopping, the car's average velocity is 10 m/s and, considering x_f above, the stopping time is ~4 seconds. These values sound quite plausible.

Is Answer Complete?

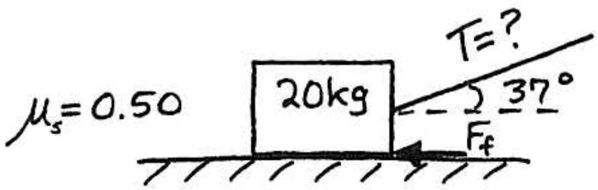
Yes. Finding the stopping distance x_f answers the question.

(extra space if needed)

Problem #6: A 20-kg crate sitting on a horizontal floor is attached to a rope that pulls 37° above the horizontal. The coefficient of static friction between the crate and the floor is 0.50. (a) Construct a force diagram for the crate. (b) Determine the least rope tension that will cause the crate to start sliding. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.29.)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the least rope tension that will cause the crate to start sliding? (This is the same as the maximum tension which will <u>not</u> start the crate moving.)

Approach

Use <u>dynamics</u>. For the crate to remain at rest the frictional force between the crate and the surface must be as great as the horizontal component of the tension.

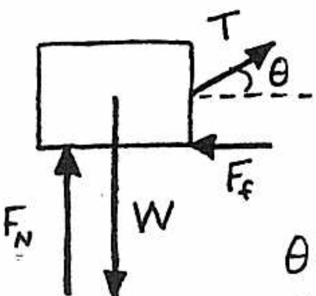
No motion so this is a <u>static</u> friction situation.

Max. tension ⇒ max. friction so use "=" in $F_f \le \mu_s F_N$.

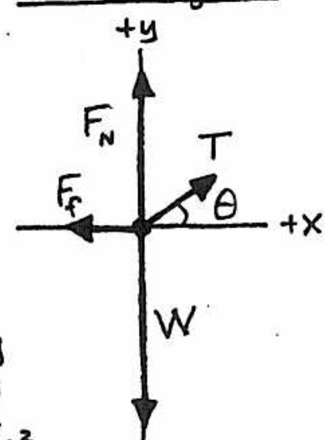
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

<u>free-body diagram</u>



<u>force diagram</u>



$\theta = 37°$
$m = 20\text{kg}$
$\mu_s = 0.50$
$g = 9.8\text{m/s}^2$

T: contact force of rope on crate
W: gravitational force of Earth on crate
F_N: normal (contact) force of floor on crate
F_s: frictional force of floor on crate

Target Quantity(ies) $\boxed{T}$

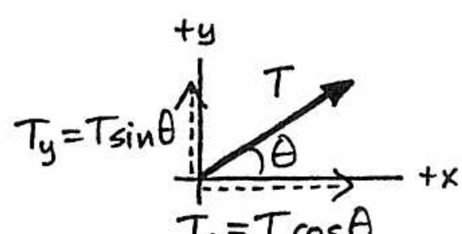
Quantitative Relationships

$$\sum F_y = F_N + T_y - W = 0$$
$$\therefore F_N + T_y - mg = 0$$
$$\sum F_x = T_x - F_f = 0$$
$$\therefore T_x - \mu_s F_N = 0$$

$W = mg$

generally: $F_f \le \mu_s F_N$
here: $F_f = \mu_s F_N$

<u>components of T</u>



PLAN the SOLUTION
Construct Specific Equations

unknowns

Find T — unknown: T

$T_x = T\cos\theta$ (1) — unknown: T_x

Find T_x

$T_x - \mu_s F_N = 0$ (2) — unknown: F_N

Find F_N

$F_N + T_y - mg = 0$ (3) — unknown: T_y

Find T_y

$T_y = T\sin\theta$ (4)

$$F_N + T\sin\theta - mg = 0$$

$$F_N = mg - T\sin\theta$$

$$T_x - \mu_s(mg - T\sin\theta) = 0$$

$$T_x = \mu_s mg - \mu_s T\sin\theta$$

$$\mu_s mg - \mu_s T\sin\theta = T\cos\theta$$

$$T\cos\theta + \mu_s T\sin\theta = \mu_s mg$$

$$\boxed{T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}}$$

Check Units

$$\frac{[\ ][kg][m/s^2]}{[\ ]+[\ ][\ ]} = [kg\ m/s^2] = [N] \text{ OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$T = \frac{(0.50)(20kg)(9.8m/s^2)}{\cos(37°) + 0.50\sin(37°)}$$

$$T = 89\text{ N}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected T came out in units of force.

Is Answer Unreasonable?

No. This modest tension is about 45% the weight of the crate. Notice that were the rope horizontal ($\theta' = 0°$) then the required tension would be $T' = (0.50)(20kg)(9.8m/s^2)/1$ $\Rightarrow T' = 98\text{ N} > T$! Less tension is needed when pulling at a modest angle because F_N (and hence F_f) is reduced.

Is Answer Complete?

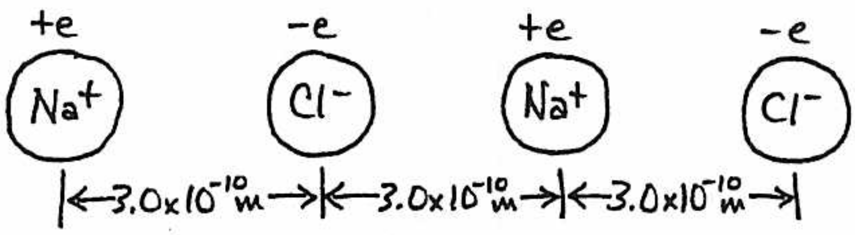
Yes. Finding T answers the question.

(extra space if needed)

Problem #7: Four ions (Na^+, Cl^-, Na^+ and Cl^-) each separated from its neighbors by 3.0×10^{-10}-m are in a row. The charge of a sodium ion is +e and that of a chlorine ion is -e. Calculate the force on the chlorine ion at the end of the row due to the other three ions. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 22.29)

FOCUS the PROBLEM

Picture and Given Information

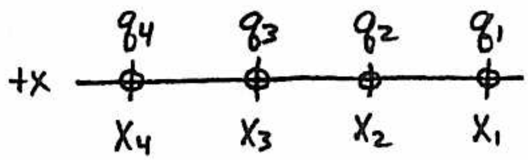


Question(s) What is the total electrostatic force on the chlorine ion on the end due to the other three ions?

Approach Use dynamics -- find the force on the chlorine ion due to each of the other ions individually then sum the forces. Remember that the electrostatic force is repulsive if the objects have the same (sign of) charge and attractive if the objects are oppositely charged.

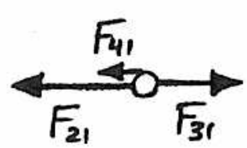
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

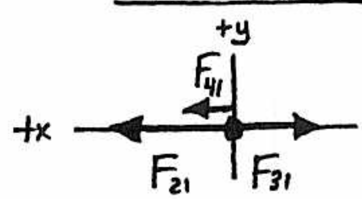


$X_1 = 0$
$X_2 = 3.0 \times 10^{-10}$m
$X_3 = 6.0 \times 10^{-10}$m
$X_4 = 9.0 \times 10^{-10}$m
$q_1 = -e$
$q_2 = e$
$q_3 = -e$
$q_4 = e$
$e = 1.6 \times 10^{-19}$C
$k_e = 9.0 \times 10^9 \frac{N\ m^2}{C^2}$

free-body diagram of ion #1



force diagram of ion #1



F_{21}, F_{31}, F_{41}: electrostatic force of ions #2, #3, #4 on ion #1

Target Quantity(ies) $\boxed{\sum F}$

Quantitative Relationships

$$\sum F_y = 0 \Rightarrow \sum F = \sum F_x$$

$$\sum F_x = F_{21} - F_{31} + F_{41}$$

$$F_{12} = \frac{k_e |q_1 q_2|}{r^2}$$

PLAN the SOLUTION
Construct Specific Equations

Unknowns

$\boxed{\sum F}$

$\boxed{\sum F_x}$

Find $\sum F$

$\sum F = \sum F_x$ (1)

Find $\sum F_x$

$\sum F_x = F_{21} - F_{31} + F_{41}$ (2) $\boxed{F_{21}, F_{31}, F_{41}}$

Find F_{21}

$$F_{21} = \frac{k_e |q_2 q_1|}{x_2^2} \quad (3)$$

$$F_{21} = \frac{k_e e^2}{x_2^2}$$

Find F_{31}

$$F_{31} = \frac{k_e |q_3 q_1|}{x_3^2} \quad (4)$$

$$F_{31} = \frac{k_e e^2}{x_3^2}$$

Find F_{41}

$$F_{41} = \frac{k_e |q_4 q_1|}{x_4^2} \quad (5)$$

$$F_{41} = \frac{k_e e^2}{x_4^2}$$

$$\sum F_x = \frac{k_e e^2}{x_2^2} - \frac{k_e e^2}{x_3^2} + \frac{k_e e^2}{x_4^2}$$

$$\boxed{\sum F = k_e e^2 \left(\frac{1}{x_2^2} - \frac{1}{x_3^2} + \frac{1}{x_4^2} \right)}$$

Check Units

$$\left[\frac{N\ m^2}{C^2}\right][C]^2\left(\frac{1}{[m]^2} - \frac{1}{[m]^2} + \frac{1}{[m]^2}\right)$$

$$= [N] \quad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$\sum F = \left(9.0\times10^{9} \frac{N\,m^2}{C^2}\right)\left(1.6\times10^{-19} C\right)^2 \times\left(\frac{1}{(3.0\times10^{-10} m)^2} - \frac{1}{(6.0\times10^{-10} m)^2} + \frac{1}{(9.0\times10^{-10} m)^2}\right)$$

$\sum F = 2.2\times10^{-9}$ N, positive $\Rightarrow$ to the left

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected $\sum F$ came out in units of force.

Is Answer Unreasonable?

No. $\sum F$ is rather small and directed toward the other ions. This makes sense because the dominant force (from the nearest ion) is attractive.

Is Answer Complete?

Yes. The total force on ion #1 has been found. This answers the question.

(extra space if needed)

4. Practice Exam Problems

Problem #1: You are about to climb up a ladder to fix a window on the second floor of your friend's house when you suddenly wonder what keeps the ladder from falling down. You know that you want to know all of the forces acting on the ladder while it is leaning against the wall before you get on it.

(a) Draw a picture of the situation indicating all of the forces on the ladder.

(b) Draw a free body diagram and force diagram of the ladder.

(c) Describe, in words, each of the forces acting on the ladder.

(d) Describe, in words, all forces which are related to the forces on the ladder by the Third law. Make sure you indicate which pairs of forces are third law pairs.

(e) Draw another picture of the situation indicating all of the forces that the ladder exerts on other objects.

Problem #2: You have been hired to design the interior of a special executive express elevator for a new office building. This elevator has all the latest safety features and will stop with an acceleration of g/3 in case of any emergency. The management would like a decorative lamp hanging from the unusually high ceiling of the elevator. You design a lamp which has three sections which hang one directly below the other. Each section is attached to the previous one by a single thin wire which also carries the electric current. The lamp is also attached to the ceiling by a single wire. Each section of the lamp weighs 7.0-N. Because the idea is to make each section appear that it is floating on air without support, you want to use the thinnest wire possible. Unfortunately the thinner the wire, the weaker it is. To determine the thinnest wire that can be used for each stage of the lamp, calculate the force on each wire in case of an emergency stop.

Problem #3: You are taking care of two small children, Sarah and Rachel, who are twins. On a nice cold, clear day you decide to take them ice skating on Lake of the Isles. To travel across the frozen lake you have Sarah hold your hand and Rachel's hand. The three of you form a straight line as you skate, and the two children just glide. Sarah must reach up at an angle of 60 degrees to grasp your hand but she grabs Rachel's hand horizontally. Since the children are twins, they are the same height and the same weight, 50-lbs. To get started you accelerate at 2.0-m/s^2. You are concerned about the force on the children's arms which might cause shoulder damage. So you calculate the force Sarah exerts on Rachel's arm, and the force you exert on Sarah's other arm. You assume that the frictional forces on the ice skates are negligible.

Problem #4: You are planning to build a log cabin in northern Minnesota. You want to pull a 205-kg log up a long smooth hill by means of a rope that is parallel to the hill surface. You need to buy a rope for this purpose so you need to know how strong the rope must be. Stronger ropes cost more. The hill surface is flat and smooth and at an angle of 30 degrees with respect to the horizontal. The coefficient of kinetic friction between the log and the hill is 0.900. When pulling the log up the hill, you will make sure that its acceleration is never more than 0.800-m/s^2. How strong a rope should you buy?

Problem #5: After graduating you get a job in Northern California. To move there, you rent a truck for all of your possessions. You also decide to take your car with you by towing it behind the truck. The instructions you get with the truck tells you that the maximum truck weight when fully loaded is 20,000-lbs and that the towing hitch that you rented has a maximum strength of 1000-lbs. Just before you leave, you weigh the fully loaded truck and find it to be 15,000-lbs. At the same time you weigh your car and find it to weigh 3000-lbs. You begin to worry if the hitch is strong enough. Then you remember that you can push your car and can easily keep it moving at a constant velocity. You know that air resistance will increase as the car goes faster but from your experience you estimate that the sum of the forces due to air resistance and friction on the car is not more than 300-lbs. If the largest hill you have to go up is sloped at 10˚ from the horizontal, what is the maximum acceleration you can safely have on that hill?

Problem #6: The quarter is almost over so you decide to have a party. To add atmosphere to your otherwise drab apartment, you decide to decorate with balloons. You buy about fifty and blow them up so that they are all sitting on your carpet. After putting most of them up, you decide to play with the few balloons left on the floor. You rub one on your sweater and find that it will "stick" to a wall. Aha! You know immediately that you are observing the electric force in action. Since it will be some time before you guests arrive and you have already made the onion dip, you decide to calculate the minimum electric force of the wall on the balloon. You know that the air exerts an upward force (the buoyant force) on the balloon which makes it almost "float". You measure the weight of the balloon minus the buoyant force of the air on the balloon to be 0.05-lb. By reading your physics book, you estimate that the coefficient of static friction between the wall and the balloon (rubber and concrete) is 0.80. Use the Focus the Problem and Describe the Physics sections of the solution sheets.

Problem #7: While working in a university research laboratory you are given the job of testing a new device, called an electrostatic scale, for precisely measuring the weight of small objects. The device is quite simple. It consists of two very light but strong strings attached to a support so that they hang straight down. an object is attached to the other end of the string. One of the objects has a very accurately known weight while the other object is the unknown. A power supply is slowly turned on to give each object an electric charge which causes the objects to slowly move away from each other (repel) because of the electric force. When the power supply is kept at its operating value, the objects come to rest at the same horizontal level. At that point, each of the strings supporting them makes a different angle with the vertical and the angle is measured. To test the device, you want to calculate the weight of an unknown sphere from the measured angles the weight of a known sphere. You use a standard sphere with a known weight of 2.0-N supported by a string which makes an angle of 10.0˚ with the vertical. The unknown sphere's string makes an angle of 20.0˚ with the vertical.

Problem #8: Finally you leave Minneapolis to get in a few days of spring break but your car breaks down in the middle of nowhere. A tow truck weighing 4000-lbs comes along and agrees to tow your car, which weighs 2000-lbs, to the nearest town. The driver of the truck attaches his cable to your car at an angle of 20˚ to the horizontal. He tells you that his cable has a strength of 500-lbs and that he plans to take 10 seconds to tow your car at a constant acceleration from rest in a straight line along the flat road until he reaches the speed limit of 45 miles per hour. If rolling friction behaves like kinetic friction, and the coefficient of rolling friction between your tires and the road is 0.10, determine if the driver can carry out his plan.

Problem #9: While visiting a friend in San Francisco you decide to drive around the city. You turn a corner and are suddenly going up a steep hill. Suddenly, a small boy runs out on the street chasing a ball. You slam on the brakes and skid to a stop leaving a 50 foot long skid mark on the street. The boy calmly walks away but a policeman watching from the sidewalk walks over and gives you a ticket for speeding. You are still shaking from the experience when he points out that the speed limit on this street is 25-mph. After you recover your wits, you examine the situation more closely. You determine that the street makes an angle of 20˚ with the horizontal and that the coefficient of static friction between your tires and the street is 0.80. You also find that the coefficient of kinetic friction between your tires and the street is 0.60. Your car's information book tells you that the mass of your car is 1570-kg. You weigh 130-lbs. Witnesses say that the boy had a weight of about 60-lbs and took 3.0 seconds to cross the 15 foot wide street. Will you fight the ticket in court?

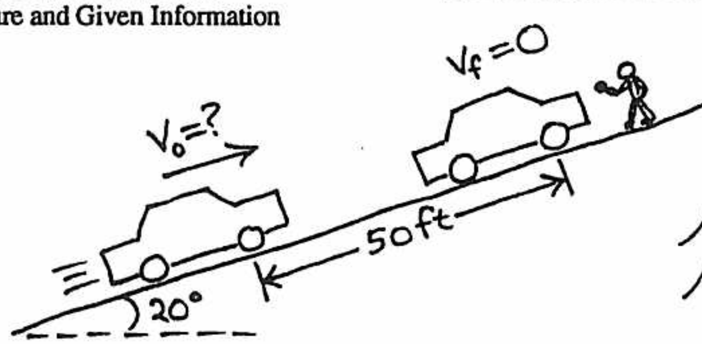
Problem #10: One morning while waiting for class to begin you are reading a newspaper article about airplane safety. This article emphasizes the role of metal fatigue in recent accidents. Metal fatigue results from the flexing of airframe parts in response to the forces on the plane especially during take off and landings. As an example, the reporter uses a plane with a take off weight of 200,000-lbs and take off speed of 200-mph which climbs at an angle of 30˚ with a constant acceleration to reach its cruising altitude of 30,000 feet with a speed of 500-mph. The 3 jet engines provide a forward thrust of 240,000-lbs by pushing air backwards. The article then goes on to explain that a plane can fly because the air exerts an upward force on the wings perpendicular to their surface called "lift". You know that air resistance is also a very important force on a plane and is in the direction opposite to the velocity of the plane. The article tells you this force is called the "drag". Although the reporter writes that some metal fatigue is primarily caused by the lift and some by the drag, she never tells you their size for her example plane. Luckily the article contains enough information to calculate them, so you do.

Dilbert / By Scott Adams

FOCUS the PROBLEM

Picture and Given Information

PRACTICE EXAM PROBLEM #9



$\mu_s = 0.80$

$\mu_k = 0.60$

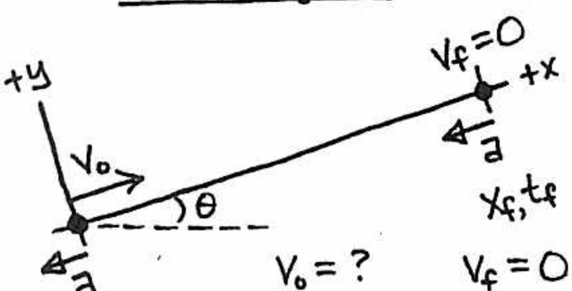
Question(s) Was your initial speed greater than the 25mph speed limit?

Approach Use dynamics to determine the net force on the car and hence its acceleration.
Use kinematics to determine the initial speed of the car.
Time: Initial time is the instant you hit the brakes.
Final time is the instant car comes to a halt.
Use μ_k not μ_s since car is moving while friction is acting.
Details about boy are unimportant in answering the question.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

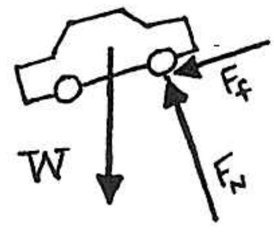
motion diagram



x_0, t_0

$V_0 = ?$	$V_f = 0$
$X_0 = 0$	$X_f = 50\,ft$
$t_0 = 0$	$t_f = ?$
	$a = ?$

free-body diagram



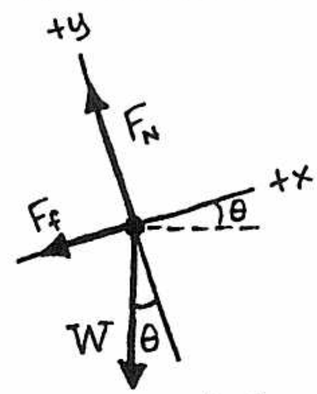
$\theta = 20°$

$g = 32\,ft/s^2$

$m_{car} = 1570\,kg$ $\quad m_{total} = M = ?$

$W_{person} = 130\,lbs$ $\quad W_{total} = W = ?$

force diagram



W: gravitational force of Earth on car

F_N: normal (contact) force of road on car

F_f: frictional force of road on car

Target Quantity(ies) $\boxed{V_0}$

Quantitative Relationships

$$\sum F_y = F_N - W_y = 0$$

$$\sum F_x = -F_f - W_x = M(-a)$$

All directed in the -x-direction.

$$F_f = \mu_k F_N$$

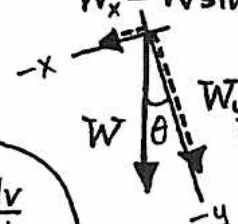
$$W = Mg$$

$$\therefore \mu_k F_N + W_x = Ma$$

Also: $-a = \frac{dv}{dt}$

components of W

$W_x = W\sin\theta = Mg\sin\theta$



$W_y = W\cos\theta$
$= Mg\cos\theta$

Constant accel. (-a) in x-direction so use

$$x_f = \tfrac{1}{2}(-a)(t_f - \cancel{t_0})^2 + V_{0x}(t_f - \cancel{t_0}) + \cancel{x_0}$$

$$\Rightarrow x_f = -\tfrac{1}{2}a t_f^2 + V_0 t_f$$

PLAN the SOLUTION
Construct Specific Equations

unknowns

Find v_o $\boxed{V_o}$

$x_f = -\frac{1}{2}at_f^2 + v_o t_f$ (1) $\boxed{a, t_f}$

Find t_f

$-a = \frac{dv}{dt} = \text{constant}$ (2)

integrate: $\int -a\, dt = \int \frac{dv}{dt} dt$

$-at + c = v$

when $t=0$: $0 + c = v_o \Rightarrow c = v_o$

$-at + v_o = v$

when $t = t_f$: $-at_f + v_o = \cancel{v_f}^{0}$

$at_f = v_o$

$t_f = v_o/a$

$x_f = -\frac{1}{2}a(v_o/a)^2 + v_o(v_o/a) = \frac{v_o^2}{2a}$

Find a

$\mu_k F_N + W_x = Ma$ (3) $\boxed{F_N, W_x, M}$

Find F_N

$F_N - W_y = 0$ (4) $\boxed{W_y}$

Find W_y

$W_y = Mg\cos\theta$ (5)

$F_N - Mg\cos\theta = 0$

$F_N = Mg\cos\theta$

$\mu_k Mg\cos\theta + W_x = Ma$

Find W_x

$W_x = Mg\sin\theta$ (6)

continued

Check Units

$\sqrt{[ft/s^2][ft]([\,])} = [ft/s]$ OK

EXECUTE the PLAN
Calculate Target Quantity(ies)

$v_o = \sqrt{2(32ft/s^2)(50ft)(0.60\cos(20°) + \sin(20°))}$

$v_o = 54\frac{ft}{s}\left(\frac{1\ mi}{5280 ft}\right)\left(\frac{3600 s}{1\ hr}\right)$

$v_o = 37\frac{mi}{hr}$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected v_o comes out in units of velocity.

Is Answer Unreasonable?

No. This is a plausible speed for a car driving on city streets.

Is Answer Complete?

No. v_o is well above the posted speed limit so you probably should not contest the ticket. This answers the question.

(extra space if needed)

$\mu_k \cancel{M} g\cos\theta + \cancel{M} g\sin\theta = \cancel{M} a$

Unknown M has cancelled out so no need to "Find M"!

$a = \mu_k g\cos\theta + g\sin\theta$

$x_f = \frac{v_o^2}{2g(\mu_k\cos\theta + \sin\theta)}$

$v_o^2 = 2gx_f(\mu_k\cos\theta + \sin\theta)$

$\boxed{v_o = \sqrt{2gx_f(\mu_k\cos\theta + \sin\theta)}}$

Chapter 5

The Conservation Approach

Introduction

Usually the difficulty in solving a problem is not in the calculations, but in deciding upon an appropriate approach by which to plan the solution. Most of the problems you will encounter can, in principle, be solved using a combination of kinematics and dynamics, but many times this approach leads to a long, convoluted, and difficult plan. Conservation principles provide an alternative approach and are a powerful tool in solving physics problems, especially when the details of the interaction between objects are not of interest.

The search for conserved quantities to model nature is one of the primary concerns of physics. Simply put, a conserved quantity is one for which you can set up an accounting procedure. Once you have chosen a system, the change in the amount of a conserved quantity in your system is always equal to the amount of that quantity that was transferred into your system from the environment or out of the system to the environment. If your system is isolated so that it does not interact with its environment, then the amount of a conserved quantity in the system can not change. Conserved quantities that you will use in this course include mass, charge, energy, momentum, and angular momentum. If X represents any conserved quantity for a system, the mathematical expression of the conservation of that quantity over a time interval between some initial time and some final time is:

$$X_f - X_i = X_{input} - X_{output}$$

where X_f is the amount of X in the system at the final time, X_i is the amount at the initial time, X_{input} is the amount that comes into the system from the environment during that time interval, and X_{output} is the amount that leaves the system to the environment during that time interval. This conservation equation can be written more compactly as:

$$\Delta X_{system} = \Delta X_{transferred}.$$

As with dynamics, it is critical to identify the system of interest, and as with kinematics, it is critical to identify the most useful initial and final times.

The first section of this chapter illustrates the use of conservation principles by describing how to use an energy conservation approach. This section concludes with some brief remarks about using the momentum conservation approach. The second section includes practice exercises from textbooks with sample solutions. The last section includes realistic practice problems from past exams.

The Energy Conservation Approach

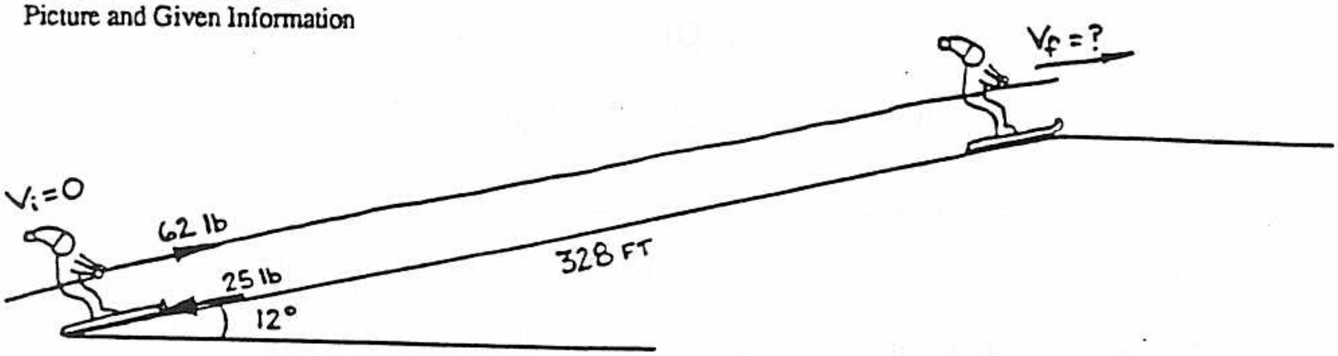
Many common problems have situations that are best suited to an energy conservation approach. For example, when the motion or the interactions of the system are quite complicated, important information about the system may be calculated without needing to know the details. No matter what happens, **energy is always conserved**. Unfortunately, in some cases you don't have

Problem #6: Every winter you hold an annual ski party. Most of your friends are good skiers and can handle the tow rope which is used to go from the lodge to the first chair lift. One of your friends, who weighs 176 pounds, usually loses his balance when a tow rope pulls him more than 5.0 mph. This tow rope pulls people up a 12 degree hill that is 328-ft long. The tow rope also exerts a 62-lb force on the skier, and the 4.0 mph wind together with the sticky snow exert a 25-lb force that opposes motion up the hill. Will your friend fall?

Similar to Jones & Childers 1992, example 6.10

FOCUS the PROBLEM

Picture and Given Information



Question(s) WILL THE SPEED OF THE SKIER BE GREATER THAN 5 mph BEFORE THE END OF THE HILL?

Approach USE ENERGY CONSERVATION. THE SYSTEM IS THE SKIER AND EARTH.
INITIAL TIME IS THE INSTANT THE SKIER STARTS ACCELERATING.
FINAL TIME IS THE INSTANT THE SKIER REACHES THE TOP OF THE HILL.

INITIAL ENERGY IS ZERO. FINAL ENERGY IS KINETIC AND GRAVITATIONAL POTENTIAL.
ENERGY TRANSFERED INTO THE SYSTEM VIA THE TOW ROPE.
ENERGY TRANSFERED OUT OF THE SYSTEM VIA SNOW AND WIND FRICTION.

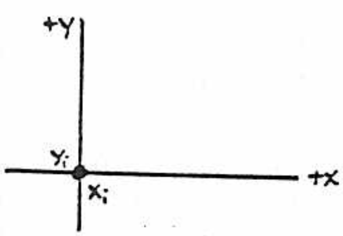
ASSUME THE FORCES DUE TO THE TOW ROPE AND THE FORCE DUE TO THE SNOW AND WIND FRICTION ARE CONSTANT FORCES.

THE NORMAL FORCE IS PERPENDICULAR TO THE DIRECTION OF MOTION SO IT DOESN'T TRANSFER ENERGY INTO OR OUT OF THE SYSTEM.

DESCRIBE the PHYSICS

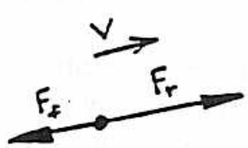
Diagram and Define Variables

INITIAL STATE



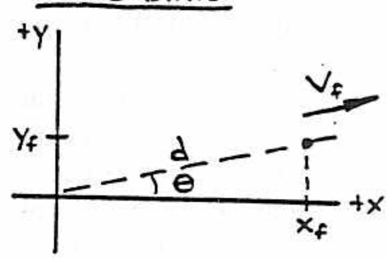
$Y_i = 0$
$V_i = 0$

ENERGY TRANSFER



$F_r = 62$ lb
$F_f = 25$ lb

FINAL STATE



$Y_f = d\sin\theta$ $\quad V_f = ?$
$X_f = d\cos\theta$ $\quad \theta = 12°$
$d = 328$ FT $\quad W = 176$ lb

Target Variable(s) $\boxed{\text{FIND } V_f}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$
$$E_i = 0 \qquad E_{in} = F_r d$$
$$E_f = KE_f + GPE_f \qquad E_{out} = F_f d$$
$$KE = \tfrac{1}{2}mV^2$$
$$GPE = mgY$$
$$W = mg$$

enough information to account for all of the energy, so that an energy conservation approach is not useful. Since energy is a scalar, results which depend on directions usually must be calculated using approaches which involve vector quantities such as kinematics, dynamics or another conserved quantity, momentum.

The example problem on the opposite page illustrates how to use an energy conservation approach. A complete solution is given later in this chapter (pages 5-20 & 5-21) together with several other examples.

Focus on the Problem

The first step in solving any problem is to draw a useful sketch of the situation. To use energy conservation, it is important that the sketch clearly show the initial and final states of the system and any interactions of the system with the environment. This sketch aids in sorting out the interactions of important objects so that you can decide which system to consider and what constitutes its initial and final state. In your sketch, include all energy transfers that affect the system. Also, include all of the relevant information given in the problem.

For the skier problem, we have chosen to draw a side view of the hill. This allows us to visualize the slope of the hill and the distance traveled by the skier. The sketch shows all of the relevant information including all of the forces on the skier. Some of this information might be superfluous to the problem solution, but since we will use the sketch to decide on the approach, it must contain everything that might be useful.

After determining the question, the next step is to decide how to approach the problem. We could use dynamics and kinematics to solve the skier problem. That would necessitate using forces to find the acceleration up the slope. The acceleration that results from those forces can then be used to find the final velocity of the skier at the top of the hill.

For this problem, it seems simpler to use conservation of energy. The skier moves up a hill with increasing speed. The skier's kinetic energy increases. The energy of the system increases because energy is being transferred into the system by its interaction with the rope. Energy is also being transferred out of the system by the frictional interaction with the snow and air resistance from the wind. The skier also interacts with the Earth via the gravitational force. If we include the Earth in the system, the Earth-skier system increases its gravitational potential energy. If we do not include the Earth in the system, the gravitational interaction causes another energy output from the skier.

Before choosing a system, you need to determine the important physical objects and how they interact . Your system might consist of some combination of those objects. In our example, the skier, the hill, the rope and the Earth are important interacting objects. A system is only useful if you are able to determine the initial amount of energy (initial energy state) of that system and the final amount of energy (final energy state) of that system. In addition you must be able to account for all energy transfer to or from that system between the initial and final times.

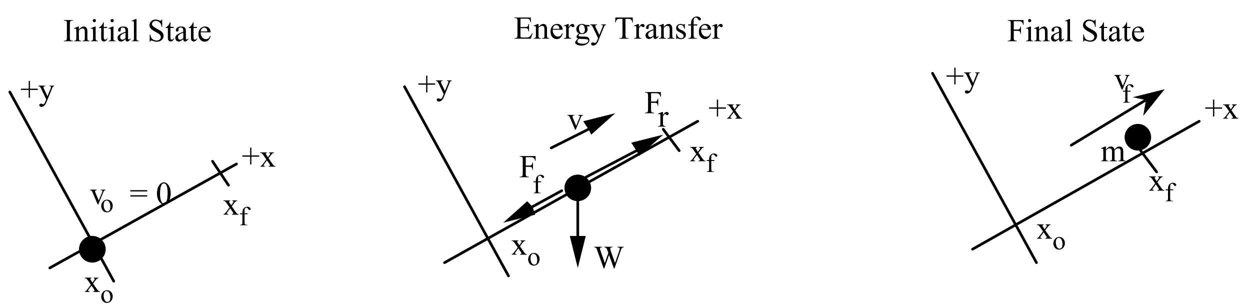
Energy Diagram

An energy diagram is very helpful to make sure that the system you choose is useful. Since we want to know the final speed of the skier, we will first consider the *skier* as our system. [Note: This is not the system shown in the example solution. That system will be discussed later in this section.] If the skier is the system, energy is then transferred by interactions with the Earth, the rope, the snow, and the air.

In addition to specifying a system, we must also specify the time interval of interest.

Now you are ready to determine the initial and final states of the system. For our example, it would be best to take the initial time to be the instant after the skier starts getting pulled up hill. Again, since we want to determine the speed of the skier at the top of the hill, the final time should be the instant the skier gets to the top of the 328-ft long the slope. Since the system is a single object, its energy state is quite simple (we ignore internal energy for the moment) -- the skier only has kinetic energy.

With the skier as our system, we must look at all of its energy transfers, inputs and outputs. Any force with a component in the direction of motion is the source of an energy input. The contact pull of the tow rope on the skier is the only force in the direction of motion. Therefore, it is the only energy input source from the environment. The frictional force of the snow and the air on the skier is opposite to the direction of the skier's motion. Consequently, this combination of forces causes an energy transfer out of the system to the environment. There is also another interaction with the environment that transfers energy from the system. The gravitational pull of the Earth on the skier (skier's weight) has a component in the direction opposite to the skier's motion. The energy diagram *for the skier as the system* is shown below with the coordinate system chosen to have one axis along the direction of motion.



$E_i = 0$

$E_{input} = F_r(x_f - x_o)$

$E_{output} = F_f(x_f - x_o) + W_x(x_f - x_o)$

$E_f = (1/2)\, m v_f^2$

Let us go back to the skier problem and choose a different system so that the gravitational force is not an external force and, therefore, can not transfer energy out of the system. Since the Earth is the object which is exerting that force on the skier, let us consider the *Earth and the skier* as our new system. Instead of an external gravitational force transferring energy from the skier, we now have a gravitational potential energy (GPE) of the Earth-skier system, which increases as the skier goes up the hill. Although the Earth is in the system from the gravitational point of view, we will still choose the snow on the hill and the air as external to the system. The tow rope, snow, and air still exert external forces which cause energy transfers to and from the system. You can now choose your y-axis to be vertical with its origin at the skiers initial elevation. Then, since $GPE_i = mgy_o$, our system has no initial gravitational potential energy. The system does have a final gravitational potential energy. The energy diagram for this system is shown on page 5-2 and in the complete solution later in the chapter (page 5-20).

As our example shows, **the energy terms for the system depend on how you choose**

your system. If an interaction is between an object in your system and an object in the environment, that interaction gives rise to an energy transfer. If you know the force, the energy transfer is just $\vec{F}\cos\theta \cdot \Delta x$ (work), where $\vec{F}$ is the force applied, θ is the angle between the force and the change in position, Δx. If an interaction is between two objects in your system, there is no energy transfer. There may be a change in the type of energy in the system such as potential energy or internal energy. It is vital that all of your energy terms are consistent with whatever system you choose.

The most convenient coordinate system may also depend on the system of objects for which you are determining the energy. With the skier as the system we chose a coordinate system with an axis along the direction of motion. With the Earth-skier system, a vertical axis was chosen to effectively show that the skier moved away from the center of the Earth and gained gravitational potential energy.

Quantitative Relationships

Next you need to write down the appropriate conservation relationship. In this case conservation of energy:

$$E_{final} - E_{initial} = E_{input} - E_{output}.$$

Identify each form of energy present in your system. These terms can be kinetic energy, gravitational potential energy, spring potential energy, internal energy, etc. The object of this section is to write down only those specific equations applicable to your system using the approach you have selected.

For the skier problem, you can choose the skier as the system and the energy terms are given with the energy diagram on page 5-4. The system's energy is only kinetic and there is energy transferred to or from that system by all of the forces in the problem except the normal force. The normal force cannot transfer any energy to the system since it is perpendicular to the direction of motion of the skier. If you choose the Earth-skier system, the energy terms are shown on page 5-20. The environment still interacts with our system via the tow rope, the snow, and the air. As before, the system (skier and Earth) loses energy to the environment through the interaction (frictional) with the snow (and air) and gains energy through the interaction with the rope. Now some of that energy transfer goes into increasing the gravitational potential energy of the system as well as its kinetic energy. The skier-Earth system will increase in kinetic energy (accelerate) up the hill only if the energy input is greater than the sum of the energy output and the potential energy increase.

One might wonder how we can have the Earth in our system yet claim that the frictional interaction of the snow is taking energy from the system. Certainly, the snow is on the Earth. The "Earth" in our system is only that part which exerts the gravitational force. The snow is not a factor in determining gravitational potential energy and so we choose to consider it outside of our system. If the snow were to be taken as part of the system, there would be no energy transfer by the frictional force. Instead, the effect of that force would be to change the internal energy of the system by heating up the surface of the skis and the snow. Since we do not have any information about this internal energy change (typically a change of temperature) it is not useful to include the snow in the system.

Plan the Solution

If you have constructed good energy diagrams and written down the energy terms which correspond to them, the mathematical plan of the solution should be very straight forward. Sometimes in conservation problems the mass of your system is an unknown and you may get one fewer equation than the number of unknowns. At a first glance this may seem like an unsolvable situation but, if each energy term depends on that unknown mass, the mass will cancel out of the conservation of energy equation. So, if you are faced with a problem where mass is an unknown, determine if each energy term is mass dependent. If each term is mass dependent, enter into the plan confident that the mass will cancel out. This is also true for dynamics problems where each force term depends on the mass of the object being accelerated.

The Momentum Conservation Approach

In some sense, using the momentum conservation approach can be even easier than using energy conservation. As with energy, **momentum is always conserved** no matter what the situation. With momentum there are no terms corresponding to potential energy or internal energy so deciding on your system is somewhat less complicated. In addition, an external force always transfers momentum to the system in the direction of that force, $\vec{p}_{transfer} = \vec{F} \cdot \Delta t$. This should be contrasted with the energy approach where energy is transferred to the system only by the component of external force along the direction of motion. A force perpendicular to the direction of motion of a system does not transfer any energy but does transfer momentum to that system.

The one complication with using the momentum approach is that momentum is a vector quantity. That means that each component of momentum is conserved independently of the other components.

$$p_{x\ final} - p_{x\ initial} = p_{x\ input} - p_{x\ output}$$

and

$$p_{y\ final} - p_{y\ initial} = p_{y\ input} - p_{y\ output}$$

and

$$p_{z\ final} - p_{z\ initial} = p_{z\ input} - p_{z\ output}.$$

Choosing a convenient coordinate system can be important in keeping your mathematical plan simple and your calculations short.

In some problems it is useful to break the situation up into separate time intervals. During one interval it may be most useful to use conservation of energy, during another conservation of momentum, and during a third both the conservation of energy and conservation of momentum. The decision on which approach to use during which time interval depends on whether or not you have enough information to determine all of the energy terms or momentum terms in the conservation equations.

2. Practice Textbook Problems

The problems listed on the next page are taken from your text. Use the five-step problem solving strategy to solve these problems. It is the most effective way to work through new problems, and it will be a useful tool on exam days. To make it easier to practice using the strategy, we include solution format sheets. These sheets mark off sections for each of the five problem-solving steps. Each section also includes brief prompts for the type of information to include in the space provided. Make copies of these sheets or sketch your own and use them to practice solving problems. This will help the strategy to become second nature for you.

Example solutions to the problems are worked out on the solution sheets, following the problem solving strategy. Do not read the example solutions before you have tried to solve the problem yourself. Your goals should be to understand (a) what kind of information belongs in each step, and (b) how one step leads logically into the next. *After* you have tried to solve a problem, you can check your understanding by comparing your solution to the example solution. When you have resolved any differences between the two solutions, go on and try to solve the next problem.

Problem #1: A 1200-kg elevator must be lifted by a cable that causes the elevator's speed to increase from zero to 4.0 m/s in a vertical distance of 6.0-m. Calculate the cable tension needed. (Based on Jones and Childers 1992, problem 6.27)

Problem #2: A 0.20-kg egg is dropped from a ladder a vertical distance of 4.0-m. The egg will break if subjected to an impulse force greater than 80-N. Over what minimum distance must a constant force be exerted to avoid breaking the egg? (Similar to Jones and Childers 1992, problem 6.76)

Problem #3: Show that the minimum distance needed to stop a car traveling at speed v is $v^2/2\mu g$, where μ is the coefficient of friction between the car and the road and g is the acceleration of gravity.

Problem #4: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55-mph. Ahead you see a sign that says, "Curve Ahead 200 feet, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your breaks at the instant you pass the sign. You slow your car down at a rate of 7-mph each second. As you reach the curve, are you traveling within the posted speed limit? (Note: This is the same problem that was solved using kinematics approach as example 3 in Chapter 2).

Problem #5: A water slide is 42-m long and has a vertical drop of 12-m. If a 60-kg person starts down the slide with a speed of 3.0 m/sec, calculate his or her speed at the bottom. A 120-N average friction force opposes the motion. (Based on Jones and Childers 1992, problem 6.41)

Problem #6: Every winter you hold an annual ski party. Most of your friends are good skiers and can handle the tow rope which is used to go from the lodge to the first chair lift. One of your friends, who weighs 176 pounds, usually loses his balance when a tow rope pulls him more than 5.0 mph. This tow rope pulls people up a 12 degree hill that is 328-ft long. The tow rope exerts a 62-lb force on the skier, and the 4.0 mph wind together with the sticky snow exert a 25-lb force that opposes motion up the hill. Will your friend fall? (Similar to Jones and Childers 1992, example 6.10)

Problem #7: A 1200-kg car traveling south at 24 m/s collides with and attaches itself to a 1200-kg truck traveling east at 16 m/s. Calculate the velocity (magnitude and direction) of the two vehicles when locked together after the collision. (Based on Jones and Childers 1992, problem 7.37)

Problem #8: A billiard ball at rest is hit head-on by a second billiard ball moving 1.5 m/s toward the east. If the collision is elastic and we ignore rotational motion, calculate the final speed of each ball.

Problem #9: An 80-g arrow moving at 80 m/s hits and embeds in a 10-kg block resting on ice. How far does the block slide on the ice following the collision if it is opposed by a 9.2-N force? (Similar to Jones and Childers 1992, problem 7.26)

Problem #10: A student shot a 10-g spitball in class. The spitball hit and stuck to a 100-g scale model of the moon that was right in front of the teacher. The model was hanging from the ceiling by a 1.5-ft string. The spitball covered the 4.0-m between the student and the model in 0.4-sec. The teacher has the ability to notice vertical displacements of more than 2-cm. Could the teacher have noticed the vertical movement of the model? (Based on Jones and Childers 1992, problem 7.26)

Problem #11: An ice-making machine removes heat from 0 Celsius water at a rate of 280 J/s. Calculate the time needed to form 2.0-kg of ice at 0 Celsius. (Similar to Jones and Childers 1992, problem 11.47)

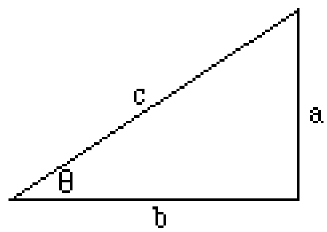
Problem #12: Calculate the amount of energy needed to change a 0.50-kg block of ice at 0 degrees Celsius into water at 20 degrees Celsius. (Similar to Jones and Childers 1992, problem 11.38)

Problem #13: The 1.0×10^7 kg of ice in a small pond has an average temperature of -5.0 degrees Celsius during the middle of winter. A movie making company wants to convert the pond to 100 degrees Celsius steam for a movie special effect. How much heat must they add to the frozen pond? (Similar to Jones and Childers 1992, problem 11.40)

Problem #14: An electric grill made of iron has a specific heat of 460 J/Kg C and a mass of 2.8-kg. To cook French toast, the grill is warmed from 20 to 350 degrees Celsius by resistive heating wires that produce thermal energy at a rate of 1500 W when connected to a 115-V potential difference. Fifty percent of the thermal energy is radiated into the room as the grill warms. How many minutes are required to warm the grill?

Problem #15: An airplane deicer melts 0.10-kg of ice from the wings of an airplane each minute. The deicer consists of resistive heating wires connected to a 24-V battery. Calculate the current through the heating wires and their resistance. Assume that the deicer transfers 100 percent of its energy to the ice.

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi R^2$

For a sphere: $A = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Fundamental Concepts and Principles:

$v_{av} = \dfrac{\Delta x}{\Delta t}$ $\quad v_{instant} = \lim(\Delta t \to 0)\dfrac{\Delta x}{\Delta t}$ $\quad a_{av} = \dfrac{\Delta v}{\Delta t}$ $\quad a_{instant} = \lim(\Delta t \to 0)\dfrac{\Delta v}{\Delta t}$

$\Sigma F = m \cdot a$ $\quad \Delta E_{system} = \Delta E_{transfer}$ $\quad KE = \frac{1}{2}mv^2$ $\quad E_{transfer} = \vec{F}\cos\theta \cdot \Delta x$

$\Delta p_{\,system} = \Delta p_{\,transfer}$ $\quad p = m \cdot v$ $\quad p_{\,transfer} = \Sigma F_r \cdot \Delta t$ $\quad I = \dfrac{\Delta q}{\Delta t}$

$V = \dfrac{PE}{q}$

Under Certain Conditions:

$x_f = \frac{1}{2}a(t_f - t_i)^2 + v_i(t_f - t_i) + x_i$ $\quad a = \dfrac{v^2}{r}$ $\quad F = G\dfrac{m_1 m_2}{r^2}$ $\quad F = k_e\dfrac{q_1 q_2}{r^2}$

$F \le \mu_s F_N$ $\quad F = \mu_k F_N$ $\quad F = k\Delta x$ $\quad PE = mg\Delta y$

$PE = \frac{1}{2}kx^2$ $\quad PE = -G\dfrac{m_1 m_2}{r}$ $\quad PE = k_e\dfrac{q_1 q_2}{r}$ $\quad R = \dfrac{\rho L}{A}$

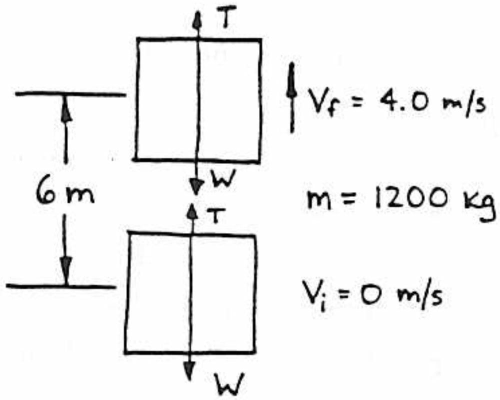
$V = IR$ $\quad P = IV$ $\quad \Delta E_{internal} = c\,m\,\Delta T$ $\quad \Delta E_{internal} = m\,L$

Useful constants: 1 mile = 5280 ft, R_E = 4000 miles, $g = 9.8\ m/s^2 = 32\ ft/s^2$,
$G = 6.7 \times 10^{-11}\ N\ m^2/kg^2$, $k_e = 9.0 \times 10^9\ N\ m^2/C^2$, $e = 1.6 \times 10^{-19}$ C

Problem #1: A 1200-kg elevator must be lifted by a cable that causes the elevator's speed to increase from zero to 4.0 m/s in a vertical distance of 6.0 m. Calculate the cable tension needed.
Based on Jones & Childers 1992, problem 6.27

FOCUS the PROBLEM

Picture and Given Information



Question(s)

WHAT UPWARD TENSION IN THE CABLE IS NEEDED TO CAUSE THE ELEVATOR TO GO FROM ZERO TO 4 m/s IN 6 m?

Approach

USE CONSERVATION OF ENERGY.

DEFINE THE SYSTEM AS THE ELEVATOR.

INITIAL TIME IS THE INSTANT THE ELEVATOR STARTS MOVING.

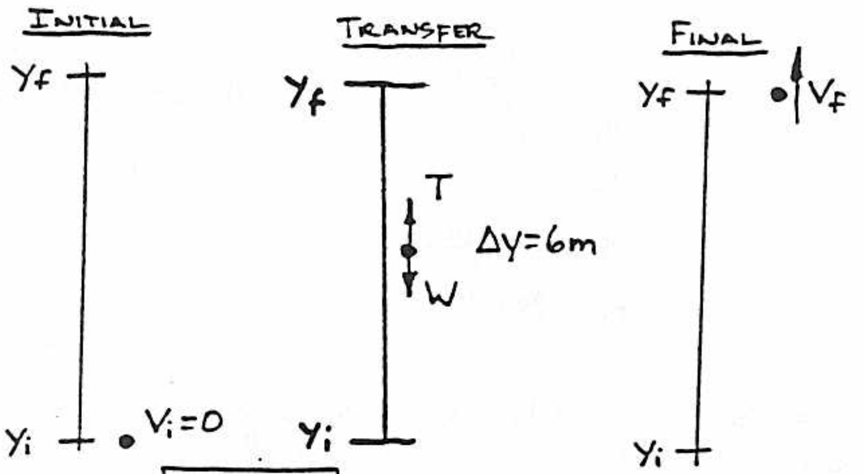
FINAL TIME IS THE INSTANT THE ELEVATOR HAS MOVED 6.0 m.

INITIAL ENERGY IS ZERO. FINAL ENERGY IS KINEMATIC.

INPUT ENERGY FROM TENSION. OUTPUT ENERGY FROM WEIGHT.

DESCRIBE the PHYSICS

Diagram and Define Variables



$y_i = 0$ $\quad y_f = 6$m
$V_i = 0$ $\quad V_f = 4$ m/s
m = 1200 kg
g = 9.8 m/s

Target Variable(s) $\boxed{\text{FIND T}}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_f = KE$ $\qquad E_{in} = T\Delta y$

$KE = \frac{1}{2}mV^2$ $\qquad E_{out} = W\Delta y$

$E_i = 0$ $\qquad W = mg$

PLAN the SOLUTION

Construct specific equations

	UNKNOWN
FIND T:	T
I $E_{in} = T\Delta y$	E_{in}
FIND E_{in}:	
II $E_f - E_i = E_{in} - E_{out}$	E_f, E_{out}
FIND E_f:	
III $E_f = \frac{1}{2} m v^2$	
FIND E_{out}:	
IV $E_{out} = W\Delta y$	W
FIND W:	
V $W = mg$	

Check for sufficiency

5 UNKNOWNS $(T, E_{in}, E_f, E_{out}, W)$

5 EQUATIONS (I, II, III, IV, V)

Outline the Math Solution

SOLVE V FOR W AND PUT INTO IV

SOLVE IV FOR E_{out} AND PUT INTO II

SOLVE III FOR E_f AND PUT INTO II

SOLVE II FOR E_{in} AND PUT INTO I

SOLVE I FOR T.

EXECUTE the PLAN

Follow the Plan

SOLVE V $W = mg$

PUT INTO IV $E_{out} = mg\Delta y$

SOLVE IV $E_{out} = mg\Delta y$

PUT INTO II $E_f - E_i = E_{in} - mg\Delta y$

SOLVE III $E_f = \frac{1}{2} m v^2$

PUT INTO II $\frac{1}{2} m v^2 - E_i = E_{in} - mg\Delta y$

SOLVE II $\frac{1}{2} m v^2 + mg\Delta y - \cancelto{0}{E_i} = E_{in}$

$\frac{1}{2} m v^2 + mg\Delta y = E_{in}$

PUT INTO I $\frac{1}{2} m v^2 + mg\Delta y = T\Delta y$

SOLVE I $\boxed{\frac{\frac{1}{2} m v^2 + mg\Delta y}{\Delta y} = T}$

CHECK UNITS:

$$= \frac{[kg][m/s]^2 + [kg][m/s^2][m]}{[m]}$$

$$= \frac{[kg][m^2/s^2]}{[m]}$$

$$= [kg][m/s^2]$$

$$= [N] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$T = \frac{\frac{1}{2}(1200\,kg)(4\,m/s)^2 + (1200\,kg)(9.8\,m/s^2)(6\,m)}{(6\,m)}$$

$\boxed{T = 13{,}360\ N}$

EVALUATE the SOLUTION

Is Solution Clear?

YES. THE ANSWER IS IN UNITS OF FORCE.

Is Answer Reasonable?

YES. THE WEIGHT OF THE ELEVATOR IS $W = mg = (1200\,kg)(9.8\,m/s^2) \cong 12{,}000\ N$.
TO ACCELERATE AS DESCRIBED WE NEED TO APPLY A FORCE GREATER THAN THE WEIGHT.

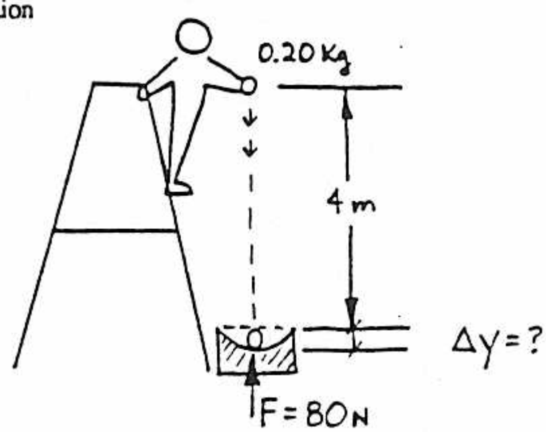
Is Answer Complete?

YES. THE QUESTION HAS BEEN COMPLETELY ANSWERED.

Problem #2: A 0.20 kg egg is dropped from a ladder a vertical distance of 4.0 m. The egg will break if subjected to an impulse force greater than 80 N. Over what minimum distance must a constant stopping force be exerted to avoid breaking the egg? `Similar to Jones & Childers 1992, problem 6.76`

FOCUS the PROBLEM

Picture and Given Information



Question(s)

WHAT IS THE MINIMUM DISTANCE (ΔY) OVER WHICH A 80N FORCE CAN BE EXERTED ON THE EGG WITHOUT BREAKING THE EGG.

Approach

USE CONSERVATION OF ENERGY.

DEFINE THE SYSTEM AS THE EARTH AND EGG.

INITIAL TIME IS THE INSTANT AFTER THE EGG IS DROPPED.

FINAL TIME IS THE INSTANT AFTER THE EGG STOPS.

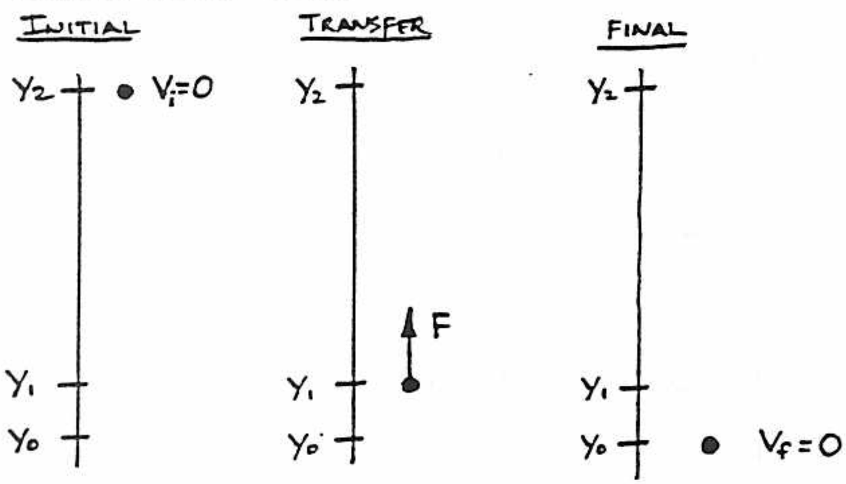
INITIAL ENERGY IS GRAVITATIONAL POTENTIAL. FINAL ENERGY IS ZERO.

INPUT ENERGY IS ZERO. OUTPUT ENERGY FROM STOPPING FORCE.

ASSUME THE EGG LANDS ON A SOFT PAD THAT EXERTS A CONSTANT STOPPING FORCE ON THE EGG.

DESCRIBE the PHYSICS

Diagram and Define Variables



$$\left.\begin{array}{l} y_2 = ? \\ y_1 = ? \end{array}\right\} \; y_2 - y_1 = \Delta y_2 = 4.0\,m$$

$$\left.\begin{array}{l} y_1 = ? \\ y_0 = 0\,m \end{array}\right\} \; y_1 - y_0 = \Delta y_1 = ?$$

$m = 0.20\,Kg$

$F = 80\,N$

Target Variable(s)

$\boxed{\text{FIND } \Delta y_1}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_f = 0$ $\qquad$ $E_{in} = 0$

$E_i = GPE_i$ $\qquad$ $E_{out} = F\Delta y_1$

$GPE_i = mgy_2$

PLAN the SOLUTION

Construct specific equations | UNKNOWN

FIND Δy_1: | Δy_1

(I) $E_{out} = F\Delta y_1$ | E_{out}

FIND E_{out}:

(II) $\cancel{E_f}^{0} - E_i = \cancel{E_{in}}^{0} - E_{out}$ | E_i

$E_i = E_{out}$

FIND E_i:

(III) $E_i = mg\,y_2$ | y_2

FIND y_2:

(IV) $\Delta y_2 = y_2 - y_1$ | y_1

FIND y_1:

$\Delta y_1 = y_1 - \cancel{y_0}^{0}$

(V) $\Delta y_1 = y_1$

Check for sufficiency

5 UNKNOWNS (Δy_1, E_{out}, E_i, y_2, y_1)

5 EQUATIONS (I, II, III, IV, V)

Outline the Math Solution

SOLVE (V) FOR y_1 AND PUT INTO (IV)

SOLVE (IV) FOR y_2 AND PUT INTO (III)

SOLVE (III) FOR E_i AND PUT INTO (II)

SOLVE (II) FOR E_{out} AND PUT INTO (I)

SOLVE (I) FOR Δy_1

EXECUTE the PLAN

Follow the Plan

SOLVE (V) $\Delta y_1 = y_1$

PUT INTO (IV) $\Delta y_2 = y_2 - \Delta y_1$

SOLVE (IV) $\Delta y_2 + \Delta y_1 = y_2$

PUT INTO (III) $E_i = mg(\Delta y_2 + \Delta y_1)$

SOLVE (III) $E_i = mg(\Delta y_2 + \Delta y_1)$

PUT INTO (II) $mg(\Delta y_2 + \Delta y_1) = E_{out}$

SOLVE (II) $mg(\Delta y_2 + \Delta y_1) = E_{out}$

PUT INTO (I) $mg(\Delta y_2 + \Delta y_1) = F\Delta y_1$

SOLVE (I) $mg\Delta y_2 + mg\Delta y_1 = F\Delta y_1$

$mg\Delta y_2 = F\Delta y_1 - mg\Delta y_1$

$mg\Delta y_2 = \Delta y_1(F - mg)$

$$\boxed{\frac{mg\Delta y_2}{F - mg} = \Delta y_1}$$

CHECK UNITS: $= \dfrac{[\mathrm{Kg}][\mathrm{m/s^2}][\mathrm{m}]}{[\mathrm{N}] - [\mathrm{Kg}][\mathrm{m/s^2}]}$

$= \dfrac{[\mathrm{N}][\mathrm{m}]}{[\mathrm{N}]-[\mathrm{N}]} = \dfrac{[\mathrm{N}][\mathrm{m}]}{[\mathrm{N}]} = [\mathrm{m}]$ O.K.

Calculate Target Variable(s)

$$\Delta y_1 = \frac{(0.20\,\mathrm{Kg})(9.8\,\mathrm{m/s^2})(4.0\,\mathrm{m})}{(80\,\mathrm{N}) - (0.20\,\mathrm{Kg})(9.8\,\mathrm{m/s^2})}$$

$$\boxed{\Delta y_1 = 0.10\,\mathrm{m}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. Δy_1 IS IN METERS, A DISTANCE UNIT.

Is Answer Reasonable?

YES. 0.10 m = 10 cm. THIS SEEMS LIKE A REASONABLE DISTANCE THAT AN EGG WOULD SINK INTO A SOFT PAD.

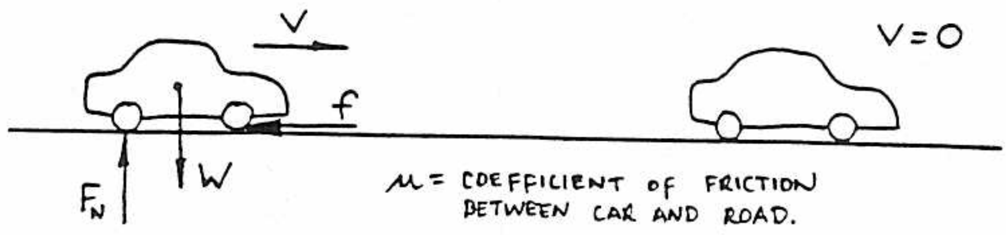
Is Answer Complete?

YES. Δy_1 MUST BE AT LEAST 0.10 M IN ORDER FOR A 80 N FORCE NOT TO BREAK THE EGG.

Problem #3: Show that the minimum distance needed to stop a car traveling at speed v is $v^2/2\mu g$, where μ is the coefficient of friction between the car and the road and g is the acceleration of gravity.

FOCUS the PROBLEM

Picture and Given Information



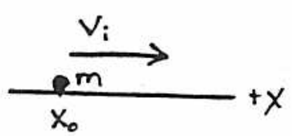
Question(s) WHAT IS THE DISTANCE REQUIRED FOR THE CAR TO STOP?

Approach USE ENERGY CONSERVATION. DEFINE THE SYSTEM TO BE THE CAR.
INITIAL TIME IS THE INSTANT THE CAR STARTS TO STOP.
FINAL TIME IS THE INSTANT THE CAR STOPS.
INITIAL ENERGY IS KINETIC. FINAL ENERGY IS ZERO.
INPUT ENERGY IS ZERO. OUTPUT ENERGY FROM FRICTION.

DESCRIBE the PHYSICS

Diagram and Define Variables

INITIAL STATE

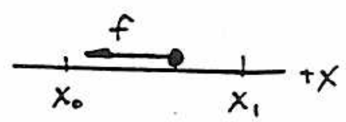


$X_o = 0$

$V_i = V$

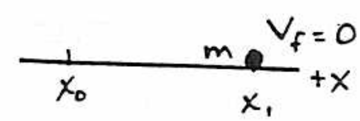
$m = ?$

ENERGY TRANSFER



$\Delta X = X_1 - X_o$

FINAL STATE



$X_1 = ?$

$V_f = 0$

$m = ?$

Target Variable(s) $\boxed{\text{FIND } \Delta X}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_f = 0$ $E_{in} = 0$

$E_i = \frac{1}{2} m V_i^2$ $E_{out} = f \Delta X$ $f = \mu F_N$ $\Sigma F_y = 0$

PLAN the SOLUTION

Construct specific equations | UNKNOWN

FIND ΔX: | ΔX

I $E_{out} = f\,\Delta X$ | E_{out}, f

FIND E_{out}:

$E_f - E_i = E_{in} - E_{out}$

$0 - E_i = 0 - E_{out}$

II $E_i = E_{out}$ | E_i

FIND E_i:

III $E_i = \frac{1}{2} m V_i^2$ | m

FIND f:

IV $f = \mu F_N$ | F_N

FIND F_N:

$\Sigma F_y = 0$

V $W - F_N = 0$ | W

FIND W:

VI $W = mg$

Check for sufficiency

7 UNKNOWNS ($\Delta X, E_{out}, f, E_i, m, F_N, W$)

6 EQUATIONS (I, II, III, IV, V, VI)

SINCE BOTH TERMS IN EQUATION III INVOLVE MASS, THE MASS SHOULD CANCEL OUT.

Outline the Math Solution

SOLVE VI FOR W AND PUT INTO V

SOLVE V FOR F_N AND PUT INTO IV

SOLVE IV FOR f AND PUT INTO I

SOLVE III FOR E_i AND PUT INTO II

SOLVE II FOR E_{out} AND PUT INTO I

SOLVE I FOR ΔX.

EXECUTE the PLAN

Follow the Plan

SOLVE VI $W = mg$

PUT INTO V $mg - F_N = 0$

SOLVE V $mg = F_N$

PUT INTO IV $f = \mu mg$

SOLVE IV $f = \mu mg$

PUT INTO I $E_{out} = (\mu mg)\,\Delta X$

SOLVE III $E_i = \frac{1}{2} m V_i^2$

PUT INTO II $\frac{1}{2} m V_i^2 = E_{out}$

SOLVE II $\frac{1}{2} m V_i^2 = E_{out}$

PUT INTO I $\frac{1}{2} m V_i^2 = \mu mg\,\Delta X$

SOLVE I $\frac{\frac{1}{2}\cancel{m} V_i^2}{\mu \cancel{m} g} = \Delta X$

$$\boxed{\frac{V_i^2}{2\mu g} = \Delta X}$$

CHECK UNITS:

$$= \frac{[\text{m/s}]^2}{[\text{m/s}^2]} = \frac{[\text{m}^2/\text{s}^2]}{[\text{m/s}^2]} = [\text{m}] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$\boxed{\Delta X = \frac{V^2}{2\mu g}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF DISTANCE.

Is Answer Reasonable?

YES. IT MAKES SENSE THAT THE GREATER THE VELOCITY OF THE CAR THE GREATER THE STOPPING DISTANCE, AND THE GREATER THE COEFFICIENT OF FRICTION THE SHORTER THE STOPPING DISTANCE.

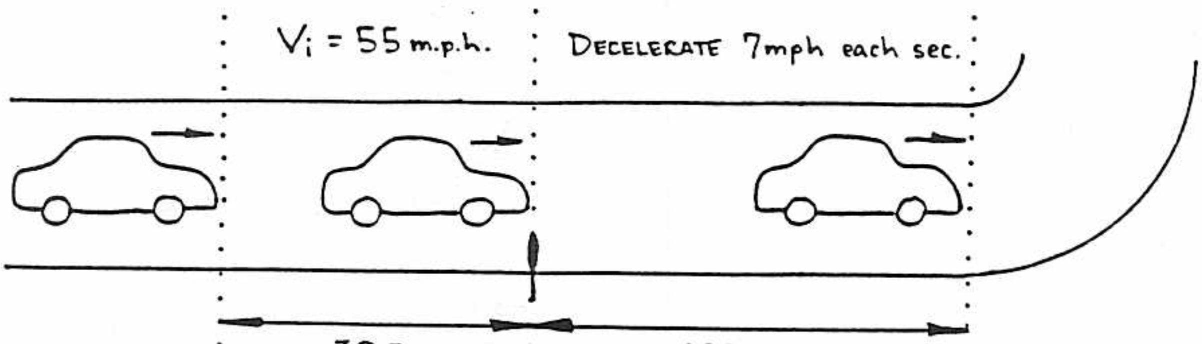
Is Answer Complete?

YES. WE SHOWED THAT THE STOPPING DISTANCE IS $V^2/2\mu g$.

Problem #4: You are a driver who always obeys posted speed limits. Late one night you are driving on a country highway at 55 mph. Ahead you see a sign that says, "Curve Ahead 200 feet, Slow to 35 mph." You are 30 feet from the sign when you first see it. You begin to apply your breaks at the instant you pass the sign. You slow your car down at a rate of 7 mph each second. As you reach the curve, are you traveling within the posted speed limit? (Note: This is the same problem that was solved using a kinematics approach as example 3 in Chapter 2).

FOCUS the PROBLEM

Picture and Given Information



Question(s)

WHAT IS THE SPEED OF THE CAR WHEN IT REACHES THE CURVE?

IS THIS SPEED LESS THAN 35 mph?

Approach

THIS PROBLEM CAN BE SOLVED USING EITHER KINEMATICS OR ENERGY CONSERVATION. USE CONSERVATION OF ENERGY. DEFINE THE SYSTEM TO BE THE CAR.

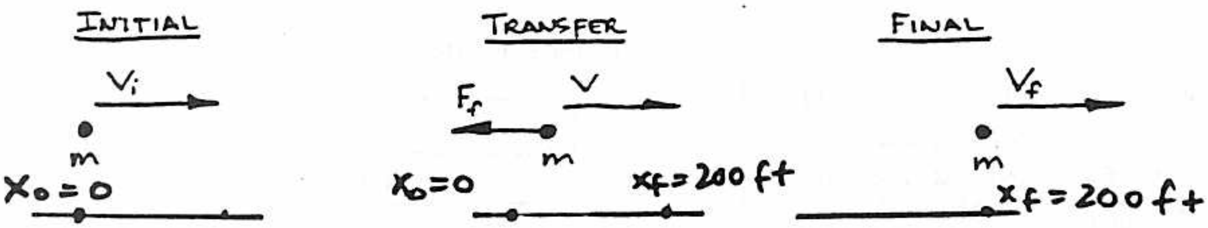
INITIAL TIME IS THE INSTANT THE DRIVER HITS THE BRAKES.
FINAL TIME IS THE INSTANT THE CAR REACHES THE CURVE.

INPUT ENERGY IS ZERO. OUTPUT ENERGY DUE TO FRICTION FORCE.

INITIAL ENERGY IS KINETIC. FINAL ENERGY KINETIC.

DESCRIBE the PHYSICS

Diagram and Define Variables



V_i = 55 mph
m = ?

V = ?
F_f = ?
m = ?

ΔX = 200 FT
$a = (7 \text{ mi/hr})/\text{sec}$

V_f = ?
m = ?

Target Variable(s) $\boxed{\text{FIND } V_f}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$$E_f = KE_f \qquad E_{in} = 0$$

$$E_i = KE_i \qquad E_{out} = F_f \Delta X \text{ (for constant force)}$$

$$KE = \frac{1}{2} m V^2$$

$$F_f = m a$$

PLAN the SOLUTION

Construct specific equations — UNKNOWN

FIND V_f: — V_f

(I) $KE_f = \frac{1}{2} m V_f^2$ — KE_f, m

FIND KE_f:

(II) $KE_f = E_f$ — E_f

FIND E_f:

$E_f - E_i = E_{in} - E_{out}$ — E_i, E_{out}

(III) $E_f - E_i = -E_{out}$

FIND E_i:

(IV) $E_i = \frac{1}{2} m V_i^2$

FIND E_{out}:

(V) $E_{out} = F_f \Delta X$ — F_f

FIND F_f:

(VI) $F_f = ma$

Check for sufficiency

7 UNKNOWNS (V_f, KE_f, m, E_f, E_i, E_{out}, F_f)

6 EQUATIONS (I, II, III, IV, V, VI)

SINCE EACH TERM IN EQUATION III INVOLVES MASS, THE MASS SHOULD CANCEL OUT.

Outline the Math Solution

SOLVE (VI) FOR F_f AND PUT INTO (V)

SOLVE (V) FOR E_{out} AND PUT INTO (III)

SOLVE (IV) FOR E_i AND PUT INTO (III)

SOLVE (III) FOR E_f AND PUT INTO (II)

SOLVE (II) FOR KE_f AND PUT INTO (I)

SOLVE (I) FOR V_f.

EXECUTE the PLAN

Follow the Plan

SOLVE (VI) $F_f = ma$

PUT INTO (V) $E_{out} = (ma)\Delta X$

SOLVE (V) $E_{out} = ma\Delta X$

PUT INTO (III) $E_f - E_i = -ma\Delta X$

SOLVE (IV) $E_i = \frac{1}{2} m V_i^2$

PUT INTO (III) $E_f - \frac{1}{2} m V_i^2 = -ma\Delta X$

SOLVE (III) $E_f = \frac{1}{2} m V_i^2 - ma\Delta X$

PUT INTO (II) $KE_f = \frac{1}{2} m V_i^2 - ma\Delta X$

SOLVE (II) $KE_f = \frac{1}{2} m V_i^2 - ma\Delta X$

PUT INTO (I) $\frac{1}{2} \cancel{m} V_i^2 - \cancel{m} a\Delta X = \frac{1}{2} \cancel{m} V_f^2$

SOLVE (I) $\frac{1}{2} V_i^2 - a\Delta X = \frac{1}{2} V_f^2$

$$V_i^2 - 2a\Delta X = V_f^2$$

$$\boxed{\sqrt{V_i^2 - 2a\Delta X} = V_f}$$

CHECK UNITS:

$$= \sqrt{[mi/Hr]^2 - \left[\frac{mi/Hr}{SEC}\right][FT]}$$

$$= \sqrt{[mi/Hr]^2 - \left[\frac{mi/Hr}{SEC}\right]\left[\frac{SEC}{Hr}\right][FT]\left[\frac{mi}{FT}\right]}$$

$$= \sqrt{[mi/Hr]^2 - [mi/Hr]^2}$$

$$= \sqrt{[mi/Hr]^2} = [mi/Hr] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$V_f = \sqrt{(55\ mi/Hr)^2 - 2\left(7\ \frac{mi/Hr}{SEC}\right)(200\ FT)}$$

$$V_f = \sqrt{(55\ mi/Hr)^2 - 2\left(7\ \frac{mi/Hr}{SEC}\right)\left(\frac{3600\ SEC}{Hr}\right)(200\ FT)\left(\frac{1\ mi}{5280\ FT}\right)}$$

$$\boxed{V_f = 33.4\ mi/Hr}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF VELOCITY.

Is Answer Reasonable?

YES. 33 mi/Hr IS VERY CLOSE TO THE POSTED SPEED OF 35 mi/Hr.

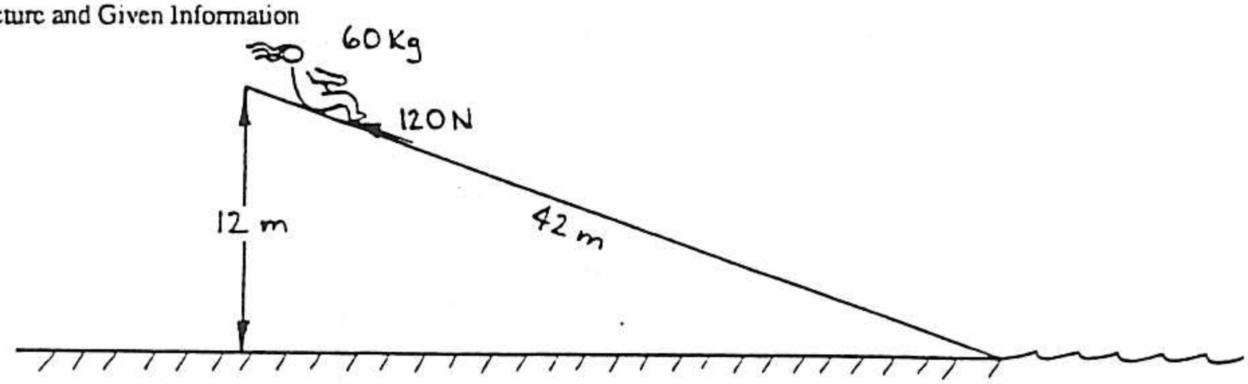
Is Answer Complete?

NO. WE DID NOT ANSWER THE QUESTION. THE CAR SLOWED TO LESS THAN 35 mi/Hr. THEREFORE, THE CAR IS TRAVELING WITHIN THE POSTED SPEED.

Problem #5: A water slide is 42 m long and has a vertical drop of 12 m. If a 60-kg person starts down the slide with a speed of 3.0 m/s, calculate his or her speed at the bottom. A 120 N average friction force opposes the motion. Based on Jones & Childers 1992, problem 6.41

FOCUS the PROBLEM

Picture and Given Information



Question(s)

WHAT IS THE INITIAL VELOCITY OF THE SLIDER?

Approach

USE CONSERVATION OF ENERGY.

DEFINE THE SYSTEM AS THE EARTH AND SLIDER.

INITIAL TIME IS THE INSTANT THE SLIDER STARTS SLIDING.

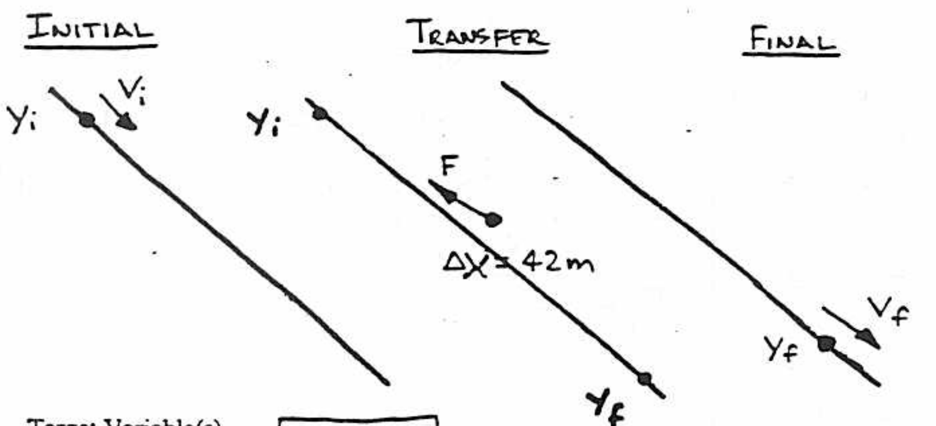
FINAL TIME IS THE INSTANT THE SLIDER REACHES THE BOTTOM OF THE SLIDE.

INITIAL ENERGY IS KINETIC AND POTENTIAL. FINAL ENERGY IS KINETIC.

INPUT ENERGY IS ZERO. OUTPUT ENERGY FROM FRICTION.

DESCRIBE the PHYSICS

Diagram and Define Variables



$y_i = 12m$ $y_f = 0m$

$V_i = 3.0\ m/s$ $V_f = ?$

$m = 60\ Kg$

$\Delta X = 42m$

$F = 120\ N$

Target Variable(s)

$\boxed{\text{FIND } V_f}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_f = KE_f$ $E_{in} = 0$

$KE = \frac{1}{2} mV^2$ $E_{out} = F\Delta y$ (CONSTANT FORCE)

$E_i = PE_i + KE_i$

$PE = mg\Delta y$

PLAN the SOLUTION
Construct specific equations

UNKNOWN

FIND V_f:

(I) $E_f = \frac{1}{2} m V_f^2$ — V_f

FIND E_f: — E_f

(II) $E_f - E_i = E_{in} - E_{out}$ — E_i, E_{out}

FIND E_i:

(III) $E_i = \frac{1}{2} m V_i^2 + mgy_i$

FIND E_{out}:

(IV) $E_{out} = F\Delta X$

Check for sufficiency

4 UNKNOWNS (V_f, E_f, E_{in}, E_{out})

4 EQUATIONS (I, II, III, IV)

Outline the Math Solution

SOLVE (IV) FOR E_{out} AND PUT INTO (II)

SOLVE (III) FOR E_i AND PUT INTO (II)

SOLVE (II) FOR E_f AND PUT INTO (I)

SOLVE (I) FOR V_f.

EXECUTE the PLAN
Follow the Plan

SOLVE (IV) $E_{out} = F\Delta X$

PUT INTO (II) $E_f - E_i = E_{in} - F\Delta X$

SOLVE (III) $E_i = \frac{1}{2} m V_i^2 + mgy_i$

PUT INTO (II) $E_f - (\frac{1}{2} m V_i^2 + mgy_i) = \cancel{E_{in}}^{\,0} - F\Delta X$

SOLVE (II) $E_f = (\frac{1}{2} m V_i^2 + mgy_i) - F\Delta$

PUT INTO (I) $\frac{1}{2} m V_i^2 + mgy_i - F\Delta X = \frac{1}{2} m V_f^2$

SOLVE (I) $\dfrac{\frac{1}{2} m V_i^2 + mgy_i - F\Delta X}{\frac{1}{2} m} = V_f^2$

$$\boxed{\sqrt{V_i^2 + 2gy_i - \frac{2F\Delta X}{m}} = V_f}$$

CHECK UNITS:

$$= \sqrt{[m/s]^2 + [m/s^2][m] - \frac{[N][m]}{[Kg]}}$$

$$= \sqrt{[m^2/s^2] + [m^2/s^2] - \frac{[\frac{Kg \cdot m}{s^2}][m]}{[Kg]}}$$

$$= \sqrt{[m^2/s^2] + [m^2/s^2] - [m^2/s^2]}$$

$$= [m/s] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$V_f = \sqrt{(3\,m/s) + 2(9.8\,m/s^2)(12\,m) - \frac{2(120\,N)(42\,m)}{(60\,Kg)}}$$

$$\boxed{V_f = 8.4\ m/s}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF VELOCITY.

Is Answer Reasonable?

YES. WE WOULD EXPECT THE SLIDER TO GO FASTER AT THE BOTTOM OF THE SLIDE. 8.4 m/s SEEMS LIKE A REASONABLE SPEED.

Is Answer Complete?

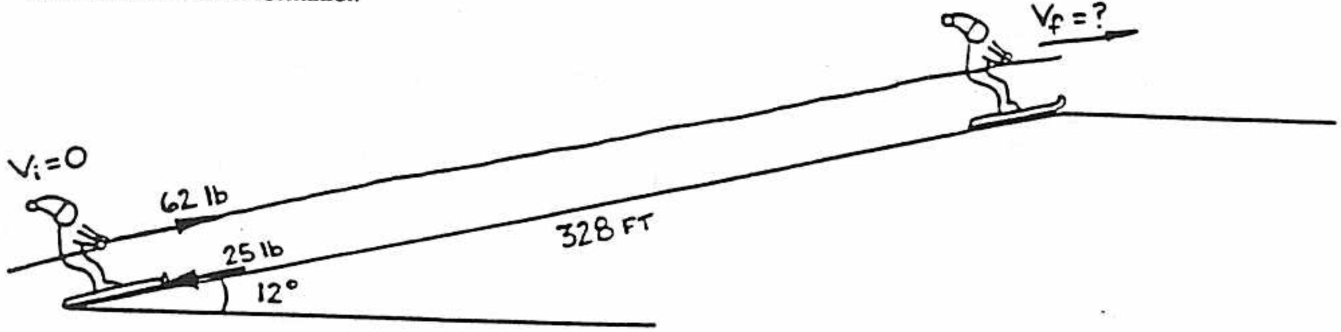
YES. WE FOUND THE SPEED OF THE SLIDER AT THE BOTTOM OF THE SLIDE.

Problem #6: Every winter you hold an annual ski party. Most of your friends are good skiers and can handle the tow rope which is used to go from the lodge to the first chair lift. One of your friends, who weighs 176 pounds, usually loses his balance when a tow rope pulls him more than 5.0 mph. This tow rope pulls people up a 12 degree hill that is 328-ft long. The tow rope also exerts a 62-lb force on the skier, and the 4.0 mph wind together with the sticky snow exert a 25-lb force that opposes motion up the hill. Will your friend fall?

Similar to Jones & Childers 1992, example 6.10

FOCUS the PROBLEM

Picture and Given Information



Question(s) WILL THE SPEED OF THE SKIER BE GREATER THAN 5 mph BEFORE THE END OF THE HILL?

Approach USE ENERGY CONSERVATION. THE SYSTEM IS THE SKIER AND EARTH.
INITIAL TIME IS THE INSTANT THE SKIER STARTS ACCELERATING.
FINAL TIME IS THE INSTANT THE SKIER REACHES THE TOP OF THE HILL.

INITIAL ENERGY IS ZERO. FINAL ENERGY IS KINETIC AND GRAVITATIONAL POTENTIAL.
ENERGY TRANSFERED INTO THE SYSTEM VIA THE TOW ROPE.
ENERGY TRANSFERED OUT OF THE SYSTEM VIA SNOW AND WIND FRICTION.

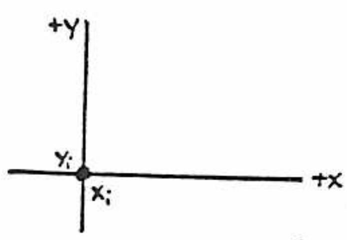
ASSUME THE FORCES DUE TO THE TOW ROPE AND THE FORCE DUE TO THE SNOW AND WIND FRICTION ARE CONSTANT FORCES.

THE NORMAL FORCE IS PERPENDICULAR TO THE DIRECTION OF MOTION SO IT DOESN'T TRANSFER ENERGY INTO OR OUT OF THE SYSTEM.

DESCRIBE the PHYSICS

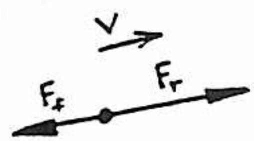
Diagram and Define Variables

INITIAL STATE



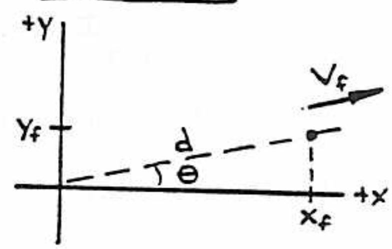
$Y_i = 0$
$V_i = 0$

ENERGY TRANSFER



$F_r = 62$ lb
$F_f = 25$ lb

FINAL STATE



$Y_f = d \sin\theta$ $V_f = ?$
$X_f = d \cos\theta$ $\theta = 12°$
$d = 328$ FT $W = 176$ lb

Target Variable(s) $\boxed{\text{FIND } V_f}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_i = 0$ $E_{in} = F_r d$

$E_f = KE_f + GPE_f$ $E_{out} = F_f d$

$$KE = \frac{1}{2} m V^2$$

$$GPE = mgy$$

$$W = mg$$

PLAN the SOLUTION

Construct specific equations | UNKNOWN

FIND V_f:

(I) $KE_f = \frac{1}{2} m V_f^2$ | KE_f, m

FIND m:

(II) $W = mg$

FIND KE_f:

(III) $E_f = KE_f + GPE_f$ | E_f, GPE_f

FIND GPE_f:

(IV) $GPE_f = m g y_f$ | y_f

FIND y_f:

(V) $y_f = d \sin\theta$

FIND E_f:

(VI) $E_f - E_i = E_{in} - E_{out}$ | E_{in}, E_{out}

FIND E_{in}:

(VII) $E_{in} = F_r d$

FIND E_{out}:

(VIII) $E_{out} = F_f d$

Check for sufficiency

8 UNKNOWNS (V_f, KE_f, m, E_f, GPE_f, y_f, E_{in}, E_{out})

8 EQUATIONS (I, II, III, IV, V, VI, VII, VIII)

Outline the Math Solution

SOLVE (VIII) FOR E_{out} AND PUT INTO (VI)

SOLVE (VII) FOR E_{in} AND PUT INTO (VI)

SOLVE (VI) FOR E_f AND PUT INTO (III)

SOLVE (V) FOR y_f AND PUT INTO (IV)

SOLVE (IV) FOR GPE_f AND PUT INTO (III)

SOLVE (III) FOR KE_f AND PUT INTO (I)

SOLVE (II) FOR m AND PUT INTO (I)

SOLVE (I) FOR V_f.

EXECUTE the PLAN

Follow the Plan

SOLVE (VIII) $E_{out} = F_f d$

PUT INTO (VI) $E_f - E_i = E_{in} - F_f d$

SOLVE (VII) $E_{in} = F_r d$

PUT INTO (VI) $E_f - \cancel{E_i}^{\,0} = F_r d - F_f d$

SOLVE (VI) $E_f = F_r d - F_f d$

PUT INTO (III) $F_r d - F_f d = KE_f + GPE_f$

SOLVE (V) $y_f = d \sin\theta$

PUT INTO (IV) $GPE_f = m g d \sin\theta$

SOLVE (IV) $GPE_f = m g d \sin\theta$

PUT INTO (III) $F_r d - F_f d = KE_f + m g d \sin\theta$

SOLVE (III) $F_r d - F_f d - m g d \sin\theta = KE_f$

PUT INTO (I) $F_r d - F_f d - m g d \sin\theta = \frac{1}{2} m V_f^2$

SOLVE (II) $W = mg$

$\frac{W}{g} = m$

PUT INTO (I) $F_r d - F_f d - \left(\frac{W}{g}\right) g d \sin\theta = \frac{1}{2}\left(\frac{W}{g}\right) V_f^2$

SOLVE (I) $d(F_r - F_f - W\sin\theta) = \frac{W}{2g} V_f^2$

$$\boxed{\sqrt{\frac{2gd(F_r - F_f - W\sin\theta)}{W}} = V_f}$$

CHECK UNITS:

$$= \sqrt{\frac{[\text{FT}/\text{s}^2][\text{FT}]([\text{lb}] - [\text{lb}] - [\text{lb}])}{[\text{lb}]}} = \sqrt{\left[\frac{\text{FT}^2}{\text{s}^2}\right]} = \left[\frac{\text{FT}}{\text{s}}\right] \text{ O.K.}$$

Calculate Target Variable(s)

$$V_f = \sqrt{\frac{2(32\ \text{FT}/\text{s}^2)(328\ \text{FT})(62\ \text{lb} - 25\ \text{lb} - 176\ \text{lb}\sin 12^\circ)}{176\ \text{lb}}}$$

$$\boxed{V_f = 7.0\ \text{FT/s}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF SPEED.

Is Answer Reasonable?

CONVERT TO MPH. $(7.0\ \text{FT/s})\left(\frac{1\ \text{mi}}{5280\ \text{FT}}\right)\left(\frac{3600\ \text{s}}{1\ \text{HR}}\right) = 4.8\ \text{mi/H}$

THIS IS SLIGHTLY FASTER THAN WALKING SPEED, WHICH SEEMS LIKE A REASONABLE SPEED FOR A TOW ROPE.

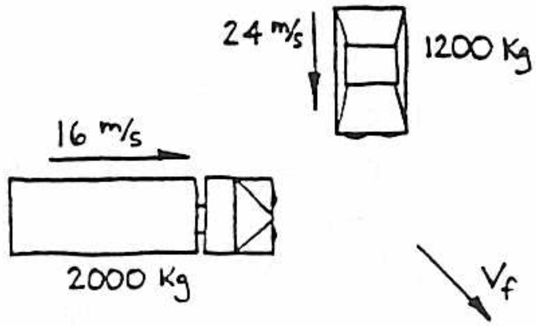
Is Answer Complete?

NO. WE MUST COMPARE THE FINAL SPEED TO 5.0 mi/H. 4.8 mi/HR IS LESS THAN 5.0 mi/HR THEREFORE, THE SKIER WOULD NOT FALL BEFORE BEING PULLED UP THE HILL.

Problem #7: A 1200-kg car traveling south at 24 m/s collides with and attaches itself to a 2000-kg truck traveling east at 16 m/s. Calculate the velocity (magnitude and direction) of the two vehicles when locked together after the collision. Based on Jones & Childers 1992, problem 7.37

FOCUS the PROBLEM

Picture and Given Information



Question(s)

WHAT IS THE VELOCITY (MAGNITUDE AND DIRECTION) OF THE TWO VEHICLES WHEN LOCKED TOGETHER AFTER THE COLLISION?

Approach

USE MOMENTUM CONSERVATION. THE SYSTEM IS THE CAR AND TRUCK.

INITIAL TIME IS THE INSTANT BEFORE VEHICLES COLLIDE.

FINAL TIME IS THE INSTANT AFTER VEHICLES COLLIDE AND MOVE AS ONE.

INITIAL MOMENTUM FROM MOVING CAR AND MOVING TRUCK.

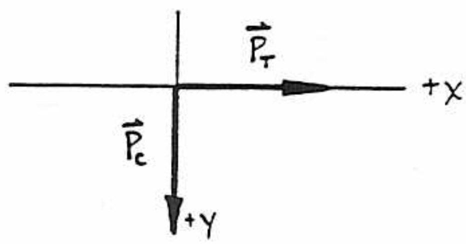
FINAL MOMENTUM FROM THE LOCKED VEHICLES MOVING AS ONE.

NO MOMENTUM TRANSFER FROM THE ENVIRONMENT.

DESCRIBE the PHYSICS

Diagram and Define Variables

INITIAL STATE

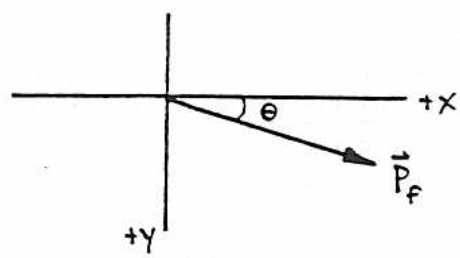


$m_T = 2000$ Kg $\quad m_c = 1200$ Kg

$V_{Txi} = 16$ m/s $\quad V_{cxi} = 0$

$V_{Tyi} = 0$ m/s $\quad V_{cyi} = 24$ m/s

FINAL STATE



$m = 3200$ Kg

$V_f = ?$

$\theta = ?$

Target Variable(s)

$\boxed{\text{FIND } V_f, \theta}$

Quantitative Relationships

$$\vec{P}_f - \vec{P}_i = \vec{P}_{transfer} \quad \begin{cases} P_{fx} - P_{ix} = P_{transfer\ x} \\ P_{fy} - P_{iy} = P_{transfer\ y} \end{cases} \quad P_{transfer} = 0$$

$$P_x = m V_x \qquad P^2 = P_x^2 + P_y^2 \qquad \sin\theta = \frac{P_y}{P}$$

$$P_y = m V_y$$

PLAN the SOLUTION

Construct specific equations

Equation	UNKNOWN
FIND V_f:	V_f
I $\quad P_f = m V_f$	P_f
FIND P_f:	
II $\quad P_f^2 = P_{fx}^2 + P_{fy}^2$	P_{fx}, P_{fy}
FIND P_{fx}:	
$P_{fx} - P_{ix} = P_{transfer\,x}$	
III $\quad P_{fx} - P_{ix} = 0$	P_{ix}
FIND P_{ix}:	
IV $\quad P_{ix} = m_T V_{Txi}$	
FIND P_{fy}:	
$P_{fy} - P_{iy} = P_{transfer\,y}$	
V $\quad P_{fy} - P_{iy} = 0$	P_{iy}
FIND P_{iy}:	
VI $\quad P_{iy} = m_c V_{cyi}$	
FIND θ:	θ
VII $\quad \sin\theta = \frac{P_{fy}}{P_f}$	

Check for sufficiency

7 UNKNOWNS (V_f, P_f, P_{fx}, P_{fy}, P_{ix}, P_{iy}, θ)

7 EQUATIONS (I, II, III, IV, V, VI, VII)

Outline the Math Solution

FIND V_f:

SOLVE VI FOR P_{iy} AND PUT INTO V

SOLVE V FOR P_{fy} AND PUT INTO II

SOLVE IV FOR P_{ix} AND PUT INTO III

SOLVE III FOR P_{fx} AND PUT INTO II

SOLVE II FOR P_f AND PUT INTO I

SOLVE I FOR V_f.

FIND θ:

SOLVE V FOR P_{fy} AND PUT INTO VII

SOLVE II FOR P_f AND PUT INTO VII

SOLVE VII FOR θ.

EXECUTE the PLAN

Follow the Plan

SOLVE VI $\quad P_{iy} = m_c V_{cyi}$

PUT INTO V $\quad P_{fy} - m_c V_{cyi} = 0$

SOLVE V $\quad P_{fy} = m_c V_{cyi}$

PUT INTO II $\quad P_f^2 = P_{fx}^2 + (m_c V_{cyi})^2$

SOLVE IV $\quad P_{ix} = m_T V_{Txi}$

PUT INTO III $\quad P_{fx} - m_T V_{Txi}$

SOLVE III $\quad P_{fx} = m_T V_{Txi}$

PUT INTO II $\quad P_f^2 = (m_T V_{Txi})^2 + (m_c V_{cyi})^2$

SOLVE II $\quad P_f = \sqrt{(m_T V_{Txi})^2 + (m_c V_{cyi})^2}$

PUT INTO I $\quad \sqrt{(M_T V_{Txi})^2 + (m_c V_{cyi})^2} = m V_f$

SOLVE I $\quad \boxed{\frac{\sqrt{(m_T V_{Txi})^2 + (m_c V_{cyi})^2}}{m} = V_f}$

SOLVE V $\quad P_{fy} = m_c V_{cyi}$

SOLVE II $\quad P_f = \sqrt{(m_T V_{Txi})^2 + (m_c V_{cyi})^2}$

PUT INTO VII $\quad \boxed{\sin\theta = \frac{m_c V_{cyi}}{\sqrt{(m_T V_{Txi})^2 + (m_c V_{cyi})^2}}}$

CHECK UNITS:

$$\frac{\sqrt{([Kg][m/s])^2 + ([Kg][m/s])^2}}{[Kg]} = \frac{[Kg][m/s]}{[Kg]} = [m/s] \text{ O.K.}$$

CHECK UNITS:

$$= \frac{[Kg][m/s]}{\sqrt{([Kg][m/s])^2 + ([Kg][m/s])^2}} = \frac{[Kg][m/s]}{[Kg][m/s]} = [\ ] \text{ O.K.}$$

Calculate Target Variable(s)

$$V_f = \frac{\sqrt{((2000\,Kg)(16\,m/s))^2 + ((1200\,Kg)(24\,m/s))^2}}{(3200\,Kg)} = \boxed{13.5\ m/s}$$

$$\sin\theta = \frac{(1200\,Kg)(24\,m/s)}{\sqrt{((2000\,Kg)(16\,m/s))^2 + ((1200\,Kg)(24\,m/s))^2}}$$

$\sin\theta = .67 \qquad \boxed{\theta = 42^\circ}$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF VELOCITY.

Is Answer Reasonable?

YES. THE TWO VEHICLES END UP 42° SOUTH OF EAST. SINCE BOTH VEHICLES HAD ABOUT THE SAME INITIAL MOMENTUM, THE FINAL ANGLE SHOULD BE CLOSE TO 45°.

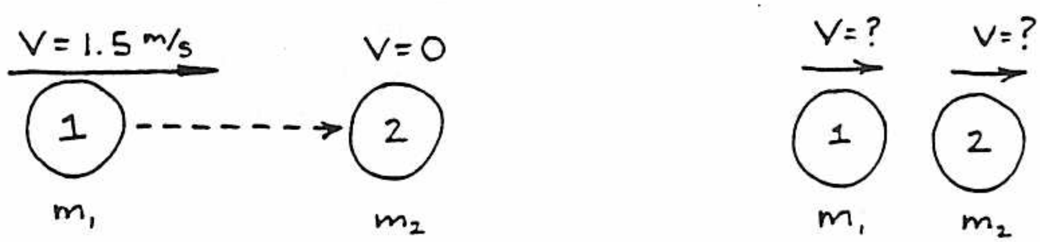
Is Answer Complete?

YES. THE ANSWER INCLUDES BOTH THE MAGNITUDE AND DIRECTION.

Problem #8: A billiard ball at rest is hit head-on by a second billiard ball moving 1.5 m/s toward the east. If the collision is elastic and we ignore rotational motion, calculate the final speed of each ball.

FOCUS the PROBLEM

Picture and Given Information



Question(s) WHAT IS THE FINAL SPEED OF EACH BALL?

Approach USE MOMENTUM CONSERVATION. THE SYSTEM IS THE TWO BILLIARD BALLS.
INITIAL TIME IS THE INSTANT BEFORE THE COLLISION.
FINAL TIME IS THE INSTANT AFTER THE COLLISION.
INITIAL MOMENTUM IS THE MOMENTUM OF THE FIRST BALL.
FINAL MOMENTUM IS THE MOMENTUM OF BOTH BALLS AFTER COLLISION.
ALSO USE ENERGY CONSERVATION.
INITIAL ENERGY IS KINETIC. FINAL ENERGY IS KINETIC.
ASSUME THAT THE MASS OF THE BALLS IS THE SAME.

DESCRIBE the PHYSICS

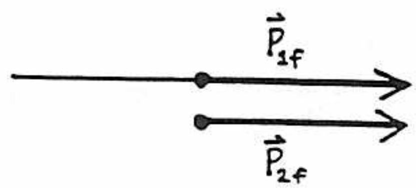
Diagram and Define Variables

INITIAL STATE

$m_1 = m$ $m_2 = m$

$V_{1i} = 1.5$ m/s $V_{2i} = 0$

FINAL STATE



$V_{1f} = ?$ $V_{2f} = ?$

Target Variable(s) $\boxed{\text{FIND } V_{1f},\ V_{2f}}$

Quantitative Relationships

$$P_{fx} - P_{ix} = P_{transfer\,x}$$

$$P_x = m V_x$$

$$P_{transfer\,x} = 0$$

$$E_f - E_i = E_{in} - E_{out}$$

$$E_f = KE_f \qquad E_{in} = 0$$

$$E_i = KE_i \qquad E_{out} = 0$$

$$KE = \frac{1}{2} m V^2$$

PLAN the SOLUTION

Construct specific equations — UNKNOWN

FIND V_{2f}: — V_{2f}

I $E_f = \frac{1}{2}mV_{1f}^2 + \frac{1}{2}mV_{2f}^2$ — E_f, V_{1f}, m

FIND E_f:

$E_f - E_i = E_{in} - E_{out}$

II $E_f - E_i = 0$ — E_i

FIND E_i:

III $E_i = \frac{1}{2}mV_{1i}^2$

FIND V_{1f}:

IV $P_{fx} = mV_{1f} + mV_{2f}$ — P_{fx}

FIND P_{fx}:

$P_{fx} - P_{ix} = P_{inx} - P_{outx}$

V $P_{fx} - P_{ix} = 0$ — P_{ix}

FIND P_{ix}:

VI $P_{ix} = mV_{1i}$

Check for sufficiency

7 UNKNOWNS ($V_{2f}, E_f, V_{1f}, m, E_i, P_{fx}, P_{ix}$)

6 EQUATIONS (I, II, III, IV, V, VI)

MASS IN EACH MOMENTUM EQUATION SHOULD CANCEL

Outline the Math Solution

SOLVE VI FOR P_{ix} AND PUT INTO V

SOLVE V FOR P_{fx} AND PUT INTO IV

SOLVE IV FOR V_{1f} AND PUT INTO I

SOLVE III FOR E_i AND PUT INTO II

SOLVE II FOR E_f AND PUT INTO I

SOLVE I FOR V_{2f}.

TO FIND V_{1f}:

SOLVE I FOR V_{2f} AND PUT INTO IV

SOLVE IV FOR V_{1f}.

EXECUTE the PLAN

Follow the Plan

SOLVE VI: $P_{ix} = mV_{1i}$

PUT INTO V: $P_{fx} - mV_{1i} = 0$

SOLVE V: $P_{fx} = mV_{1i}$

PUT INTO IV: $mV_{1i} = mV_{1f} + mV_{2f}$

SOLVE IV: $\cancel{m}V_{1i} = \cancel{m}V_{1f} + \cancel{m}V_{2f}$

$V_{1i} = V_{1f} + V_{2f}$

$\boxed{V_{1i} - V_{2f} = V_{1f}}$

PUT INTO I: $E_f = \frac{1}{2}m(V_{1i} - V_{2f})^2 + \frac{1}{2}mV_{2f}^2$

SOLVE III: $E_i = \frac{1}{2}mV_{1i}^2$

PUT INTO II: $E_f - \frac{1}{2}mV_{1i}^2 = 0$

SOLVE II: $E_f = \frac{1}{2}mV_{1i}^2$

PUT INTO I: $\cancel{\frac{1}{2}m}V_{1i}^2 = \cancel{\frac{1}{2}m}(V_{1i} - V_{2f})^2 + \cancel{\frac{1}{2}m}V_{2f}^2$

SOLVE I:

$V_{1i}^2 = (V_{1i} - V_{2f})^2 + V_{2f}^2$

$\cancel{V_{1i}^2} = \cancel{V_{1i}^2} - 2V_{1i}V_{2f} + V_{2f}^2 + V_{2f}^2$

$2V_{1i}V_{2f} = 2V_{2f}^2$

$\boxed{V_{1i} = V_{2f}}$

SOLVE I: $V_{2f} = V_{1i}$

PUT INTO IV: $V_{1i} - V_{1i} = V_{1f}$

$\boxed{0 = V_{1f}}$

CHECK UNITS: $= [m/s]$ O.K.

Calculate Target Variable(s)

$$\boxed{\begin{array}{l} V_{1f} = 0 \\ V_{2f} = 1.5\ m/s \end{array}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. BOTH ANSWERS ARE IN UNITS OF VELOCITY.

Is Answer Reasonable?

YES. THE FIRST BALL TRANSFERS ALL OF ITS MOMENTUM TO THE SECOND BALL.

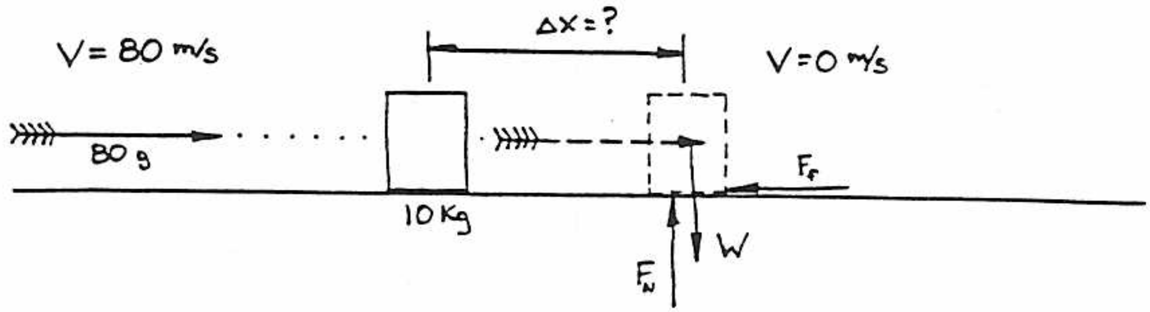
Is Answer Complete?

YES. WE HAVE FOUND THE FINAL SPEED OF BOTH BALLS.

Problem #9: An 80- g arrow moving at 80 m/s hits and embeds in a 10-kg block resting on ice. How far does the block slide on the ice following the collision if it is opposed by a 9.2-N force?
Based on Jones & Childers 1992, problem 7.26

FOCUS the PROBLEM

Picture and Given Information



Question(s) HOW FAR DOES THE BLOCK MOVE AFTER THE ARROW HITS THE BLOCK?

Approach USE MOMENTUM CONSERVATION. DEFINE SYSTEM AS THE ARROW AND BLOCK.
INITIAL TIME IS THE INSTANT BEFORE THE ARROW HITS THE BLOCK.
FINAL TIME IS THE INSTANT AFTER THE ARROW HITS THE BLOCK.
INITIAL MOMENTUM DUE TO ARROW. FINAL MOMENTUM DUE TO ARROW AND BLOCK.
NO MOMENTUM TRANSFER FROM THE ENVIRONMENT.

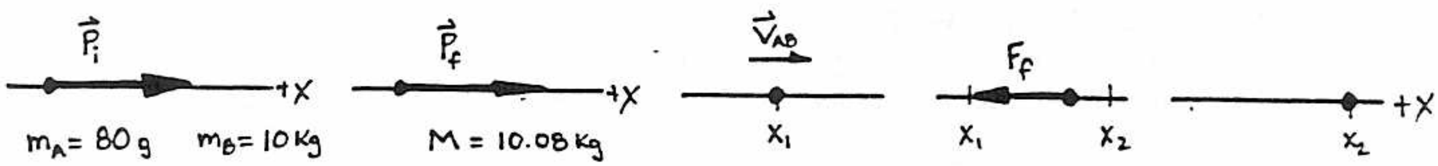
ALSO USE ENERGY CONSERVATION.
INITIAL TIME IS THE INSTANT AFTER THE ARROW HITS THE BLOCK.
FINAL TIME IS THE INSTANT AFTER THE ARROW AND BLOCK STOP MOVING.
INITIAL ENERGY IS KINETIC. FINAL ENERGY IS ZERO.
ENERGY TRANSFER TO ENVIRONMENT THROUGH THE FRICTIONAL FORCE.

DESCRIBE the PHYSICS

Diagram and Define Variables

MOMENTUM DIAGRAM ENERGY DIAGRAM

INITIAL STATE FINAL STATE INITAL STATE TRANSFER FINAL STATE



$m_A = 80\text{ g}$ $m_B = 10\text{ Kg}$ $M = 10.08\text{ Kg}$

$V_{iA} = 80\text{ m/s}$ $V_{iB} = 0$ $V_{AB} = ?$ $X_1 = 0$ $X_2 - X_1 = \Delta X$ $X_2 = ?$

$V_{AB} = ?$ $\Delta X = ?$ $V = 0$

Target Variable(s) $\boxed{\text{FIND } \Delta X}$

Quantitative Relationships

$$P_{fx} - P_{ix} = P_{transferX} \qquad E_f - E_i = E_{in} - E_{out}$$
$$P_{fx} = m_A V \qquad E_f = 0 \quad E_{in} = 0$$
$$P_{ix} = M V \qquad E_i = KE_i \quad E_{out} = F_f \Delta X$$
$$P_{transferx} = 0 \qquad KE = \tfrac{1}{2} m V^2$$

PLAN the SOLUTION

Construct specific equations

	UNKNOWN
FIND ΔX:	ΔX
(I) $E_{out} = F_f \Delta X$	E_{out}
FIND E_{out}:	
$E_f - E_i = E_{in} - E_{out}$	
$0 - E_i = 0 - E_{out}$	
(II) $E_i = E_{out}$	E_i
FIND E_i:	
(III) $E_i = \frac{1}{2} M V_{AB}^2$	V_{AB}
FIND V_{AB}:	
(IV) $P_{fx} = M V_{AB}$	P_{fx}
FIND P_{fx}:	
$P_{fx} - P_{ix} = P_{inx} - P_{outx}$	
(V) $P_{fx} - P_{ix} = 0$	P_{ix}
FIND P_{ix}:	
(VI) $P_{ix} = m_A V_{iA}$	

Check for sufficiency

6 UNKNOWNS (ΔX, E_{out}, E_i, V_{AB}, P_{fx}, P_{ix})
6 EQUATIONS (I, II, III, IV, V, VI)

Outline the Math Solution

SOLVE (VI) FOR P_{ix} AND PUT INTO (V)
SOLVE (V) FOR P_{fx} AND PUT INTO (IV)
SOLVE (IV) FOR V_{AB} AND PUT INTO (III)
SOLVE (III) FOR E_i AND PUT INTO (II)
SOLVE (II) FOR E_{out} AND PUT INTO (I)
SOLVE (I) FOR ΔX.

EXECUTE the PLAN

Follow the Plan

SOLVE (VI) $P_{ix} = m_A V_{iA}$

PUT INTO (V) $P_{fx} - m_A V_{iA} = 0$

SOLVE (V) $P_{fx} = m_A V_{iA}$

PUT INTO (IV) $m_A V_{iA} = M V_{AB}$

SOLVE (IV) $\frac{m_A V_{iA}}{M} = V_{AB}$

PUT INTO (III) $E_i = \frac{1}{2} M \left(\frac{m_A V_{iA}}{M}\right)^2$

SOLVE (III) $E_i = \frac{1}{2} M \left(\frac{m_A V_{iA}}{M}\right)^2$

PUT INTO (II) $\frac{1}{2} M \left(\frac{m_A V_{iA}}{M}\right)^2 = E_{out}$

SOLVE (II) $\frac{1}{2} M \left(\frac{m_A V_{iA}}{M}\right)^2 = E_{out}$

PUT INTO (I) $\frac{1}{2} M \left(\frac{m_A V_{iA}}{M}\right)^2 = F_f \Delta X$

SOLVE (I) $\boxed{\frac{M}{2 F_f} \left(\frac{m_A V_{iA}}{M}\right)^2 = \Delta X}$

CHECK UNITS:

$$= \frac{[Kg]}{[N]} \left(\frac{[Kg][m/s]}{[Kg]}\right)^2$$

$$= \frac{[Kg]}{[Kg][m/s^2]} \left(\frac{[m^2]}{[s^2]}\right)$$

$$= [m] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$\Delta X = \frac{(10.08 \text{ Kg})}{2(9.2 \text{ N})} \left(\frac{(.08 \text{ Kg})(80 \text{ m/s})}{(10.08 \text{ Kg})}\right)^2$$

$$\boxed{\Delta X = .22 \text{ m}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF LENGTH.

Is Answer Reasonable?

YES. SINCE THE BLOCK IS RELATIVELY MASSIVE THE ARROW SHOULDN'T PUSH THE BLOCK VERY FAR.

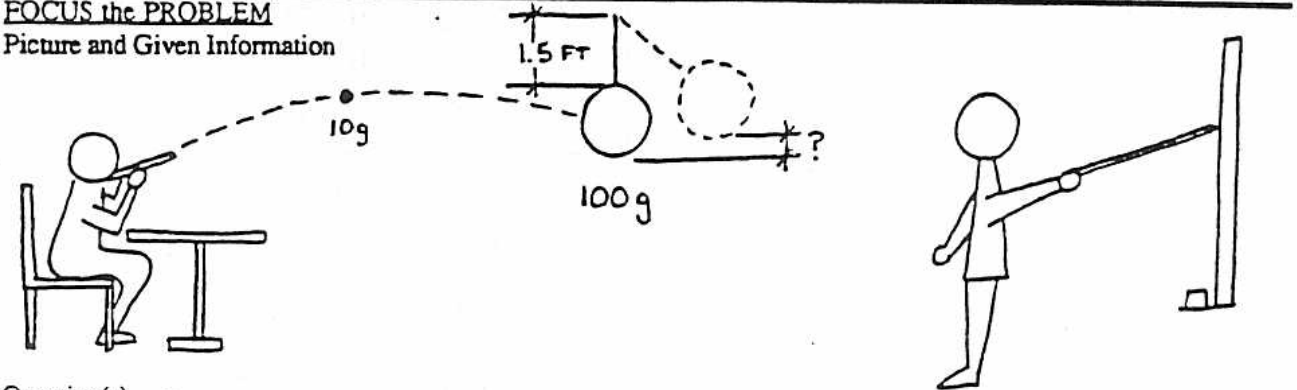
Is Answer Complete?

YES. WE HAVE DETERMINED HOW FAR THE BLOCK WILL MOVE.

Problem #10: A student shot a 10-g spitball in class. The spitball hit and stuck to a 100-g scale model of the moon that was right in front of the teacher. The model was hanging from the ceiling by a 1.5-ft string. The spitball covered the 4.0-m between the student and the model in 0.4-sec. The teacher has the ability to notice vertical displacements of more than 2-cm. Could the teacher have noticed the vertical movement of the model?
Based on Jones & Childers 1992, problem 7.29

FOCUS the PROBLEM

Picture and Given Information



Question(s) DID THE MODEL RISE MORE THAN 2 CM ABOVE ITS RESTING HEIGHT?

Approach USE KINEMATICS TO FIND HORIZONTAL VELOCITY OF SPITBALL.
INITIAL TIME IS INSTANT AFTER LAUNCH. FINAL TIME IS INSTANT BEFORE COLLISION.

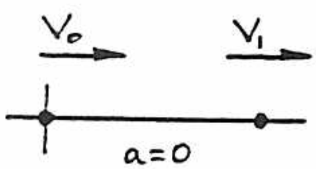
USE MOMENTUM CONSERVATION TO FIND SPEED OF MODEL AND SPITBALL AFTER COLLISION.
SYSTEM IS THE SPITBALL, MOON MODEL, AND EARTH.
INITIAL TIME IS INSTANT BEFORE COLLISION. FINAL TIME IS INSTANT AFTER COLLISION.
INITIAL MOMENTUM IS DUE TO SPITBALL. FINAL MOMENTUM DUE TO SPITBALL AND MODEL.
MOMENTUM TRANSFER FROM SPITBALL TO MODEL IN HORIZONTAL DIRECTION ONLY BECAUSE STRING WILL EXERT AN OPPOSING FORCE IN VERTICAL DIRECTION.

USE ENERGY CONSERVATION TO FIND FINAL HEIGHT.
INITIAL TIME IS INSTANT AFTER LAUNCH.
FINAL TIME IS INSTANT MODEL IS AT ITS MAXIMUM HEIGHT.
INITIAL ENERGY IS KINETIC. FINAL ENERGY IS GRAVITATIONAL POTENTIAL.
NO ENERGY TRANSFER FROM ENVIRONMENT.

DESCRIBE the PHYSICS

Diagram and Define Variables

MOTION DIAGRAM

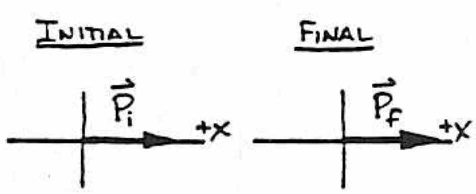


$X_0 = 0$ $\quad$ $x_1 = 4\,m$

$t_0 = 0$ $\quad$ $t_1 = .4\,sec$

$V_0 = 0$ $\quad$ $V_1 = ?$

MOMENTUM DIAGRAM

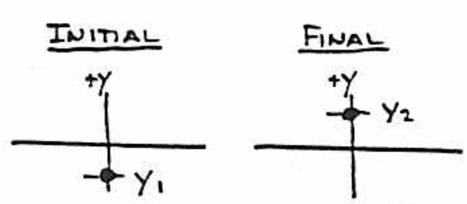


$m_s = 10g$ $\quad$ $M = 110g$

$m_m = 100g$ $\quad$ $V_{sm} = ?$

$V_1 = ?$

ENERGY DIAGRAM



$Y_1 = 0$ $\quad$ $Y_2 = ?$

$V_f = 0$

Target Variable(s) $\boxed{\text{FIND } Y_2}$

Quantitative Relationships

$$\bar{V} = \frac{X_1 - X_0}{t_1 - t_0}$$

$$\bar{V} = V_1 \text{ for } a = 0$$

$$P_{fx} - P_{ix} = P_{transfer\,x}$$

$$P_{fx} = mV_f$$

$$P_{ix} = mV_i$$

$$P_{transfer\,x} = 0$$

$$E_f - E_i = E_{in} - E_{out}$$

$$E_f = GPE_f \qquad E_{in} = 0$$

$$E_i = KE_i \qquad E_{out} = 0$$

$$KE = \tfrac{1}{2}mV^2$$

$$GPE = mgy$$

PLAN the SOLUTION

Construct specific equations

	UNKNOWN
FIND Y_2:	
I $E_f = Mgy_2$	Y_2, E_f
FIND E_f:	
$E_f - E_i = E_{in} - E_{out}$	
II $E_f - E_i = 0$	E_i
FIND E_i:	
III $E_i = \frac{1}{2} M V_{sm}$	V_{sm}
FIND V_{sm}:	
IV $P_{fx} = M V_{sm}$	P_{fx}
FIND P_{fx}:	
$P_{fx} - P_{ix} = P_{transfer\,x}$	
V $P_{fx} - P_{ix} = 0$	P_{ix}
FIND P_{ix}:	
VI $P_{ix} = m_s V_1$	V_1
FIND V_1:	
VII $V_1 = \bar{V}$	$\bar{V}$
FIND $\bar{V}$:	
$\bar{V} = \frac{x_1 - x_0}{t_1 - t_0}$	
VIII $\bar{V} = \frac{x_1}{t_1}$	

Check for sufficiency

8 UNKNOWNS ($Y_2, E_f, E_i, V_{sm}, P_{fx}, P_{ix}, V_1, \bar{V}$)

8 EQUATIONS (I, II, III, IV, V, VI, VII, VIII)

Outline the Math Solution

SOLVE VIII FOR $\bar{V}$ AND PUT INTO VII

SOLVE VII FOR V_1 AND PUT INTO VI

SOLVE VI FOR P_{ix} AND PUT INTO V

SOLVE V FOR P_{fx} AND PUT INTO IV

SOLVE IV FOR V_{sm} AND PUT INTO III

SOLVE III FOR E_i AND PUT INTO II

SOLVE II FOR E_f AND PUT INTO I

SOLVE I FOR Y_2.

EXECUTE the PLAN

Follow the Plan

SOLVE VIII $\bar{V} = \frac{x_1}{t_1}$

PUT INTO VII $V_1 = \frac{x_1}{t_1}$

SOLVE VII $V_1 = \frac{x_1}{t_1}$

PUT INTO VI $P_{ix} = m_s\left(\frac{x_1}{t_1}\right)$

SOLVE VI $P_{ix} = m_s\left(\frac{x_1}{t_1}\right)$

PUT INTO V $P_{fx} - m_s\left(\frac{x_1}{t_1}\right) = 0$

SOLVE V $P_{fx} = m_s\left(\frac{x_1}{t_1}\right)$

PUT INTO IV $m_s\left(\frac{x_1}{t_1}\right) = M V_{sm}$

SOLVE IV $\frac{m_s}{M}\left(\frac{x_1}{t_1}\right) = V_{sm}$

PUT INTO III $E_i = \frac{1}{2} M \left(\frac{m_s x_1}{M t_1}\right)^2$

SOLVE III $E_i = \frac{1}{2} M \left(\frac{m_s x_1}{M t_1}\right)^2$

PUT INTO II $E_f - \frac{1}{2} M \left(\frac{m_s x_1}{M t_1}\right)^2$

SOLVE II $E_f = \frac{1}{2} M \left(\frac{m_s x_1}{M t_1}\right)^2$

PUT INTO I $\frac{1}{2}\cancel{M}\left(\frac{m_s x_1}{M t_1}\right)^2 = \cancel{M} g y_2$

SOLVE I $\boxed{\frac{1}{2g}\left(\frac{m_s x_1}{M t_1}\right)^2 = Y_2}$

CHECK UNITS:

$$= \frac{1}{[m/s^2]}\left(\frac{[kg][m]}{[kg][s]}\right)^2$$

$$= \frac{[m]^2/[s]^2}{[m]/[s]^2} = [m] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$Y_2 = \frac{1}{2(9.8\,m/s^2)}\left(\frac{(10g)(4.0m)}{(110g)(.4\,sec)}\right)^2$$

$$\boxed{Y_2 = .042\text{ m} = 4.2\text{ cm}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF LENGTH.

Is Answer Reasonable?

YES. WE OULD NOT EXPECT A SPITBALL TO MOVE A MODEL VERY MUCH.

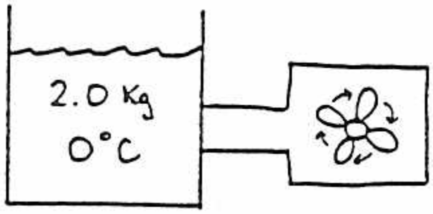
Is Answer Complete?

NO. WE HAVE NOT ANSWERED THE QUESTION. THE TEACHER HAS THE ABILITY TO NOTICE VERTICAL DISPLACEMENTS OF MORE THAN 2.0 cm. SINCE THE MODEL ROSE 4.2 cm, THE TEACHER COULD HAVE NOTICED THE MOVEMENT OF THE MODEL.

Problem #11: An ice-making machine removes heat from 0 degrees Celsius water at a rate of 280 J/s. Calculate the time needed to form 2.0 kg of ice at 0 degrees Celsius. Similar to Jones & Childers 1992, prob. 11.47

FOCUS the PROBLEM

Picture and Given Information



REMOVES HEAT AT A RATE OF 280 J/s.

Question(s) HOW MUCH TIME DOES IT TAKE TO CONVERT 2 kg OF 0°C LIQUID WATER TO 2 Kg OF 0°C ICE?

Approach USE ENERGY CONSERVATION. SYSTEM IS THE WATER.
INITIAL TIME IS THE INSTANT THE ICE-MAKER IS SWITCHED ON.
FINAL TIME IS THE INSTANT ALL OF THE LIQUID WATER TURNS TO ICE.
INITIAL ENERGY IS INTERNAL. FINAL ENERGY IS INTERNAL.
ENERGY TRANSFER TO ENVIRONMENT VIA ICE-MAKER.
ASSUME WATER ONLY LOSES HEAT TO ICE-MAKER AND THERE IS NO EXTERNAL HEAT LOSS.

DESCRIBE the PHYSICS

Diagram and Define Variables

INITIAL STATE	ENERGY TRANSFER	FINAL STATE
(LIQUID)	E_{out} $r = 280$ J/s	(SOLID)
$m = 2.0$ Kg		$m = 2.0$ Kg
$T_i = 0°C$		$T_f = 0°C$
		$L_f = 3.3 \times 10^5$ J/Kg

Target Variable(s) $\boxed{\text{FIND } \Delta t}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$$E_i = (IE_{ice})_i \qquad E_{in} = 0$$

$$E_f = (IE_{ice})_f \qquad E_{out} = r\Delta t$$

$$\Delta IE_{ice} = (IE_{ice})_f - (IE_{ice})_i$$

$$\Delta IE_{ice} = -mL_f$$

PLAN the SOLUTION
Construct specific equations

UNKNOWN

FIND Δt: — Δt

I $E_{out} = r\Delta t$ — E_{out}

FIND E_{out}:

$E_f - E_i = E_{in} - E_{out}$

$E_f - E_i = 0 - E_{out}$

II $E_f - E_i = -E_{out}$ — E_f, E_i

FIND E_f:

III $E_f - E_i = \Delta IE_{ice}$ — ΔIE_{ice}

FIND ΔIE_{ice}:

IV $\Delta IE_{ice} = -mL_f$

Check for sufficiency

5 UNKNOWNS (Δt, E_{out}, E_f, E_i, ΔIE_{ice})

4 EQUATIONS (I, II, III, IV)

TWO EQUATIONS CONTAIN E_f AND E_i. HOPEFULLY ONE WILL CANCEL OUT.

Outline the Math Solution

SOLVE IV FOR ΔIE_{ice} AND PUT INTO III

SOLVE III FOR E_f AND PUT INTO II

SOLVE II FOR E_{out} AND PUT INTO I

SOLVE I FOR Δt.

EXECUTE the PLAN
Follow the Plan

SOLVE IV $\Delta IE_{ice} = -mL_f$

PUT INTO III $E_f - E_i = -mL_f$

SOLVE III $E_f = E_i - mL_f$

PUT INTO II $\cancel{E_i} - mL_f - \cancel{E_i} = -E_{out}$

SOLVE II $-mL_f = -E_{out}$

$mL_f = E_{out}$

PUT INTO I $mL_f = r\Delta t$

$$\boxed{\frac{mL_f}{r} = \Delta t}$$

CHECK UNITS:

$$= \frac{[Kg][J/Kg]}{[J/s]} = \frac{[\cancel{Kg}][\cancel{J}][s]}{[\cancel{Kg}][\cancel{J}]} = [s] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$\Delta t = \frac{(2.0\,Kg)(3.3\times10^5\,J/Kg)}{(280\,J/s)} = 2357\ \text{sec}$$

$$\Delta t = (2357\ \text{sec})\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = \boxed{39\ \text{min} = \Delta t}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF TIME.

Is Answer Reasonable?

YES. I WOULD EXPECT TO WAIT OVER A HALF HOUR TO MAKE OVER 4 POUNDS OF ICE (2 Kg).

Is Answer Complete?

YES. WE HAVE FOUND THE AMOUNT OF TIME NEEDED TO CONVERT 2 Kg OF LIQUID WATER TO ICE.

Problem #12: Calculate the amount of energy needed to change a 0.50-kg block of ice at 0 degrees Celsius into water at 20 degrees Celsius. Similar to Jones & Childers 1992, problem 11.38

FOCUS the PROBLEM

Picture and Given Information

Question(s) HOW MUCH ENERGY MUST BE ADDED TO 0.50 Kg OF ICE AT 0°C TO OBTAIN WATER AT 20°C?

Approach USE ENERGY CONSERVATION. THE SYSTEM IS THE WATER.

INITIAL TIME IS THE INSTANT ENERGY IS ADDED TO THE SYSTEM.
FINAL TIME IS THE INSTANT THE WATER REACHES 20°C.

INITIAL ENERGY IS INTERNAL. FINAL ENERGY IS INTERNAL.

ENERGY TRANSFER FROM ENVIRONMENT TO THE SYSTEM.

ASSUME NO ENERGY TRANSFER FROM THE SYSTEM.

DESCRIBE the PHYSICS

Diagram and Define Variables

INITIAL STATE

(ICE)

$T_i = 0°C$

$m = 0.50$ Kg

$C_{water} = 4180$ J/Kg °C

$L_f = 3.35 \times 10^5$ J/Kg

ENERGY INPUT

Q_{input}

FINAL STATE

(LIQUID)

$T_f = 20°C$

Target Variable(s) $\boxed{Q_{input}}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$$E_i = (IE_{water})_i + (IE_{ice})_i \qquad E_{in} = Q_{input}$$

$$E_f = (IE_{water})_f + (IE_{ice})_f \qquad E_{out} = 0$$

$$\Delta IE(KE) = mC\Delta T$$

$$\Delta IE(PE) = mL_f$$

PLAN the SOLUTION

Construct specific equations

	UNKNOWN
FIND Q_{input}:	
I $E_{in} = Q_{input}$	Q_{input}, E_{in}
FIND E_{in}:	
$E_f - E_i = E_{in} - E_{out}$	
II $E_f - E_i = E_{in}$	E_f, E_i
FIND E_f:	
III $E_f - E_i = \Delta IE_{ice} + \Delta IE_{water}$	ΔIE_{ice}, ΔIE_{water}
FIND ΔIE_{ice}:	
IV $\Delta IE_{ice} = \Delta IE(PE)_{ice}$	$\Delta IE(PE)_{ice}$
FIND $\Delta IE(PE)_{ice}$:	
V $\Delta IE(PE)_{ice} = mL_f$	
FIND ΔIE_{water}:	
VI $\Delta IE_{water} = \Delta IE(KE)_{water}$	$\Delta IE(KE)_{water}$
FIND $\Delta IE(KE)_{water}$:	
VII $\Delta IE(KE)_{water} = mC_{water}\Delta T$	

Check for sufficiency

8 UNKNOWNS ($Q_{input}, E_{in}, E_f, E_i, \Delta IE_{ice}, \Delta IE_{water}, \Delta IE(PE)_{ice}, \Delta IE(KE)_{water}$)

7 EQUATIONS (I, II, III, IV, V, VI, VII)

TWO EQUATIONS CONTAIN E_f AND E_i. HOPEFULLY ONE WILL CANCEL OUT.

Outline the Math Solution

SOLVE VII FOR $\Delta IE(KE)_{water}$ AND PUT INTO VI

SOLVE VI FOR ΔIE_{water} AND PUT INTO III

SOLVE V FOR $\Delta IE(PE)_{ice}$ AND PUT INTO IV

SOLVE IV FOR ΔIE_{ice} AND PUT INTO III

SOLVE III FOR E_f AND PUT INTO II

SOLVE II FOR E_{in} AND PUT INTO I

SOLVE I FOR Q_{input}.

EXECUTE the PLAN

Follow the Plan

SOLVE VII	$\Delta IE(KE)_{water} = mC_{water}\Delta T$
PUT INTO VI	$\Delta IE_{water} = mC_{water}\Delta T$
SOLVE VI	$\Delta IE_{water} = mC_{water}\Delta T$
PUT INTO III	$E_f - E_i = \Delta IE_{ice} + mC_{water}\Delta T$
SOLVE V	$\Delta IE(PE)_{ice} = mL_f$
PUT INTO IV	$\Delta IE_{ice} = mL_f$
SOLVE IV	$\Delta IE_{ice} = mL_f$
PUT INTO III	$E_f - E_i = mL_f + mC_{water}\Delta T$
SOLVE III	$E_f = mL_f + mC_{water}\Delta T + E_i$
PUT INTO II	$mL_f + mC_{water}\Delta T + E_i - E_i = E_{in}$
SOLVE II	$mL_f + mC_{water}\Delta T = E_{in}$
PUT INTO I	$mL_f + mC_{water}\Delta T = Q_{input}$
SOLVE I	$\boxed{m(L_f + C_{water}\Delta T) = Q_{input}}$

CHECK UNITS:

$$= [\text{Kg}]\left([\text{J/Kg}] + [\text{J/Kg}\cdot{}^\circ\text{C}][{}^\circ\text{C}]\right)$$

$$= [\text{Kg}]([\text{J/Kg}])$$

$$= [\text{J}] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$Q_{input} = (.50\,\text{Kg})\left[(3.35\times10^5\,\text{J/Kg}) + (4180\,\text{J/Kg}\cdot{}^\circ\text{C})(20\,{}^\circ\text{C})\right]$$

$$\boxed{Q_{input} = 2.1\times10^5\ \text{J}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN JOULES.

Is Answer Reasonable?

YES. PHASE CHANGES TAKE MUCH MORE ENERGY PER Kg THAN SIMPLY RAISING THE TEMPERATURE. THEREFORE, THE ANSWER SHOULD BE CLOSE TO THE PHASE CHANGE ENERGY PER Kg.

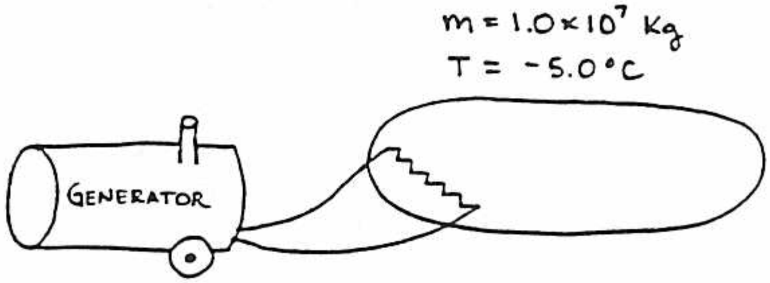
Is Answer Complete?

YES. WE DETERMINED HOW MUCH ENERGY IS NEEDED TO CHANGE A .50 Kg BLOCK OF ICE AT 0°C TO WATER AT 20°C.

Problem #13: The 1.0×10^7 kg of ice in a small pond has an average temperature of -5.0 degrees Celsius during the middle of winter. A movie making company wants to convert the pond to 100 degree Celsius steam for a movie special effect. How much heat must they add to the frozen pond? Similar to Jones & Childers 1992, prob.11.

FOCUS the PROBLEM

Picture and Given Information



Question(s)

HOW MUCH HEAT MUST BE ADDED TO THE POND IN ORDER TO TURN IT INTO 100°C STEAM?

Approach

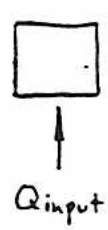
USE ENERGY CONSERVATION. SYSTEM IS THE POND AND WATER.

INITIAL TIME IS THE INSTANT BEFORE GENERATOR IS TURNED ON.
FINAL TIME IS THE INSTANT THAT ALL THE LIQUID WATER HAS TURNED TO STEAM.

INITIAL ENERGY IS INTERNAL. FINAL ENERGY IS INTERNAL.
ENERGY INPUT FROM ENVIRONMENT DUE TO HEAT FROM GENERATOR.
ASSUME NO HEAT TRANSFER FROM SYSTEM TO ENVIRONMENT.

DESCRIBE the PHYSICS

Diagram and Define Variables



INITIAL STATE:
$T = -5°C$
$m = 1.0 \times 10^7$ Kg
$L_f = 3.3 \times 10^5$ J/Kg
$C_{ice} = 2090$ J/Kg °C

FINAL STATE:
$T = 100°C$
$L_v = 2.3 \times 10^6$ J/Kg
$C_{water} = 4180$ J/Kg °C

Target Variable(s)

$\boxed{Q_{input}}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$
$$E_i = (IE_{ice})_i + (IE_{water})_i + (IE_{steam})_i$$
$$E_f = (IE_{ice})_f + (IE_{water})_f + (IE_{steam})_f$$
$$\Delta IE = (IE)_f - (IE)_i$$
$$\Delta IE(KE) = mC\Delta T$$
$$\Delta IE(PE) = mL$$

$$E_{in} = Q_{input}$$
$$E_{out} = 0$$

PLAN the SOLUTION

Construct specific equations

UNKNOWN

FIND Q_{input}:

I $Q_{input} = E_{in}$ — Q_{input}, E_{in}

FIND E_{in}:

$$\Delta IE_{ice} + \Delta IE_{water} + \Delta IE_{steam} = E_{in} - E_{out}$$

II $\Delta IE_{ice} + \Delta IE_{water} + \Delta IE_{steam} = E_{in}$ — ΔIE_{ice}, ΔIE_{water}, ΔIE_{steam}

FIND ΔIE_{ice}:

III $\Delta IE_{ice} = \Delta IE(KE)_{ice} + \Delta IE(PE)_{ice}$ — $\Delta IE(KE)_{ice}$, $\Delta IE(PE)_{ice}$

FIND $\Delta IE(KE)_{ice}$:

IV $\Delta IE(KE)_{ice} = m C_{ice} \Delta T_{ice}$

FIND $\Delta IE(PE)_{ice}$:

V $\Delta IE(PE)_{ice} = m L_f$

FIND ΔIE_{water}:

VI $\Delta IE_{water} = \Delta IE(KE)_{water}$ — $\Delta IE(KE)_{water}$

FIND $\Delta IE(KE)_{water}$:

VII $\Delta IE(KE)_{water} = m C_{water} \Delta T_{water}$

FIND ΔIE_{steam}:

VIII $\Delta IE_{steam} = \Delta IE(PE)_{steam}$ — $\Delta IE(PE)_{steam}$

FIND $\Delta IE(PE)_{steam}$:

IX $\Delta IE(PE)_{steam} = m L_v$

Check for sufficiency

9 UNKNOWNS (Q_{input}, E_{in}, ΔIE_{ice}, ΔIE_{water}, ΔIE_{steam}, $\Delta IE(KE)_{ice}$, $\Delta IE(PE)_{ice}$, $\Delta IE(KE)_{water}$, $\Delta IE(PE)_{steam}$)

9 EQUATIONS (I, II, III, IV, V, VI, VII, VIII, IX)

Outline the Math Solution

SOLVE IX FOR $\Delta IE(PE)_{steam}$ AND PUT INTO VIII

SOLVE VIII FOR ΔIE_{steam} AND PUT INTO II

SOLVE VII FOR $\Delta IE(KE)_{water}$ AND PUT INTO VI

SOLVE VI FOR ΔIE_{water} AND PUT INTO II

SOLVE V FOR $\Delta IE(PE)_{ice}$ AND PUT INTO III

SOLVE IV FOR $\Delta IE(KE)_{ice}$ AND PUT INTO III

SOLVE III FOR ΔIE_{ice} AND PUT INTO II

SOLVE II FOR E_{in} AND PUT INTO I

SOLVE I FOR Q_{input}.

EXECUTE the PLAN

Follow the Plan

SOLVE IX $\Delta IE(PE)_{steam} = m L_v$

PUT INTO VIII $\Delta IE_{steam} = m L_v$

SOLVE VIII $\Delta IE_{steam} = m L_v$

PUT INTO II $\Delta IE_{ice} + \Delta IE_{water} + m L_v = E_{in}$

SOLVE VII $\Delta IE(KE)_{water} = m C_{water} \Delta T_{water}$

PUT INTO VI $\Delta IE_{water} = m C_{water} \Delta T_{water}$

SOLVE VI $\Delta IE_{water} = m C_{water} \Delta T_{water}$

PUT INTO II $\Delta IE_{ice} + m C_{water} \Delta T_{water} + m L_v = E_{in}$

SOLVE V $\Delta IE(PE)_{ice} = m L_f$

PUT INTO III $\Delta IE_{ice} = \Delta IE(KE)_{ice} + m L_f$

SOLVE IV $\Delta IE(KE)_{ice} = m C_{ice} \Delta T_{ice}$

PUT INTO III $\Delta IE_{ice} = m C_{ice} \Delta T_{ice} + m L_f$

SOLVE III $\Delta IE_{ice} = m C_{ice} \Delta T_{ice} + m L_f$

PUT INTO II $m C_{ice} \Delta T_{ice} + m L_f + m C_{water} \Delta T_{water} + m L_v = E_{in}$

SOLVE II $m C_{ice} \Delta T_{ice} + m L_f + m C_{water} \Delta T_{water} + m L_v = E_{in}$

PUT INTO I $m C_{ice} \Delta T_{ice} + m L_f + m C_{water} \Delta T_{water} + m L_v = Q_{input}$

SOLVE I $m C_{ice} \Delta T_{ice} + m L_f + m C_{water} \Delta T_{water} + m L_v = Q_{input}$

$$\boxed{m(C_{ice} \Delta T_{ice} + L_f + C_{water} \Delta T_{water} + L_v) = Q_{input}}$$

CHECK UNITS:

$$= [\mathrm{Kg}]([\mathrm{J/Kg\,^\circ C}][^\circ\mathrm{C}] + [\mathrm{J/Kg}] + [\mathrm{J/Kg\,^\circ C}][^\circ\mathrm{C}] + [\mathrm{J/Kg}])$$

$$= [\mathrm{Kg}]([\mathrm{J/Kg}]) = [\mathrm{J}] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$Q_{input} = (1.0\times10^7\,\mathrm{Kg})\big[(2090\,\mathrm{J/Kg\,^\circ C})(5^\circ\mathrm{C}) + (3.3\times10^5\,\mathrm{J/Kg}) + (4180\,\mathrm{J/Kg\,^\circ C})(100^\circ\mathrm{C}) + (2.3\times10^6\,\mathrm{J/Kg})\big]$$

$$\boxed{Q_{input} = 3.1\times10^{13}\,\mathrm{J}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF ENERGY.

Is Answer Reasonable?

YES. IT TAKES ABOUT TEN TIMES MORE ENERGY PER Kg TO CONVERT WATER TO STEAM THAN TO CONVERT ICE TO WATER. OUR ANSWER IS ABOUT TEN TIMES GREATER THAN EXAMPLE 9.6

Is Answer Complete?

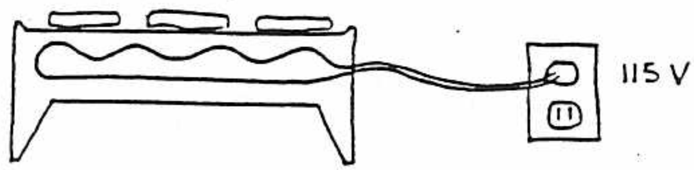
YES. WE HAVE DETERMINED HOW MUCH HEAT MUST BE ADDED TO THE POND.

Problem #14: An electric grill made of iron has a specific heat of 460 J/kg·C and a mass of 2.8-kg. To cook French toast, the grill is warmed from 20 to 350 degrees Celsius by resistive heating wires that produce thermal energy at a rate of 1500 W when connected to a 115-V potential difference. Fifty percent of the thermal energy is radiated into the room as the grill warms. How many minutes are required to warm the grill?

FOCUS the PROBLEM

Picture and Given Information

$m = 2.8$ Kg
$C = 460$ J/kg·C



Question(s) HOW MANY MINUTES DOES IT TAKE TO WARM THE GRILL?

Approach USE ENERGY CONSERVATION. SYSTEM IS THE COOKING SURFACE OF THE GRILL.

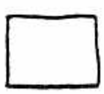
INITIAL TIME IS THE INSTANT AFTER THE GRILL IS SWITCHED ON.
FINAL TIME IS THE INSTANT THE GRILL REACHES 350°C.

INITIAL ENERGY IS INTERNAL. FINAL ENERGY IS INTERNAL.
ENERGY TRANSFER FROM ELECTRIC HEATING COIL TO GRILL SURFACE.
50% OF THE THERMAL ENERGY RADIATES TO THE ROOM.

DESCRIBE the PHYSICS

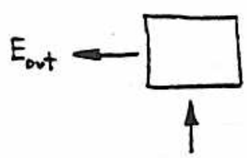
Diagram and Define Variables

INITIAL STATE



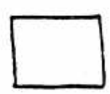
$m = 2.8$ Kg
$T_i = 20°C$
$C_{iron} = 460$ J/kg·C

ENERGY TRANSFER



$P = 1500W$

FINAL STATE



$T_f = 350°C$

Target Variable(s) $\boxed{\text{FIND } \Delta t}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$E_i = (1E_{iron})_i$ $E_{in} = \Delta E_{electric}$
$E_f = (1E_{iron})_f$ $E_{out} = .5\,E_{in}$

$$\frac{\Delta E_{electric}}{\Delta t} = P$$

PLAN the SOLUTION

Construct specific equations

	UNKNOWN
FIND Δt:	Δt
I $\frac{\Delta E_{electric}}{\Delta t} = P$	$\Delta E_{electric}$
FIND $\Delta E_{electric}$:	
II $\Delta E_{electric} = E_{in}$	E_{in}
FIND E_{in}:	
III $E_f - E_i = E_{in} - E_{out}$	E_f, E_i, E_{out}
FIND E_f:	
IV $E_f - E_i = \Delta IE(KE)_{iron}$	$\Delta IE(KE)_{iron}$
FIND $\Delta IE(KE)_{iron}$:	
V $\Delta IE(KE)_{iron} = mC_{iron}\Delta T$	
FIND E_{out}:	
VI $E_{out} = .5E_{in}$	

Check for sufficiency

7 UNKNOWNS ($\Delta t, \Delta E_{electric}, E_{in}, E_f, E_i, E_{out}, \Delta IE(KE)_{iron}$)

6 EQUATIONS (I, II, III, IV, V, VI)

SINCE E_f AND E_i ARE IN TWO EQUATIONS HOPEFULLY ONE WILL CANCEL OUT.

Outline the Math Solution

SOLVE VI FOR E_{out} AND PUT INTO III

SOLVE V FOR $\Delta IE(KE)_{iron}$ AND PUT INTO IV

SOLVE IV FOR E_f AND PUT INTO III

SOLVE III FOR E_{in} AND PUT INTO II

SOLVE II FOR $\Delta E_{electric}$ AND PUT INTO I

SOLVE I FOR Δt.

EXECUTE the PLAN

Follow the Plan

SOLVE VI $E_{out} = .5E_{in}$

PUT INTO III $E_f - E_i = E_{in} - .5E_i$

$E_f - E_i = .5E_{in}$

SOLVE V $\Delta IE(KE)_{iron} = mC_{iron}\Delta T$

PUT INTO IV $E_f - E_i = mC_{iron}\Delta T$

SOLVE IV $E_f = mC_{iron}\Delta T + E_i$

PUT INTO III $mC_{iron}\Delta T + E_i - E_i = .5E_{in}$

SOLVE III $mC_{iron}\Delta T = .5E_{in}$

$2mC_{iron}\Delta T = E_{in}$

PUT INTO II $\Delta E_{electric} = 2mC_{iron}\Delta T$

SOLVE II $\Delta E_{electric} = 2mC_{iron}\Delta T$

PUT INTO I $\frac{2mC_{iron}\Delta T}{\Delta t} = P$

SOLVE I $\boxed{\frac{2mC_{iron}\Delta T}{P} = \Delta t}$

CHECK UNITS:

$$= \frac{[Kg][J/kg\cdot c][°c]}{[W]}$$

$$= \frac{[J]}{[W]} = \frac{[J]}{[J/s]} = [s] \quad \text{O.K.}$$

Calculate Target Variable(s)

$$\Delta t = \frac{2(2.8\text{ Kg})(460\text{ J/kg}\cdot°c)(330°c)}{(1500\text{ W})} = 570\text{ SEC}$$

$$\Delta t = (570\text{ sec})\left(\frac{1\text{ min}}{60\text{ SEC}}\right) = \boxed{9.5\text{ min} = \Delta t}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

YES. THE ANSWER IS IN UNITS OF TIME.

Is Answer Reasonable?

YES. I WOULD EXPECT TO WAIT 9.5 MINUTES FOR A GRILL TO WARM UP.

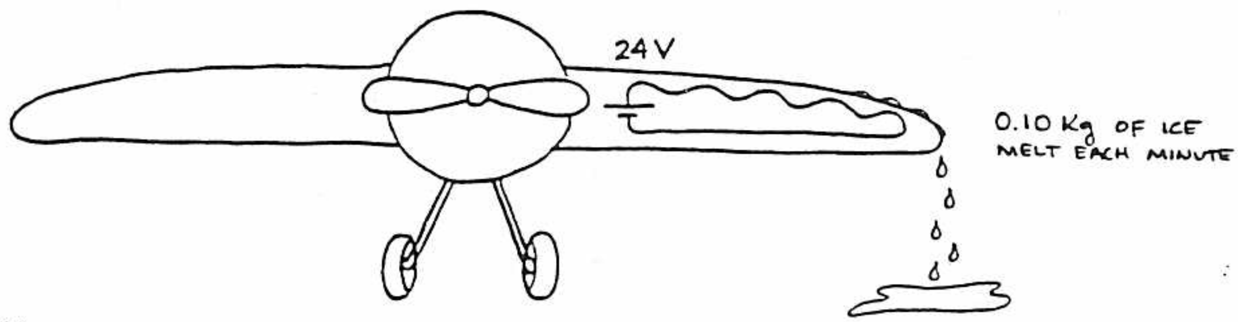
Is Answer Complete?

YES. WE DETERMINED HOW LONG IT TAKES FOR THE GRILL TO WARM UP.

Problem #15: An airplane deicer melts 0.10-kg of ice from the wings of an airplane each minute. The deicer consists of resistive heating wires connected to a 24-V battery. Calculate the current through the heating wires and their resistance. Assume that the deicer transfer 100 percent of its energy to the ice.

FOCUS the PROBLEM

Picture and Given Information



Question(s) WHAT IS THE CURRENT AND RESISTANCE IN THE HEATING WIRES?

Approach

USE ENERGY CONSERVATION. THE SYSTEM IS A 0.10 Kg PIECE OF ICE.

INITIAL TIME IS THE INSTANT THE HEATING WIRES ARE WARMED UP. FINAL TIME IS EXACTLY ONE MINUTE AFTER INITIAL TIME.

INITIAL ENERGY IS INTERNAL. FINAL ENERGY IS INTERNAL. ENERGY TRANSFER TO SYSTEM FROM HEATING WIRES.

USE ELECTRICAL TRANSFER RELATIONS AND OHM'S LAW.

ASSUME ALL ELECTRICAL ENERGY FROM THE HEATER GOES TO MELTING THE ICE.

DESCRIBE the PHYSICS

Diagram and Define Variables

INITIAL STATE	ENERGY TRANSFER	FINAL STATE
(SOLID)		(LIQUID)
$m = 0.10\ \text{Kg}$	$E_{in} = \Delta E_{electric}$	$T_f = 0°C$
$T_i = 0°C$		$t_f = 1\ \text{min}$
$L_f = 3.3 \times 10^5\ \text{J/Kg}$		
$t_i = 0$		

Target Variable(s) $\boxed{\text{FIND } R \ \& \ I}$

Quantitative Relationships

$$E_f - E_i = E_{in} - E_{out}$$

$$E_f = (IE_{ice})_f \qquad E_{in} = \Delta E_{electric}$$

$$E_i = (IE_{ice})_i \qquad E_{out} = 0$$

$$\Delta IE_{ice} = m L_f$$

$$\frac{\Delta E_{electric}}{\Delta t} = \frac{(\Delta V)^2}{R} \qquad \Delta V = IR$$

PLAN the SOLUTION

Construct specific equations

	Unknowns
Find I:	I
(I) $\Delta V = IR$	R
Find R:	
(II) $\dfrac{\Delta E_{electric}}{\Delta t} = \dfrac{(\Delta V)^2}{R}$	$\Delta E_{electric}$
Find $\Delta E_{electric}$:	
(III) $E_{in} = \Delta E_{electric}$	E_{in}
Find E_{in}:	
$E_f - E_i = E_{in} - E_{out}$	
(IV) $E_f - E_i = E_{in}$	E_f, E_i
Find E_f:	
(V) $E_f - E_i = \Delta IE_{ice}$	ΔIE_{ice}
Find ΔIE_{ice}:	
(VI) $\Delta IE_{ice} = mL_f$	

Check for sufficiency

7 Unknowns ($I, R, \Delta E_{electric}, E_{in}, E_f, E_i, \Delta IE_{ice}$)

6 Equations (I, II, III, IV, V, VI)

Two equations contain E_f and E_i. Hopefully one will cancel out.

Outline the Math Solution

Solve (VI) for ΔIE_{ice} and put into (V)

Solve (V) for E_f and put into (IV)

Solve (IV) for E_{in} and put into (III)

Solve (III) for $\Delta E_{electric}$ and put into (II)

Solve (II) for R and put into (I)

Solve (I) for I.

EXECUTE the PLAN

Follow the Plan

Solve (VI) $\Delta IE_{ice} = mL_f$

Put into (V) $E_f - E_i = mL_f$

Solve (V) $E_f = mL_f + E_i$

Put into (IV) $mL_f + E_i - E_i = E_{in}$

Solve (IV) $mL_f = E_{in}$

Put into (III) $mL_f = \Delta E_{electric}$

Solve (III) $mL_f = \Delta E_{electric}$

Put into (II) $\dfrac{mL_f}{\Delta t} = \dfrac{(\Delta V)^2}{R}$

Solve (II) $\boxed{R = \dfrac{\Delta t(\Delta V)^2}{mL_f}}$

Put into (I) $\Delta V = I\left(\dfrac{\Delta t(\Delta V)^2}{mL_f}\right)$

Solve (I) $\Delta V = \dfrac{I\Delta t(\Delta V)^2}{mL_f}$

$$\frac{\Delta V m L_f}{\Delta t(\Delta V)^2} = I$$

$$\boxed{\frac{mL_f}{\Delta t \Delta V} = I}$$

Check units:

$I = \dfrac{[kg][J/kg]}{[sec][V]}$ $\qquad R = \dfrac{[sec][V]^2}{[kg][J/kg]}$

$I = [J/V\cdot sec] = [amp]$ O.K. $\qquad R = \left[\dfrac{V^2 sec}{J}\right] = [ohm]$ O.K.

Calculate Target Variable(s)

$$I = \frac{(.10\,kg)(3.3\times10^5\,J/kg)}{(60\,sec)(24\,V)} = \boxed{23\,J/V\cdot sec = I}$$

$$R = \frac{(60\,sec)(24\,V)^2}{(.10\,kg)(3.3\times10^5\,J/kg)} = \boxed{1.0\,\frac{V^2\cdot sec}{J} = R}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. The answers are in units of current and resistance.

Is Answer Reasonable?

Yes. We would expect a small resistance and a large current so that the energy transfer rate is high and the ice melts quickly.

Is Answer Complete?

Yes. We have found the current and resistance of the heating wires.

3. Practice Exam Problems

Problem #1: You are watching a National Geographic Special on television. One segment of the program is about archer fish, which inhabit streams in southeast Asia. This fish actually "shoots" water at insects to knock them into the water so it can eat them. The commentator states that the archer fish keeps its mouth at the surface of the stream and squirts a jet of water from its mouth at 13 feet/second. You watch an archer fish shoot a juicy moth off a leaf into the water. You estimate that the leaf was about 2.5 feet above a stream. You wonder what the minimum angle from the horizontal that the water can be ejected from the fish's mouth to hit the moth. Since you have time during the commercial, you quickly calculate it.

Problem #2: Your artist friend is designing a kinetic sculpture and asks for your help since she knows that you have had physics. Part of her sculpture consists of a 6.0-kg object (you can't tell what it is supposed to be but it's art) and a 4.0-kg object which hang straight down from opposite ends of a very thin, flexible wire. This wire passes over a smooth, cylindrical, horizontal, stainless steel pipe 3.0 meters above the floor. The frictional force between the rod and the wire is negligible. The 6.0-kg object is held 2.0 meters above the floor and the other object hangs 0.50-m above the floor. When the mechanism releases the 6.0-kg object, both objects accelerate and one will eventually hit the floor but they don't hit each other. To determine if the floor will be damaged, calculate the speed of the object which hits the floor.

Problem #3: Super Dave has just returned from the hospital where he spent a week convalescing from injuries incurred when he was "shot" out of a cannon to land in an airbag which was too thin. Undaunted, he decides to celebrate his return with a new stunt. He intends to jump off a 100-ft tall tower with an elastic cord tied to one ankle and the other end tied to the top of the tower. This cord is very light but very strong and stretches so that it can stop him without pulling his leg off. Such a cord exerts a force with the same mathematical form as the spring force. He wants it to be 75 feet long so that he will be in free fall for 75 feet before the cord begins to stretch. To minimize the force that the cord exerts on his leg, he wants it to stretch as far as possible. You have been assigned to purchase the cord for the stunt and must determine the elastic force constant which characterizes the cord that you should order. Before the calculation, you carefully measure Dave's height to be 6.0-ft and his weight to be 170-lbs. For maximum dramatic effect, his jump will be off a diving board at the top of the tower and, from tests you have made, you determine that his maximum speed coming off the diving board is 10-ft/sec. Neglect air resistance in your calculation. Let Dave worry about that.

Problem #4: In a weak moment you have volunteered to be a human cannonball at an amateur charity circus. The "cannon" is actually a 3.0-ft diameter tube with a big stiff spring inside which is attached to the bottom of the tube. A small seat is attached to the free end of the spring. The ringmaster, one of your soon to be ex-friends, gives you your instructions. He tells you that just before you enter the mouth of the cannon, a motor will compress the spring to 1/10 its normal length and hold it in that position. You are to gracefully crawl in the tube and sit calmly in the seat without holding on to anything. The cannon will then be raised to an angle such that your speed through the air at your highest point is 10-ft/sec. When the spring is released, neither the spring nor the chair will touch the sides of the 12-ft long tube. After the drum roll, the spring is released and you go flying through the air with the appropriate sound effects and smoke. With the perfect aim of your gun crew you will fly through the air over a 15-ft wall and land safely in the net. You are just a bit worried and decide to calculate how high above your starting position you will be at your highest point. Before the rehearsal, the cannon is taken apart for maintenance. You see the spring which is now removed from the cannon and hanging straight down with one end attached to the ceiling. You determine that it is 10-ft. long. When you hang on its free end without touching the ground, it stretches by 2.0-ft. Is it possible for you to make it over the wall?

Problem #5: As a concerned citizen, you have volunteered to serve on a committee investigating injuries to Junior High School students participating in sports programs. Currently your committee is investigating the high incidence of ankle injuries on the basketball team. You are watching the team practice looking for activities which can result in large horizontal forces on the ankle. Observing the team practice jump shots gives you an idea so you try a small calculation. A 40-kg student jumps 1.0-m straight up and shoots the 0.80-kg basketball at his highest point. From the trajectory of the basketball, you deduce that the ball left his hand at 30° from the horizontal at 20-m/s. What is his horizontal velocity when he hits the ground?

Problem #6: You are a volunteer at the Campus Museum of Natural History. Because of your interest in the environment and your physics experience, you have been asked to assist in the production of an animated film about the survival of hawks in the wilderness. In the script, a 1.5-kg hawk is hovering in the air so it is stationary with respect to the ground when it sees a goose flying below it. The hawk dives straight down. When it strikes the goose and digs its claws into the goose's body, it has a speed of 60-km/hr. The goose, which has a mass of 2.5-kg, was flying north at 30-km/hr just before it was struck by the hawk and killed instantly. The animators want to know the velocity (magnitude and direction) of the hawk and dead goose just after the strike.

Problem #7: You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has been damaged so it no longer works. She is 200 meters from the shuttle and moving with it. That is, she is not moving with respect to the shuttle. But she is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10-kg tool kit and throw it away with all her strength, so that it has a speed of 8-m/s. In the script, she survives, but is this correct? Her mass, including her space suit, is 80-kg.

Problem #8: Because of your concern that incorrect science is being taught to children when they watch cartoons on TV, you have joined a committee which is reviewing a new cartoon version of Tarzan. In this episode, Tarzan is on the ground in front of a herd of stampeding elephants. Jane, who is up in a tall tree, sees him just in time. She grabs a convenient vine and swings towards Tarzan, who has twice her mass, to save him. Luckily, the lowest point of her swing is just where Tarzan is standing. When she reaches him, he grabs her and the vine. They both continue to swing to safety over the elephants up to a height which looks to be about 1/2 that of Jane's original position. To decide if you are going to approve this cartoon, calculate the maximum height Tarzan and Jane can swing as a fraction of her initial height.

Problem #9: Your friend has just been in a traffic accident and is trying to negotiate with the insurance company of the other driver to pay for fixing her car. She believes that the other car was speeding and therefore the accident was the other driver's fault. She knows that you have a knowledge of physics and hopes that you can prove her conjecture. She takes you out to the scene of the crash and describes what happened. She was traveling North when she entered the fateful intersection. There was no stop sign so she looked in both directions and did not see another car approaching. It was a bright, sunny, clear day. When she reached the center of the intersection, her car was struck by the other car which was traveling East. The two cars remained joined together after the collision and skidded to a stop. The speed limit on both roads entering the intersection is 50-mph. From the skid marks still visible on the street, you determine that after the collision the cars skidded 56 feet at an angle of 30˚ north of east before stopping. She has a copy of the police report which gives the make and year of each car. At the library you determine that the weight of her car was 2600-lbs and that of the other car was 2200-lbs where you included the driver's weight in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding to convince the insurance company. She must also show that she was under the speed limit.

Problem #10: You have been able to get a part time job with a medical physics group investigating ways to treat inoperable brain cancer. One form of cancer therapy being studied uses slow neutrons to knock a particle (either a neutron or a proton) out of the nucleus of the atoms which make up cancer cells. The neutron knocks out the particle it collides with in an inelastic collision. The heavy nucleus essentially does not move in the collision. After a single proton or neutron is knocked out of the nucleus, the nucleus decays killing the cancer cell. To test this idea, your research group decides to measure the change of internal energy of a nitrogen nucleus after a neutron collides with one of the neutrons in its nucleus and knocks it out. In the experiment, one neutron goes into the nucleus with a speed of 2.0×10^7-m/s and you detect two neutrons coming out at angles of 30˚ and 15˚. You can now calculate the change of internal energy of the nucleus.

Calvin and Hobbes / By Bill Watterson