

Physics 1402 Lab Notes



Preface

Basic Expectations:

Laboratories will be held each week starting in the second week of the course. There will be **six** lab units of two weeks duration: waves, electrostatics, capacitors and dc circuits, magnetostatics, Faraday's law, and ac circuits. A detailed time-table will be provided at your first laboratory meeting. Attendance at all laboratory sessions is required. **YOU MUST COMPLETE ALL SIX LABORATORY UNITS IN ORDER TO PASS THIS COURSE**. Your final lab grade will be based on your lab logbook (see the first chapter of these notes) as well as two written reports that will be completed over the course of the term. Note that although you will be working on both data collection and analysis in groups, you will maintain your own logbook. Written reports will also be prepared individually. The expectations for reports will be introduced by your laboratory instructor.

STATEMENT ON ACADEMIC HONESTY

Note that the College of Science and Engineering policy on academic honesty applies to both the laboratory and lecture sections of this course. It can be found at the following web address:

http://cse.umn.edu/services/advising/CSE_CONTENT_188716.php

In particular, in submitting your logbook and written reports for grading, it is understood that all data collection and analysis have been carried out by you and your lab partners. You may not use data that was collected by some third party. Written reports may include data and analysis carried out in collaboration with lab partners but must otherwise be your own work. Note that data fabrication, which refers to making up data that were not actually collected by you using your experimental apparatus, is a serious academic offense. The minimum penalty for data fabrication is a failing grade (F) for the entire course. In preparing your written reports, please be aware of the usual standards for the use and citation of reference materials. In particular, the use of text or graphics from others without appropriate citation is plagiarism, for which the minimum penalty is a failing grade (F) for the entire course.

Warning on Computers and Saving Data

The computers in the 1402 lab are maintained by the department's instructional staff. Individual accounts are not created, and all data are deleted when you log out. For this reason, you must either save your data on a memory stick, upload them to the web, or email them to yourself *before* logging out at the end of the lab period. With few exceptions, data should also be recorded into your logbook (either by hand or by printing out a spreadsheet) before leaving the laboratory.

Contents

Preface	2
Contents	4
Experimental Precision and Uncertainty	11
Laboratory 1: Waves	19
Laboratory 2: Electrostatics	25
Laboratory 3: RC Circuits	32
Laboratory 4: Magnetostatics	38
Laboratory 5: Faraday's Law	51
Laboratory 6: RLC Circuits and AM Radio	56
Appendix A: The Oscilloscope	64
Appendix B: Other Equipment	66
Appendix C: Statistics and Error Propagation	74
Appendix D: Excel Spreadsheets	77

Laboratory Notebook

All raw data, calculations, and conclusions are to be recorded in a lab logbook. The lab book must be a bound notebook containing quadrille ruled paper and should be used only for this course. Lined logbooks, looseleaf binders, or scraps of paper are not allowed. You will paste in data, computer generated graphs, etc. when they are taken. Think of this lab book as a journal in which you will record all activities related to the lab, including calculations or analysis that are carried out at home. (This allowance for using a lab book at home is different from the standard practice in most companies and even some academic labs, in which it is often forbidden to remove a logbook from the laboratory.)

Each page of your logbook should be numbered and each entry should be dated. It is useful to keep a few pages at the beginning of the book blank in order to fill them in as a "table of contents". For the purposes of organization, when we change from one unit to the next, skip a few pages, and start the unit with a title page such as "Kinematics."

You should never tear out pages from your logbook. Whenever you make some sort of mistake (we all make them), just tidily cross out that section of your logbook. Make sure that it is still possible to read whatever is under the cross-out; it is quite common to find out later that what is there is not actually wrong, or that it includes some useful information.

This logbook is very important. It should contain enough information so that, after a period of, say, 6 months, you would be able to completely reconstruct all the steps you took during the experiment. In an academic laboratory, we regularly consult logbooks from over five years ago. All raw data must be included. All arrangements of equipment, settings of instruments, etc., must be included. All axes and tables should be carefully labeled with the correct units. Words of explanation should be present wherever necessary.

A lab book is not a mini-report! Unless otherwise instructed it is more important to list the relevant points than to write in complete paragraphs. Text should be limited to a few short sentences at a time, and use of bullet points and the like should be used to maximize information while minimizing distracting words.

The technique of keeping a good lab notebook is a very useful skill to develop. Obviously, tidiness and clear writing are essential attributes; it is possible that others may also need to be able to decipher what you have done in your experiment. Note that a ruler and pens of a couple different colors will help you keep a well-organized lab book. A pair of scissors and a roll of tape or a glue stick will also be useful for cutting out graphs and tables from printouts for insertion into your lab book. A simple hand calculator is also useful.

Much of the data you collect in the course will be obtained with the assistance of computers. In most cases, the data should be printed out and pasted into your lab book, along with the corresponding graphs. There may be an exceptional case in which the amount of data generated is so large that printing it out would obviously be a waste of space and paper. In that case, you should print out the graphs only, but record in your lab-book where the original data are stored.

Things to include in your notebook:

- 1. A detailed set-up of the experiment with enough information that you could acquire the materials and re-create the set-up if you had to.
- 2. All relevant equations and useful derivations of the equations with the variables clearly defined. Make sure you can understand the equations if you look at them on a later date.
- 3. **All raw data taken**. Often you will end up creating a data table in a spreadsheet, but remember to also include constant values (like a mass or length that did not change over multiple trials).
- 4. Plots should be clearly labeled, and fully analyzed (data fit with equations, those equations compared to the relevant physics, etc.). A plot is not a substitution for a data table! Only when the data table is too large (over 50 data points per trial) should you not have a data table.
- 5. Uncertainties for all measured points, and propagation of uncertainty to final results should be clearly displayed.
- 6. A conclusion based on the numerical results. You should state a clear conclusion on the results of the experiment, but you must also clearly reference the data that quantitatively supports that conclusion.

DO NOT LOSE YOUR LAB BOOK! Put your name, course number, and email address on the cover just in case.

Sample Notebook Pages

There is no single "correct" way to keep a lab book. These examples illustrate some things to think about. Clear sections and labeling make a lab write up easy to read, and easy to write.

Lab 1.1 Standing Waves on a String Title: Purpose! Use the speed of sound on a wire to determine the ratio of tension to mass density per length Set-up: wire oulley function generator 0000 mechanica driver mass Procedure:) Varied frequency (f) to find nth harmonic of wire for as many values of n as possible. Tension was provided by a hanging mass. The measured frequencies were used to determine the ratio I, where T = tension and M M = mass density per length. This was then compared to the directly measured T of the wire. We did three trials. Trial 1: Red wire with D. 151 kg of hanging mass. Trial 2: Red wire with 0.251 kg of hanging mass. Trial 3: White wire with 0.254 kg of hanging mass. Examples of modes (n): etc. ch n=2 1=1

Equations: Equations. Speed of sound V= NF, N= 2L for a wire V= JH, Measured = Muire Lwire 223333333333333333333333333333333929262666 prediction equation $\frac{T}{\mu} = \left(\frac{2LP}{D}\right)^{a}$ Data: Uncertainty of Measured Value of The: Amwire = = 0.0005 kg AL = = 0.005 m Ammass = = 0.0005 kg $\frac{\Delta \pi}{T} = \left(\frac{\Delta m_{wire}}{m_{wire}}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta m_{mass}}{m_{mass}}\right)^2 = (1.25\%)$ Measured Value of In : red wire m=0.01 kg L= 2.03 m T= 0.151 kg-9.81 m/s² = 1.485 N = 1.485 N Trial 1: Trial 2: red wire m= 0.01kg L=2.03m T=0.251 kg-9.81m/s² = 2.466 N T= (26 + 8 m²/s²) I = 626 + 8 m²/s2) Trial 3: white wire m= 0.02 kg L= 2.02 m T= 0.254 kg - 9.81 m/s2 = 2.492 N I = 222 ± 3 mg/2

Make sure to use units on all numbers that have them.

Experimental value of th: L= 1.540 m (for all trials) Trial 1: Trial 2: 412tz 4124° f(Hz) 1 n f(Hz) 1 6.34 8.3 381.3 2 653.5 12.7 382.5 23450 3 637.9 19.1 382.5 643, 4 379.5 25.3 32.9 56 31.7 38.1 641,8 381.3 644,1 382.5 49.0 7 45.0 392.0 78 51.3 8 390.1 65.7 639,8 9 57.8 391.3 9 74,8 655.3 10 63.3 380.1 Avg. 412f2 = 642 ± 9 m2/22 4 21.0 395.2 Avg. 4Lªfa = 385+5 ma Trial 3: Observation: 4L2f2 f(Hz) na As tension and M increased, 4.78 9.7 216.8 waves became more 2345 223.1 14.7 226.2 difficult to see due to 19.6 227.8 24.6 229.6 decreased amplitude. 235,6 Avg. 412f2 = 227 +3 m2/52 4L2f2 Uncertainty of nº AL= = 0.001m Af= = 0.1 Hz $\frac{\Delta + L^{2} f^{2}}{4L^{2} f^{2}} = \sqrt{2 \left(\frac{\Delta L}{L}\right)^{2}} + 2 \left(\frac{\Delta f}{f}\right)^{2} = \left(\frac{1.49}{5}\right)^{2}$

For the sake of space only three trials were done for this lab notebook. In normal execution of this lab you would likely do one for each type of wire available or multiple tensions. Your write ups should include ALL data taken.



Experimental Precision and Uncertainty

The experimentalist is responsible for communicating his or her results in a manner that conveys the appropriate experimental precision and uncertainty. For example, the apparatus used in this course is capable of measuring *g* with a precision of at best about 0.1 - 1%. If, at the end of my calculations, I write down a number in my lab book that appears on my calculator as *g* = 9.80451 m/s², the reader can only assume that all of the digits reported are *significant* and that I am claiming a precision of around 0.001%. This would obviously be absurd. If the precision were 1 %, a somewhat better choice would be to report the result as 9.8 m/s². Even this format, however, leaves some ambiguity about the precision of the result. By careful analysis of experimental uncertainty, let's assume that I have determined that the uncertainty in my result is 0.2 m/s^2 . If I then write down my result as $9.8 \pm 0.2 \text{ m/s}^2$, it is absolutely clear to the reader both *what* I have measured and *how well* I have measured it.

To summarize: an experimental measurement should always be recorded along with its uncertainty. For the purposes of this course, we will rely more on common sense rather than statistical techniques, but always take time to consider the sources of experimental error and discuss them in your lab book.

Uncertainty

Uncertainty is an important part of understanding a lab because it lets you quantify what your margin of error actually is. If you find yourself writing a conclusion and listing 'sources of error' you should also be including those sources into the presentation of your data and results.

Uncertainty is not a hard limit. If your uncertainty is ±1mm this does not mean that 0.9mm error from prediction is perfectly acceptable, but 1.1mm error is completely unacceptable. Uncertainty is not a wall that cannot be passed, it is more akin to the standard deviation of a bell-shaped curve. On a bell-shaped curve 68% of the curve is within one standard deviation, 95% is within two standard deviations, and 99.7% is within three standard deviations. Likewise, you should understand that in a correctly done experiment uncertainty will only be greater than difference from prediction "68%" of the time (the actual percent is not precise or relevant, but should be understood to represent "most" of the time), but two uncertainties should cover "95%" of the time. Error falling within one uncertainty is great, but error falling within two uncertainties should still be seen as possibly agreeing, but errors that fall outside of two uncertainties is right out.

There are many causes of uncertainty; here we will look at three common ones: Instrumental precision, human error, and systematics. In general, uncertainties should only have one significant figure.

Instrumental Precision

Instrumental precision is a fairly easy uncertainty to understand and a fairly simple one to determine. In general, the uncertainty from instrumental precision is half of the smallest marking or digit displayed on the measurement device. This in general leads to things like ± 0.5 mm on a meter stick or ± 0.005 sec on a stopwatch.

While easy to determine, it is often the smallest contribution. More often than not instrumental precision is overwhelmed by uncertainty from human error. DO NOT simply look at instrumental precision when determining uncertainty unless you are sure human error is not a factor.

Human Error

Human error is by far the most likely source of uncertainty for most measurements. While a ruler may have a precision of ± 0.5 mm, the ability of humans to see lines that small, hold a ruler at the correct spot, look from the correct angle, deal with curved edges or difficult to reach locations means that the actual uncertainty of a measurement is noticeable higher. As a rule of thumb, the smallest markings are usually the extent of human ability to gauge so expect ± 1 mm from a meter stick due to human error as a minimum (it may increase if the object measured is at all difficult).

A decent method of determining human error is to vary the measurement a set amount and ask yourself "could I believe this is the correct measurement". The range through which you can believably move the measurement will show you what human error adds.

Another type of human error is easily seen in the use of a stopwatch. While the instrumental precision may be 0.005 sec, human reaction time is more like 0.2 sec, and thus a stop watch far more likely has an uncertainty of ± 0.2 sec.

Do not add instrumental precision and human error, simply choose the larger one.

Systematics

In the above examples, I have been discussing *random* errors. We are assuming that the measured values, although they may not be perfect, fluctuate about the correct value. Unfortunately, we often deal with apparatus that is not properly calibrated or an experimental approach that has some implicit bias which is not easily recognized. For example, if I measured falling objects with a stopwatch that was miscalibrated (so that it ran fast), I might find

values for *g* coming out as 10.2, 10.3, 10.6, 10.4, 10.2, and 10.3 m/s². Averaging these, I would find $g = 10.3 \pm 0.1$ m/s², which is clearly higher (by 5 error bars) than the expected value of 9.8 m/s². This very large discrepancy is an example of a *systematic* error, which can be eliminated only by fixing the apparatus (in this case the bad stopwatch). In other cases, systematic errors may result from a flaw in technique, as you may find out the first time you use a video camera to measure a falling object. Note that in an extreme case, one can make a measurement with very small random errors, so that the result appears to be extremely *precise*, but the *accuracy*, which is determined by the systematic error, could be very poor. If you notice this sort of behavior, it is your job to try and track down and quantify the likely source of the systematic in your data.

Propagation of Uncertainty

Once we have established the uncertainty of a measurement, we need to make sure that uncertainty is applied to all values derived from that measurement. Suppose that we make two independent measurements $A \pm \Delta A$ and $B \pm \Delta B$, where ΔA and ΔB are the uncertainties in each measurement. What is the error in the quantity y=A+B? What about for y=A/B?

Addition and subtraction:

In the cases of y = A+B and y=A-B, the uncertainty of y (Δ y) is: $\Delta y = \sqrt{(\Delta A)^2 + (\Delta B)^2}$

(often one is told $\Delta y = \Delta A + \Delta B$, but the above version is correct for the case when ΔA and ΔB are statistically random)

Multiplication and Division:

When y=AB or y=A/B

$$\frac{\Delta y}{y} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Note that this gives a fractional uncertainty, the result will be unitless and you will need to be multiplied by y to get Δy . Sometimes it is instead written

$$\Delta y = y \times \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Let's see what happens for the case of a rectangle, for which $L = 4.5 \pm 0.1$ cm and W = 7.5 ± 0.1 cm. Letting y = LW be the area of the rectangle, we find

$$\frac{\Delta y}{y} = \sqrt{\left(\frac{0.1}{4.5}\right)^2 + \left(\frac{0.1}{7.5}\right)^2} = 0.13,$$

so that $\Delta y = 0.026 \times 33.75 \text{ cm}^2 = 0.878 \text{ mm}^2$ (which should be shortened to just 0.9 cm² to obey the limitations on significant figures in uncertainties).

A slightly more general form for multiplication and division is for the case of $y=A^nB^m$:

$$\frac{\Delta y}{y} = \sqrt{n^2 \left(\frac{\Delta A}{A}\right)^2 + m^2 \left(\frac{\Delta B}{B}\right)^2}$$

General Rule:

The above rules can be used to find the uncertainty for probably 90% of the equations you will face in these labs, but if you find yourself in need of a general formula, for any function y(A,B):

$$\Delta \mathbf{y} = \sqrt{\Delta \mathbf{A}^2 \left(\frac{\partial y}{\partial A}\right)^2 + \Delta B^2 \left(\frac{\partial y}{\partial B}\right)^2}$$

Which when applied to each of the special cases above will return the provided equations.

At this point, you might be completely confused or simply depressed at how tedious the propagation of experimental errors in a calculation can be. Do not worry! The goal of this course is for you to begin getting used to thinking about uncertainties and start you getting comfortable with including them in your lab work. To begin with, make sure you always note uncertainty on any measurement you make. Next, look at which uncertainties will propagate to the final results, and consider how big of an effect they should make (it's is often a good idea to think in terms of percentages). Then begin propagating uncertainties in a simple form you can get used to using. Perhaps start by using $\frac{\Delta y}{y} = \frac{\Delta A}{A}$, where A is your greatest source of uncertainty and see what

you get. Then try blindly applying $\Delta y = y \times \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$ (in most situations this will at least get you the correct order of magnitude) until you become more familiar with the procedure and expectations.

See Appendix B for more information on uncertainty and propagation.

Significant Figures

You will be carrying out calculations based on your raw data. At times you will end up with a lot of digits (especially when using computer based timing or dividing numbers). It is important that you understand how many of those digits are relevant, and that you show that understanding by only ever reporting those digits. As a rule of thumb, you should never write down more than 4 significant figures without being <u>very</u> sure that you can claim to be that precise. But let's consider the actual way you should determine significant figures.

Suppose that I am asked to calculate the area of a rectangle, based on my measurements of $L = 4.5 \pm 0.1$ cm and W = 7.5 ± 0.1 cm. It is trivial to calculate the product on a calculator, which will duly record 33.75 cm² as the answer. Is this what I should report?

Previously we calculated that the uncertainty of this area was 0.9 cm^2 , and this limits how much importance we can place on any digit less than 0.1 cm^2 . As such, I would write down in my lab book: "The area of the rectangle is $33.8 \pm 0.9 \text{ cm}^2$." Note that the number of significant digits in the result is determined by the error, but that the error itself should be recorded with just a single significant figure. It would not accurate to state the result as " $33.75 \pm 0.87 \text{ cm}^2$ ".

Read the previous paragraph as many times as is necessary to understand the fundamental point: *there is no sense in reporting the results of a calculation with a precision better than that of any of the inputs into that calculation*.

Note that handling significant figures (and many other things) can be simpler if you use scientific notation. For example, imagine that you write down the number 1560 cm³ in your lab-book. Is the last zero significant or not? All confusion would be avoided if the number were written 1.56×10^3 cm³, if you did not mean the zero to be significant, or 1.560×10^3 cm³, if you did. Note that to record an experimental uncertainty in scientific notation, you should place it before the "x" sign. For example, 1560 ± 20 cm³ would be written as $(1.56 \pm 0.02) \times 10^3$ cm³. The uncertainty is <u>always</u> packaged with the root inside parentheses.

Analyzing Data Plots

Making plots is a natural part of most labs. Plots are useful for visualizing the behavior of data, and they allow us to easily fit the data with a line. In honors physics it is not enough to simply make a plot and slap a fit line on it, we expect that you will use plots to their full potential for not just displaying data, but for interpreting it as well.

Linearizing plots

A very common technique when making a plot is to linearize it, that is to adjust the data so that when it is plotted it makes a straight line. For example, if we are plotting distance (y) vs time fallen (t), the equation should be:

 $y = y_0 + \frac{g}{2}t^2$ which when plotted as *y* vs. *t* will give a parabola. Parabolas are very hard to fit by hand. They are also hard to examine for how well a fit line matches the data. If instead you were to plot *y* vs t^2 , you would find the data should make a straight line of the form y=mx+b, where m=g/2 and b=y₀ and x= t^2 . You will find that linearizing the data will make the graphs you plot easier to fit, and easier to analyze. In the labs of this course you will likely have to fit data that is in the form y=1/x or y=e^x. You will find the data much easier to interpret if you plot them as *y* vs 1/x and ln(y) vs x.



Figure 1.1: Data (symbols) and best-fit line (solid line) for the hypothetical experiment discussed in the text. The equation for the best-fit line is also indicated. This particular result was generated by Excel's "Trendline" function, which does not estimate errors in the slope and the intercept. Note that the program has spit out five digits, but not all of these are significant!

Goodness of Fit

When comparing a plot of data to a predicted equation, it is not immediately apparent how to quantify how well the measurements agree with predictions. The simplest way is to fit the measurement with its own equation and compare to the predicted equation, and this is a reasonable first step. The "Goodness of Fit" equation (also called a reduced χ^2 (chi-squared), though this is a very general term that applies to many similar calculations) is

$$\chi^2 = \frac{1}{N} \Sigma_i \frac{(M_i - P_i)^2}{\Delta^2}$$

where N is the number of data points, M are the measured values, P is the predicted value and Δ is the total uncertainty at that point. This provides an

average comparison between the uncertainty and the distance from prediction for every point along a line. While χ^2 values of 0 to 1 imply good agreement ('0' implies a perfect match, a value of '1' or lower implies that the prediction is within the uncertainty of the measurement), values of 1.1 to 4 are still believable (a value of '4' implies you were within two uncertainties of the prediction on average), and it is really only values of 4+ that imply bad agreement.

A note on standard deviation: In general, standard deviation is not as good a gauge of uncertainty as other methods described earlier. Either it will be about the same size as your estimated uncertainty (because the standard deviation will be due to that uncertainty) or it will be notably smaller due to consistency of measurement. If standard deviation is notably larger than your estimated uncertainty, it likely means you neglected a major source of uncertainty in your estimation. Bottom line: most of the time standard deviation will be smaller than your actual uncertainties.

A note on R^2 : If you use a trend-line fit, R^2 is never a useful number. Never ever. It is a measure of correlation, and in physics your data will always be well correlated, even if it is otherwise not very good. This value does not tell you how correct your data is, and is not remotely a gauge of uncertainty. Never use R^2 . **Never.**

Fitting a Line by "hand"

Imagine taking a ruler and putting it down on the plot, shifting and rotating the ruler in order to find the "best" line that goes through all of the data points. Can we quantify what you are trying to do visually? Choosing the best line is equivalent to minimizing the average distance between the line determined by the ruler and each of the data points. Let us consider the case in which the value of *x* for each data point is known exactly. Then the best fit line y = a+bx will have an intercept *a* and slope *b* that *minimize* the quantity

$$D = \sum_{i=1}^{N} (y - y_i)^2 = \sum_{i=1}^{N} (a + bx_i - y_i)^2 ,$$

where the sum is over all N data points. The quantity *D* is the sum of the (squares) of the deviations of each data point y_i from the fitting function $y = a + bx_i$, and the problem of selecting the best choice of intercept *a* and slope *b* is equivalent to *minimizing D*. Note that if the line passed through every data point exactly, then *D* would be zero. Since all of the data points (x_i , y_i) are known (they are your data!), *D* is just a mathematical function of the two parameters *a* and *b*. Minimizing *D* is merely a problem in differential calculus (evaluating the derivatives with respect to *a* and *b* and then setting them equal to zero). There is little value in reproducing the calculation here. Be aware, however, that Excel's "Trendline" function carries out this minimization automatically.

you with values for *a* and *b*, often with many insignificant digits, and no estimation of the errors! Think about a way to estimate the error in the slope of the line shown in Fig. 1. Hint: given the error bars shown, draw the lines of largest and smallest slope that a "reasonable" person would draw through the data.

Better routines for fitting lines will estimate the errors Δa and Δb as well as account for the fact that the errors Δy_i in each data point are not necessarily the same. Accounting for the effects of unequal errors is carried out by *weighting* each term in the sum that makes up *D*.

Before leaving this issue, note that the general concept of *fitting* data to a function can be extended to other functions besides lines. The fitting procedure is always a matter of minimizing the average deviation of the fitted function from the data. For polynomials (quadratics, cubics, etc.), it turns out that this problem can always be solved in closed form, and the minimization procedure goes by the somewhat confusing name of *linear least squares*. An extension, known as *non-linear least squares*, applies to arbitrary functions.

Laboratory 1: Waves

This lab will serve as an introduction to the topic of waves, a concept that is very important in virtually all fields of science and engineering.

Problem 1.1: Standing Waves on a String

Goal: Plan and carry out a quantitative experiment that compares predictions made from the provided wave equations to measurements made in the lab.

Suggested Reading: Read the section of your textbook on standing waves.

Background

As an introduction to this section, we will first explore the simplest case: waves moving in onedimension. One of the best examples and visualizations of a standing wave is the transverse vibrations on a string. We are interested in the way in which these waves propagate along the medium (i.e. the wave's velocity along the string). You are most likely familiar with the expression for the velocity, *v*, of a wave as given by the equation

$$v = \lambda \cdot f , \qquad (1.1)$$

where λ is the wavelength, and *f* is the wave's frequency. For this lab, there is another expression for the wave's velocity, particularly for that on a string. Your text has already derived for you the expression for the velocity, *v*, of these transverse waves:

$$v = \sqrt{T/\mu} . \tag{1.2}$$

Notice that the velocity is dependent on the wire's tension, T, and mass density per-unit-length, μ .

Experiment

In this experiment, we will use several types of wire as our "string". Determine the relationship between the tension in a wire and its mass density per-unit-length (that is, to verify equation 1.2). The following materials will be provided for you:

- a mechanical driver
- wires of various sizes
- a collection of masses
- a pulley
- a Pasco function generator
- a meter stick
- a mass balance

You will have to vary the tension T in the wire the mass density per-unit-length, μ , and the frequency f of the mechanical driver. You need to collect enough data so that you can make plots of the relationships between variables, such as those in equations 1.1 and 1.2. You need to collect an appropriate amount of data in this experiment, keeping in mind that at least 5 data points are usually required to fit a line with any degree of reliability.

Note: the Pasco function generator can be very sensitive to how fast you turn the dial. Turn it slowly when trying to find a specific frequency.

1. Using the materials listed above, set up an experiment to observe standing waves on a wire. Use figure 1.1 to help guide your setup.



Figure 1.1: The standing wave setup.

2. Once your apparatus is prepared, begin exploring what happens by varying the frequency of the driver. Make some quick, qualitative observations about what happens if you vary the other variables in equations 1.1 and 1.2.

How many different standing wave modes can you observe? Make a careful sketch showing the positions of the nodes for the first few modes. We normally identify these modes with the number of standing $\frac{1}{2}$ waves between the fixed ends of the wire. Does the location of the mechanical driver matter?

Do not dwell too long in this step of the experiment. This should only take 10 to 15 minutes.

- 3. After making your initial observations, it is time to go ahead with the plan for your experiment that you have made. You will need to verify the prediction in equation 1.2. Decide on a how you will test this equation. What final quantities will you compare?
- 4. To begin the experiment, choose a wire, and apply a fixed tension to it. Determine the velocity of the waves, *v*, on the wire by making a suitable linear plot of your data. Remember to account for the uncertainty in your measurements of the frequency, *f*, of each mode.
- 5. Repeat this process for several different tensions, *T*, using the same wire.
- 6. In order to test the relationship in equation 1.2, you will also have to vary the linear mass density, keeping the tension fixed.

7. Think of a good way to plot and fit your data in order to verify the prediction. You may have to make several plots of your data.

Problem 1.2: Standing Waves of Sound

Goal: Identify the "unknown" gases using measurements and equations related to the speed of sound.

Suggested Reading: Read the sections in your text about ideal gases and the speed of sound (they may be in separate sections). Problem 1.1 also deals with standing waves, so reading it may provide helpful information. Also, read *Appendix C* to introduce you to the oscilloscope.

Background

For this portion of the lab, we have 3 sets of tubes, each filled with an "unknown" gas. Your task is to figure out what gas is in each tube. You can assume that the gases obey the ideal gas law.

You will find in you textbook that the speed of sound, v, in a gas with a density, ρ , and under pressure, P, is

$$v = \sqrt{\gamma P / \rho} , \qquad (1.3)$$

where γ is the *specific heat ratio* of the gas. Note the similarity of this equation to that of Eq. 1.2.

Before you begin, ask yourself this question: Does the speed of sound depend on the pressure in the tube? Can you recast the expression for v so that P is eliminated?

You may want to consult your text book for more information about ideal gases and the speed of waves in an ideal gas. Show that for a given γ , which you can look up, and the molar mass *M* of the gas, you could calculate *v*. Alternatively, if you measure *v*, you can determine either γ or *M* if you know the other.

You may assume that the gas in each tube is either a) one of the noble gases or b) a constituent of our atmosphere. This gives you a countable number of possibilities for γ and M. You can use the fact that for an ideal monatomic gas, $\gamma = 5/3$, while for a diatomic gas, $\gamma = 7/5$.

Experiment

To complete this exercise, you will be provided with the following materials: a BK function generator,

- an oscilloscope,
- BNC cables,
- banana cables,
- BNC to banana adapters, and
- 3 tubes with transducers at ends.

The gas tubes are sealed and have piezoelectric transducers at each end. The transducers can be used as either speakers or microphones. In this case, one will be used to transmit your wave-form through the gas, and the second will be used to measure those waves on the other end.

NOTE: The tubes are fragile. Please do not drop them or apply force to the ends. They tend to snap near the transducers. The gas inside is not dangerous, but broken tubes may be.

Using BNC cables, hook one end of the tube up to a function generator and the other end up to Channel 2 of an oscilloscope. If you have not used an oscilloscope before, play around with it to become familiar. The power switch for the small Tektronix digital scopes is on top. (Do not worry about damaging the oscilloscope. Just about the only way to destroy one is to drop it.) See Appendix C for notes on the oscilloscope.

- 2. Hook up the "TTL/CMOS" output of the function generator to Channel 1 of the scope. Display both Channel 1 and Channel 2 on the screen. Trigger the scope off of Channel 1. If, after some tinkering you cannot figure out how to both Channel 1 and Channel 2, ask your TA or a classmate for assistance. Hitting the "Auto Set" button should do the trick. When properly triggered, you should see a square wave on the oscilloscope from the "TTL/CMOS" output of your function generator and a sine wave from your experiment. Use the "Measure" button to explore the additional information the oscilloscope can give you.
- 3. Before you begin taking data, make your preliminary qualitative observations. Sweep through the frequencies on the function generator and look at what happens on the oscilloscope. Do you see anything interesting? Search for resonances of the tube. As you approach a resonance, you will probably need to change the sensitivity of your scope as the signal increases.

Locate a resonance. How precisely can you determine the resonant frequency? Make an estimate of this uncertainty which you will include in your measurements of *f*.

Additional exercise: By looking at your oscilloscope carefully, observe what happens to the phase of the output signal as you tune the frequency through the resonance. For ONE of the resonances around 1 - 2 kHz in frequency, plot the amplitude and phase as a function of frequency for frequencies near the resonance. Do these graphs look like the corresponding plots for a driven simple harmonic oscillator? What is the quality factor Q of the resonance? What is the phase shift between drive and response as the frequency passes through the resonance?

- 4. Be sure that you have a plan for what and how much data you will collect in order to identify the gas in the tube. You should be able to find at least 8 10 resonances for each tube. Make sure that one of these is the fundamental mode. Some resonances may be hard to see due to low amplitude, but higher ones will become more visible again. Make a note if you think you 'missed' a resonance, but continue on to the next expected resonance to ensure you take as much data as possible. By plotting your data vs. mode number *n*, (where *n*=1 is the fundamental, *n*=2 is the first harmonic, etc.), determine the velocity of sound in each tube.
- 5. Now that you have determined the velocities, what gas is in each tube? Note that there is nothing about the experiment that directly tells you the value of γ for a particular gas. To make the correct conclusion, you may have to test possible values of γ that you can obtain from tables.

When writing down your conclusions, make sure you write in your lab-book the code number or symbol that appears on each tube so that the TA can determine whether you are correct.

Problem 1.3: Interference of Visible Light

Goal: Measure the wavelength of a laser using the data you collect using several different methods.

Suggested Reading: Read the section of your textbook that covers interference from two coherent sources. Some of this material may be in the chapters on optics. While it is your responsibility to read all labs ahead of time, this lab in particular will be much more difficult if you have not read it and prepared the equations ahead of time.

Background

We will be using light from a laser and a set of slits to create an interference pattern.

The laser and slits are positioned on an optical rail and the interference pattern is observed on a screen at a distance of a **couple meters**. In this case, the angular position of the interference maxima (in radians) is always small relative to 1. Read about single slit diffraction and double-slit interference in your textbook.

What is the condition for constructive interference of the waves emitted through two slits? What is the condition for destructive interference? What about in the case of single slit diffraction? Where will the maxima and minima appear on the screen in each situation?

Experiment

To complete this exercise, you will be provided with the following materials:

- a red Laser,
- slides with single, double and multiple slits,
- an optical track , and
- a white screen

Your main objective is to determine the wavelength of the laser from the data you collect. You will have additional objectives of measuring the width of a human hair, and making a graph of the expected intensity of the pattern.

Warning: You will be working with lasers for this lab. You should never look directly into the light of the laser. Even though the power in these lasers is relatively small, you can severely damage your eyes with a short exposure to the light. **Do not intentionally misuse the lasers**.

Read the section of your textbook that covers diffraction and interference from coherent sources. Some of this material may be in chapters on optics. Determine the conditions for interference (constructive and destructive) for single and double slits. Make sure you understand how the positions of specific types of interference depend on the wave length of the light, as well as other measurable conditions.

1. Single-slit diffraction: Since we want to understand diffraction first, mount the slide with the single slits and project the diffraction pattern of one slit onto the screen. Tape a piece of graph paper to the screen and mark the positions of the minima and maxima for the diffraction pattern from each slit. The pattern you will see from this is because light emanating from different portions of a *single slit* also generates its own interference pattern, a phenomenon known as *diffraction*. A crude way of thinking about diffraction is to note that light from one half of the slit interferes with light from the other half. When the path length difference between the two halves is equal to half a wavelength, they interfere destructively. Make sure to record the slit width for each pattern as well as the distance from the slit to the screen.

A. Plot the positions of the minima and maxima of the diffraction pattern for each slit.

B. Test to see if slit width has an effect on the positions of the minima.

C. Use your data to determine the wavelength of the light emitted by the laser. What is the uncertainty in your result?

2. *Measuring the width of a hair (yours!)*: Note that a thin wire is the geometric *complement* of a slit. If I want to make a slit, I need to cut out a thin strip from a sheet of paper. Since the sum of the open slit and the strip is a full sheet of paper, which blocks all of the light, you can use the superposition principle to show that the diffraction patterns of a slit and the corresponding strip are exactly the same when viewed a large distance from the source. (To do this, consider the fact that the sheet of paper blocks all of the light and remember that the intensity which you observe is the square of the amplitude of the wave.) For this section, use the known value of the laser's wavelength.

Mount a strand of hair so that it is in the path of the laser beam, and report on the results. **Use the resulting diffraction pattern to measure the thickness of your hair.**

3. *Double-slit interference pattern*: We now know enough to examine the double-slit interference pattern. Set the double-slit card in the holder and project the pattern onto the screen, to which you have taped a fresh sheet of graph paper. Mark the positions of the intensity maxima (there are a lot). Use these data to determine the wavelength of the laser. Does the spacing between the fringes depend on the slit separation *d* as you expect?

Notice that in addition to the minima that fall between each of the interference maxima, there are broader minima due to diffraction. These will cause interference maxima to be very dim or to disappear. For a given slit width and spacing you should be able to make sense of the resulting pattern. If the screen is too close to the slit you may not see the double slit patterns, make sure you see both the large diffraction pattern and much smaller interference pattern clearly.

4. Check your results for (3) quantitatively: The intensity of a double slit diffraction pattern is given as

$$I = 4I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha, \qquad (1.4)$$

where $= \pi d \sin \theta / \lambda$, *d* is the *separation* of the two slits, $\beta = \pi w \sin \theta / \lambda$, and w is the *width* of each slit. Note that θ is the angular position on the screen, with zero at the center of the pattern. For a large distance from the slits to the screen, as in your experiment, you can use the approximation $\sin \theta = x/L$, where *L* is the distance from the slits to the screen and *x* is the distance on the screen measured from the center of the interference pattern. You can determine everything that goes into this equation (I_0 is an arbitrary initial intensity, set it to 1 or some other simple value). Note that α and β are both proportional to *x*, and so you can rewrite Eq. 1.4 so that *I* is an ordinary function of *x*. Using Excel, make a plot of I as a function of *x*, where *x* is the distance from the center of the pattern. How does it compare with what you observe on the screen? Which interference maxima are suppressed because of diffraction? Does this correspond to what you observe?

5. *If you have time:* try looking at the interference pattern of 3, 4, or 5 slits. How do these compare with the two-slit interference pattern? Try to think of them as combinations of multiple double slit interferences to see if that is a reasonable approximation.

Laboratory 2: Electrostatics

In this laboratory, you will explore electric fields by "building" different configurations of charged objects and mapping their electric fields. In problems 3 & 4, you will measure the behavior of electrons as they move through an electric field and compare this behavior to your calculations and your experience with gravitational fields.

Problem 2.1: Electric Field Vectors

Goal: In this part of the lab, you will qualitatively <u>and</u> quantitatively predict the electric field from simple charge configurations and see if the results correspond to your expectations.

Background

This part of the lab is based on computer simulations. Before you begin using the simulation, sketch the electric field vector around a positive point charge in your lab notebook. Write down the equation that relates the magnitude of the electric field to the distance to the point charge.

Repeat the above steps for the other two cases (negative charge and a dipole). For the dipole, remember that the total electric field from multiple point charges is the vector sum of the electric fields due to each point charge. For this configuration, assume that a positive and a negative charge are separated a distance d apart. Find the equation for the magnitude and direction of an electric field for any point some distance r from the center of the di-pole configuration. This is easiest to compute on the axese of symmetry.

Experiment (Simulation)

You should now attempt to confirm the predictions that you have made for each of the charge configurations. While you can save an image of your work, that image will contain little useful information for later analysis. You will need to make a table to record your measurements made with the simulation program.

On the desktop, open the <u>Electrostatics 3D</u> program. At the top of the window is a bar of tools. The most important buttons are the "add a charge" and "manual E lines".



Place a point charge. You can select its charge and its location. The program will automatically draw a few field lines and equipotential circles. You can remove these by clicking on the "Automatic" menu and deselecting "E-Lines" and "Equipotential Surface".

d New Point Charge	
/alue of new charge: ✓ Draw Automatic E lines from this charge ─ Place charges on an equilateral polygon Number of Charges: 2	Type of charge
OK Cancel	Help

Part 1: Electric Field Vectors

Once you have your charge placed, click the "Electric Field Line" tool and move the cursor around in the workspace. You should see a green arrow representing the direction and magnitude of the E-field at the point of the cursor. At the lower left hand area of the window you should see an information bar reporting the (x,y) position of the cursor, the E-field at that point, and the electric potential. If you click with the cursor, the arrow will disappear and the E-field line passing through that point will be drawn. Make a table of position, magnitude, and direction of the simulated E-field (note: the point of x=0 cm, y=0 cm is in the upper left hand corner of the workspace. Make sure to record the (x,y) position of any charges you place.) You should be able to predict the **direction and magnitude** of the electric field for the configuration you've made and compare that to the out-put of the simulation. Take approximately 10 data points.

Clear the screen (click on individual charges and hit delete to remove them, use the "Edit" menu to remove unwanted field lines) and repeat this exercise for a single negatively charged point object. Note any observations that you make.

Part 2: Simulation of a Dipole

Clear the screen again and create a dipole by dragging two equal, but oppositely charged point objects onto the workspace. Measure and compare the electric field at points along all axes of symmetry of the dipole. Fill out the field lines so you can see the shape for this set up. You may want a print-out of this for comparison in Lab 2.2.

If you have time, prepare for lab 2.2 further by exploring another type of di-pole. Using the perpendicular infinite wire of charge (two buttons to the right of the "add a point charge") make another 'di-pole'. Repeat the measurements you made for the point charge version and note the differences in magnitude, direction, and shape. Why are they different? Which di-pole will Lab 2.2 resemble?

Part 3: The Electrice Field from a Line of Charge

Now you will simulate the electric field due to a uniformly charged rod of length L. Before starting, calculate the electric field at a point a short distance from the middle of the rod along a line perpendicular to it, and also at a point a short distance from the end of the rod along its axis. If the line were infinite in length what would the field be?

On the desktop, open the <u>Electrostatics</u> 3D program. Use the line of charge button (next to the point charge) to make a line of charge of small to medium length. Map out the magnitude and direction of the E-field along the x- and y-axis. Now make one as long as you can. Repeat the measurements.

As you vary the distance r from the rod along the line bisecting the center of the rod, how should the electric field vary with r in the two limits $r \ll L$ and $r \gg L$? How close to approximating an infinite line is it? What regions match this approximation best?

Part 4: Electric Potential from Multiple Point Charges

Clear your simulation work area. Place three point charges of various values on the corners of a square. Place the cursor at the last corner of the record its electric potential (read-out from the bottom left corner of the window). Also measure the potential at the center of the square, and a few other points of interest. You can add equipotential lines if you like. Vary the length of the sides of the square and the scale of the charges.

Collect quantitative data and compare the results to your predictions for your configuration. Some important points to look at may lie along axes of symmetry. Also make any qualitative observations about the shape of the equipotential, i.e. the surfaces (closed curves in two dimensions) on which the potential is the same.

1. Given that

$$E_x = -dV/dx$$
 $E_y = -dV/dy$ and $E_z = -dV/dz$

what should the relationship be between the equipotential surfaces and the electric field lines? If you have had multivariable calculus, you should be able to predict the answer. Otherwise, see if you can figure it out by exploring both the electric field and the potential for the same configuration of charges using the simulation program.

2. (Optional) Write down an expression for the electrostatic potential of a point charge q surrounded by a ring of radius R with a charge per unit length of

$$-q/2\pi R$$

Do your best to determine how the potential will behave in the limit r >> R. If you are feeling really ambitious, consider a spherical coordinate system in which the polar axis passes through the axis of the ring. How will *V* depend on *r* and θ ? If you are feeling extraordinarily ambitious, what is the electric field? What type of charge configuration is this?

Problem 2.2: Electric Field from a Dipole

Goal: Create a two-dimensional dipole field with the provided set-up and measure the electric field in the plane of the conducting paper.

Suggested Reading: Read the section of your text on electric fields and di-poles. Lab 2.1 also involves dipoles, and can be used as an additional reference.

Background

For this problem, you will create a two-dimensional dipole field by giving two parallel metal rods opposite charges with a battery while their tips are in contact with a sheet of conducting paper. You will then measure the electric field in the plane of the paper.

Experiment

Equipment:

- conductive paper
- 2 brass rods
- Plastic rod stands
- Digital Multi-Meter
- Electric field probe
- 6-volt battery
- Various wires and connectors

You will be using the conductive paper setup described in Appendix D. There is a coordinate grid drawn on the conductive paper. Two brass rods (electrodes) stand upright with their tips in contact with the conductive paper and connected to opposite terminals of a battery or power supply. The electric field probe is connected to a digital multimeter (DMM) set to read volts. You will also have the EM Field program. A white sheet of paper with a grid similar to the grid on the conducting paper is useful for recording the field (do not write on the conductive paper).



Overhead view of conductive paper for this problem.

Appendix D explains how to use the digital multimeter (DMM) and the power supply. Follow the instructions given there to set up the conductive paper. Remember that the DMM measures *voltage*, not the electric field. How are the two related?

Once the rods are connected to the power supply, set the DMM to volts and turn it on. Place the tips of the probe on the conductive paper midway between the tips of the two rods.

Explore the setup and make your qualitative observations. If you cannot get regular readings, make sure that the rods are in good contact with the paper. It may be necessary to apply some weight and/or to clean off the ends of the rods with sandpaper.

Place your probe down somewhere on the paper. What is the direction of the electric field at that position? Rotate the probe so that the <u>center</u> of the probe stays in the same spot.

Ask yourself these questions. Do the values change (pay attention to the sign)? Is there a minimum or maximum value? Are there any symmetries in this data? If there are large fluctuations, determine how you

will measure consistently. Describe how you will use the probe to determine the field direction at other points.

Analysis

Make a map of the electrostatic field from the dipole configuration noting direction and magnitude. Show how the magnitude of the electrostatic field is related to the distance from one of the "charges". What effect does the contact paper being only two-dimensional have on the pattern you see? Compare this to the results from the simulations in Lab 2.1. Which di-pole type best agrees with the map you made?

Problem 2.3: Deflection of an Electron Beam by an Electric Field

Goal: Explore and predict the deflection of an electron beam by an electric field.

Suggested Reading: Read sections in your text about the force on charged particles by electric fields. You may also want to look up the potential energy of charged particles in electric fields and review energy conservation and kinematics from the previous semester.

Background

For this problem, you will use a Cathode Ray Tube (CRT), in which electrons are emitted at one end of an evacuated glass tube and are detected by their interaction with a phosphorous screen on the other end. You will calculate the deflection of an electron that begins with an initial horizontal velocity, passes between a pair of short metal plates that produce a constant electric field between them, and then continues through a region with no electric field until hitting the screen. Your result could depend on the strength of the electric field, the electron's initial velocity, intrinsic properties of the electron, the length of the metal plates that produce the vertical electric field, and the distance from the end of the metal plates to the screen. NOTE: In the next problem you will measure the effect of changing the electron's initial speed.

Before you begin the experiment, you should explore your apparatus a little bit. Turn on the CRT. Note the position of the electron beam on the screen. Now pick up the CRT. Move it around and rotate it. Does the position of the dot change? If so, why?

Can you measure the effect of the Earth's gravitational force on the motion of the electrons in the CRT? To do this, compare the magnitude of the expected deflection from both gravitational and electrostatic forces. If gravity is not responsible for the observed deflection when you change the orientation of the tube, then what are the other reasonable candidates? Make sure you answer this question before proceeding, and come up with a strategy for minimizing the effects of this additional deflection. In particular, do not move your tube around during the experiment!

Use the diagram of the CRT in Appendix D to predict the deflection of the beam as a function of the potential across the deflection plates in the tube. You will need to use your kinematic equations. Note that you will not be able to directly measure the electron's initial velocity at the one end of the tube. You will, however, know the voltage that accelerates the electrons, V_{acc} . Use conservation of energy to express the electron's initial velocity in terms of V_{acc} .

Experiment

Equipment:

- a CRT tube
- a CRT power supply box

- a standard DC power supply
- a Digital Multi-Meter (DMM)
- various wires

Essentially, a CRT consists of a filament for producing electrons, which is biased with respect to a cathode (usually a plate with a hole in it), which accelerates the electrons. After this there are one or two sets of deflection plates to which a voltage is applied.

WARNING: You will be working with equipment that generates large electric voltages. Improper use can cause painful burns. To avoid danger, the power must be turned off and you must wait at least one minute before any wires are disconnected from or connected to the power supply. **Never touch the conducting metal of any wire.**



Note: the values for the X-deflection plates will be different.

Check to see that the connections from the power supply to the high voltage and the filament heater are correct, before you turn the power supply on. You should have between 250 and 500 volts of electric potential between the cathode and anode. After a moment, you should see a spot that you can adjust with the knob labeled "Focus". If your connections are correct and the spot still does not appear, inform your lab instructor.

Before you turn on the electric field between the Y-deflection plates, make a note of the position of the spot on the screen. The deflections you measure will be in relation to this point. **Make sure not to change the position of the CRT since external fields may affect the position of the spot.**

Measure the position of the beam spot as you change the voltage applied to the Y-deflection plates. Note: Be sure to record your measurements with the appropriate number of significant figures and with your estimated uncertainty. Accumulate 10 - 15 data points or so.

Draw a graph of the measured deflection of the electron beam as a function of the voltage difference across the deflector plates. Analyze the data quantitatively in terms of the predictions made earlier in the lab. This should include error analysis.

Now switch to the X-deflection plates. These plates are in front of the Y-deflection plates (closer to the screen) and have a slightly different configuration. Accumulate 10-15 data points using the X-deflection plates. Using this data, determine the values of D and L. Assume that S stays the same and that the start of the X-plate is 0.5 cm after the end of the Y-plate.

Problem 2.4: Deflection of an Electron Beam and Velocity

Goal: To determine how velocity affects the deflection of a charged particle in an electric field.

Suggested Reading: Same as lab 2.3

Background

For this problem, you will use a Cathode Ray Tube (CRT), just as in Problem 3. Now consider what happens to the deflection of the beam as the initial velocity of the electrons is varied by changing the acceleration voltage, V_{acc} .

Experiment

Equipment:

- a CRT tube
- a CRT power supply box
- a standard DC power supply
- a Digital Multi-Meter (DMM)
- various wires
- Pasco function generator

Measure the deflection of the beam spot as you change the acceleration voltage of the electrons, keeping the electric field between the deflection plates constant. Be sure to determine the uncertainty in your measurements.

Note that in this case the only way to change the acceleration voltage is to switch the connections to the power supply among the different dc voltages available. See the warnings above! Turn off the power supply before touching any of the connections.

Draw a graph of your measurements of the deflection of the electron beam as a function of the acceleration voltage. In order to do this with a linear fit, you will need to transform the acceleration voltage to initial velocity. Did your data agree with your prediction of how the electron beam would deflect due to changes in the acceleration voltage? Repeat for several values of deflection voltage and for similar values on the other deflection plate. Include quantitative conclusions and error analysis.

Optional Exploration: Lissajuous Figures

Using two Pasco function generators available in the lab, experiment with how to connect the outputs to the x and y plates of your CRT to make the electron beam draw a circle on the screen. You may not see a perfect circle on your screen. What is going on? Can you draw (approximately) a "figure 8"? If you need some ideas, look up the term "Lissajous Figure." By varying the frequency of one generator relative to the first, see if you can draw figures with a progressively larger number of loops.

Laboratory 3: RC Circuits

Problem 3.1: Simple RC Circuits

Goal: Explores the behavior of resistors and capacitors in simple RC circuits and measure their time constant.

Suggested Reading: Read the section in your text about RC circuits and time constants.

Background

In this part of the lab, you will verify the results you derived in class for the behaviors of capacitors and resistors in circuits. For example, you expect the voltage across a capacitor in an RC circuit to increase and decay exponentially when it turned on or off. Make sure you understand what these functional dependences are and how they depend on R and C.

Experiment

Equipment:

- Vernier differential probe
- labPro input box
- A computer with LoggerPro
- 6-volt battery
- Various resistor in the $k\Omega$ to $M\Omega$ range
- Various capacitors in the µF to nF range
- Various wires

Part 1: Single RC circuits

Start by hooking up a 1 μ F capacitor and 100 k Ω resistor to a power supply and a Vernier differential voltage probe as shown. Make you're your differential voltage probe is plugged into the LabPro box (and that the box is plugged into the computer). Make sure that the LabPro software starts properly. If it does not, ask your TA. For this lab, you should set LabPro so that it takes data as fast possible (at least several hundred points per second). Because of the construction of the voltage probe (it does not have a ground), you can use it to measure the voltage between any two points in the circuit. This is why it is called a "differential" probe

Hook up the voltage leads across the resistor. After starting data collection with the LabPro box, turn on the power supply (within a second or so). Watch what happens. You should see a jump followed by a nice exponential decay on your computer. If you do not, discuss with your lab mates what is going on and try to fix it. Note that the differential voltage probe has a limited voltage range (it usually maxes out around 6 volts).



Figure: 3.1.1 Basic set-up for Problem 1. You will be changing the values of R and C as well as the location of the voltage probes.

Once you are sure that things are working properly, do the following:

- For several different values of R and C, measure the voltage as a function of time after the power supply is switched on. From each of the data sets (after the initial jump), determine the *time constant* of the decay and compare with the predictions based on the known value of R and C. Does the time constant depend on R and C as expected? To do this, you will have to make *logarithmic* plots and perform a linear least squares fit.
- 2. Now switch the probe so that it measures the voltage across the capacitor. For at least one choice of R and C, measure the voltage across each of the circuit components. Plot these separately and then add the results together (as a function of time) in your spreadsheet. Do you observe what you expect, given Kirchoff's Laws? Which of the two quantities is proportional to the current flowing in the circuit? Which is proportional to the charge stored on the capacitor?

Part 2: Circuits with Multiple Capacitors and Resistors

In class, you have derived the rules for the addition of capacitance and resistance when components are placed in either "series" or "parallel." Review this before coming to lab, so that you can make predictions for the following cases.

- 1. Replace the resistor in the circuit above with two resistors in series. What effect should this have on the time constant? Draw the appropriate circuit in your lab book. Do the experiment and verify the time constant.
- 2. Replace the resistor in the above circuit with two resistors in *parallel*. What effect should this have on the time constant? Draw the appropriate circuit in your lab book. Do the experiment and verify the time constant.
- 3. Now what about the capacitors? Replace the capacitor with two capacitors in series. What effect should this have on the time constant? Draw the appropriate circuit in your lab book. Do the experiment and verify the time constant.
- 4. Repeat for two capacitors in parallel. Predict, measure, and compare the time constant.
- 5. Now assume you have three resistors and three capacitors. How can you hook them up to maximize the time constant? Confirm experimentally. Verify explicitly that Kirchoff's laws are obeyed in this case.

Problem 3.2: Measurements of Capacitance

Goal: Make a parallel plate capacitor and measure its behavior.

Suggested Reading: Read the section in your text on parallel plate capacitors and the section on dielectrics in capacitors.

Background

In the laboratory we have high resolution electronic balances, aluminum plates, power supplies, and various sheets of dielectric materials. We can use these materials to make some very accurate measurements. In particular, you will be able to measure the force between two charged plates and use it to determine the capacitance of the parallel plates (comparing with predictions) as well as the dielectric constant of materials that you will insert between the plates.

Experiment

Equipment:

- Electric scale
- Bottom plate
- Top plate with translation stage attachment
- DC to DC converter
- Power supply
- Super thin wire
- Conductive tape
- Various wires
- Plexiglas dielectric plate (round)

Consider a simple, symmetric parallel-plate capacitor. Each plate has a surface area, *A*, and the plates are separated by a distance *d*. What would you expect the capacitance to be? (Use this as your quantitative prediction)

A charge +Q is applied to one plate, and -Q is applied to the other. What is the electric field, *E*, between the plates? What is the potential, *V*, between the plates?

What is the force, *F*, between the plates? Be careful. Chances are that you may be off by a factor of 2 if you do not take time to think about the problem. Put your result into terms of variables you can measure, such as *V*, *d*, and *A*.

Make a quantitative prediction of how the force varies with the separation. How can you determine the capacitance of the plates?

Measuring Capacitance

Use the materials available to set up an experiment so that you can measure this force as a function of the potential, *V*, and separation, *d*. You will place one of the plates on the electronic balance, and another will be suspended above the bottom plate. Apply a potential across the plates and measure the resulting force.

This lab can be done with high precision, and so take a lot of data (10 to 20 points for each case). Be sure to record the uncertainties in your measurements. Be careful making your measurements of the distance between the plates. Make sure that the plates are parallel – a small bubble level will help.

Some fine wire is provided for making connections with the high voltage output of the power supply. Make sure that these wires do not move during the experiment! They can be attached to the outside of your

plates using conducting tape. Measure the resistance of the connections to make sure the plates are actually connected to the circuit.

We have made some DC to DC converters to take a 0-12 V signal from your regular power-supplies and convert it to a 0-1.2 kV output. This signal can be measured using one of the 0 - 1000 V multimeters, but do not go above 1000 V. Attach the voltmeter in parallel with the plates and *leave it attached during your measurements*.

WARNING: You will be working with equipment that generates large electric voltages. Improper use can cause painful burns. To avoid danger, do not touch the plates when the high voltage is on. Make sure the plates are discharged after turning off your power supply. Turn the supply off before disconnecting any wires. Never touch the conducting metal of any wire or plate!

You will need to make two plots: one for the force, *F*, as a function of the potential, *V*, and one for the force, *F*, as a function of the plate separation, *d*. Compare your results with your predictions.

Determine a good way to plot your data in order to test your predictions. If $y = kx^a$, where *a* is an exponent, there are at least two possible ways to *linearize* your data. Assuming that you know *a*, you can plot y vs. x^a . Alternatively, you can plot the natural logarithms: ln(y) vs. ln(x), and a will be given by the slope of this "log-log" plot.

If you find significant deviations from theory, you might want to look up the "edge corrections" for the capacitance of parallel plates.

Further Exploration: Measuring the Dielectric Constant

In this part of the experiment, you will fill the gap between your plates with a dielectric plexiglass plate, which is used to increase capacitance. Go back to the *Background* section of this problem. Answer the questions again, this time accounting for a dielectric between the plates. What should the force between the plates be?

For the dielectric provided, a Plexiglas (Lucite) disk, measure the dielectric constant using the apparatus you have constructed. Be careful. You will need to place the dielectric on the bottom plate and then set the gap between the top plate and the dielectric as small as possible (Why?).

Determine which data taking method will give the best results and, thus, the best value for the dielectric constant. Make a graph and determine the dielectric constant of the material. Your result should be reported with a corresponding uncertainty. You may notice that the reading on the scale changes with time. When should you take the reading?

Problem 3.3: RC Circuits with AC

Goal: To set up and examine several RC circuits with alternating current.

Suggested Reading: Read the sections in your text dealing with RC circuits' with an alternating current supply. Consider looking up Integrator and differentiator circuits to be prepared for what you will observe.

Background

For this problem, we return to the simple RC circuit. But, instead of running a DC signal through, we will be applying an Alternating Current (AC) signal.

Experiment

Equipment:

- BK precision function generator
- Oscilloscope .
- Coaxial adapter
- various wires and connectors
- Collection of resistors and capacitors

Integrator and Differentiator Circuits

Set up the circuit shown in the figure. Pick a of $10 - 15 \text{ k}\Omega$ and a capacitor of about 10 nF. function generator to apply the input signal. Set



resistor Use a the

function generator so that it makes a square wave with an initial frequency to 500 Hz. Take a look at the output of the circuit with an oscilloscope. If you need some assistance with the oscilloscope, ask a TA or refer to appendix C. Look at the output on an oscilloscope.

Note that you must have only one ground point in your circuit. This is the negative terminal of the oscilloscope, which should be connected to the negative terminal of the function generator. This is different than when using the differential voltage probes with the Vernier boxes.

- 1. Sketch the output waveform in your lab book. The shape should be familiar (if you don't recognize it, look at your results to Problem 2 of this lab). Measure the time constant of the circuit.
- 2. Now vary the frequency of the input. What happens when the frequency becomes very large relative to 1/RC? Sketch the waveform for various frequencies.
- 3. Analyze the circuit quantitatively by making a plot of the amplitude of the output as a function of the frequency
- 4. Can you determine why in this limit this circuit is referred to as an "integrator"? Try replacing your square wave input with a sine wave and look at the input and output of your circuit. Does what you see make sense?
- 5. Reduce the time constant of your circuit by using a 100 Ω resistor and a small capacitance (suggested 270 pF). What does the output of the circuit look like at low frequencies (relative to 1/RC)?
6. Now reverse the positions of the capacitor and resistor (keeping the same low time constant as step 5) so that you are looking at the voltage across the resistor. Why is this circuit referred to as a "differentiator?" What should happen if you drive the circuit with a triangle wave? Try it. Sketch the output that you see on the oscilloscope for a few different waves. Vary the wave shape and frequency.



Figure 3.3.2: The Differentiator circuit. Note the positions of the capacitor and resistor. This is important when using the scope, for which the negative terminal is always ground!

Laboratory 4: Magnetostatics

Problem 4.1: Magnetic Field of One Coil

Goal: To use a multi-turn coil of wire to create a magnetic field and explore its behavior.

Suggested Reading: Read in your text about the Biot-Savart law and magnetic fields from loops of current.

Background

In this section of the lab, you will map the magnetic field of a coil of wire carrying a current. Before you begin the actual experiment, set up the coil and run a current through it. The current must not exceed 2 Amps.

Use a compass to make a qualitative map of the field in and around the coil. Do not spend much time on this, so take about 5 minutes to make the map. How does the profile of the field compare to that of a bar magnet?

From the diagram below, you will see that there are two axes of the coil that we are interested in. The x-axis is perpendicular to the coil, and the y-axis lies in the plane of the coil. You will investigate the field along both of these axes.





Use the Biot-Savart law to determine how the magnetic field along the x-axis (perpendicular to the plane of the coil) depends on the distance from the center of the coil. You will have to measure the diameter of the coil and use the given number of turns. Approximate what the equation would be when the distance along this axis is much larger than the radius of the coil.

Make another prediction for how the field depends on the distance along the y-axis from the center of the coil. In this case, your prediction will not be quantitative, although you can say something about points very close to the edges of the coil and also very far away. Note that there are two regimes (inside and outside of the coil).

Experiment

Equipment:

- Power Supply
- 200 turn coil
- Hall probe
- labPro input box
- Digital Multi-Meter
- Compass
- Various wires and connectors

Set up the apparatus if you have not done so yet. Connect the large coil to the power supply and adjust the current. Use the DMM to measure the current through the coils.

Connect the Hall probe according to the directions in Appendix D. Find a way to position the Hall probe so that you can accurately measure the magnetic field along one of the axes (don't forget to zero the probe!). Explore the ranges that the magnetic field will cover and think about how to distribute your data points

NOTE: The Hall probes have yellow (sometimes black) sleeves around them to keep them from bending at a joint. Bending the probe at that joint, even just the small amount the sleeve allows, will short out the probes ability to take data. Always hold the probe by its non-data-taking end and never apply pressure near the bend.

Remember that the range on the computer must match the range on the Hall sensor. Be aware that the field sensor will simply read a constant value when exposed to a field larger than its range. Think of this if you find a magnetic field that appears to be constant when it should not be so! Furthermore, make sure you zero out your probe or you will also be measuring a significant background field.

- 1. Make a plot of the field along the x-axis as a function of the distance from the center of the coil. Plot your prediction on the same graph as your measurements.
- 2. Now measure the field along the radius (y-axis), working from the inside to the outside of the coil. Make a plot of the data, with uncertainties. How do your data compare with your qualitative predictions for points close to the edges of the coil as well as very far away? Does the magnetic field behave as you expect it to when you approach the edges of the coil radially?

Problem 4.2: Measuring the Magnetic Field of Two Parallel Coils

Goal: To use a set-up of Helmholtz coils to create a magnetic field and explore its behavior.

Suggested Reading: Same as 4.1.

Background

You have two nearly identical large coils of wire mounted so that the distance between them equals their radii. There is something quite useful about this particular geometry, as it turns out that it optimizes the homogeneity of the magnetic field in the region between the coils.

Look at the diagram below. Using the Biot-Savart law, derive the equation for the magnetic field along the x-axis directly between the two coils. Assume that the 200-turn coils are separated a distance d, have a radius, R, and each carry a current, I.

Connect two coils to a power supply so that each coil has the same current. Each coil has 200 turns.

You will have a digital Multimeter (DMM), a compass, a meter stick, and a Hall probe. A computer with Vernier software is used for data acquisition.



Before coming to lab: You should have an expression for the magnetic field of the double-coil as a function of *x* as well as the distance L between the coils. Show that the first three derivatives of the magnetic field with respect to z vanish when L = R.

Experiment

Equipment:

- Power Supply
- Helmholtz 200 turn coils
- Hall probe
- labPro input box
- Digital Multi-Meter
- Compass
- Various wires and connectors

For this problem you will demonstrate what it is meant by a pair of coils being Helmholtz coils. To do this, you will need the materials listed in the figure above. You can connect the coils in series or in parallel – it really doesn't matter, but it will affect the current that you will use for your predictions. Use the DMM to measure the current through he coils.

Measure the magnetic field along the x-axis of the coils in their Helmholtz configuration (separated by a distance equal to their radii). Compare the results quantitatively with a prediction based on Biot-Savart. Plot your data and your prediction. Describe the limitations on the accuracy of your measurements and analysis.

At the point midway between the two coils, measure the axial field (i.e. component of the field along the z- or y-axis) as you move the probe off the x-axis. Graph your results. There are no predictions for this part. Without spending too much time, determine the volume over which the magnetic field varies less than 20% from the value found exactly halfway between the coils.

Problem 4.3: Magnetic Force on a Moving Charge

Goal: To explore how magnetic fields affect moving charged particles.

Suggested Reading: Read the section in your text about the Lorentz force. Pay attention to parts that mention the radius of curvature due to this force.

Background (Complete this before coming to lab!)

Recall Lab 2, Problems 3 and 4. You used a Cathode Ray Tube (CRT) and deflection plates to deflect an electron beam from its original trajectory. You were also able to predict the deflection based on the potential across the plates. Now we are interested in a similar problem – one in which we can make a very important measurement. You are being asked to measure the electron's charge-to-mass ratio, *e/m*. You will not be able to do this with great accuracy (on a good day within 10%), but you will be able to verify important aspects of the Lorentz force law.

You will be using the cathode ray tube (CRT) described in Appendix D. The magnetic field will be provided by connecting the Helmholtz coils to the power supply and placing the CRT between the coils. You can safely assume that the path of the electron beam in the CRT fits in the Helmholtz coils' region of nearly uniform field.



Magnetic field out of page

Figure 4.3.1: CRT in a magnetic field. The solid arc shows the electron beam trajectory with a magnetic field, which points out of the page. The distance, D_{tot} , is measured from the end of the acceleration region. See Appendix D for values.

Assume a distance L between the anode (where the electron leaves the acceleration region) and the front surface of the tube. Assume that the beam is in a uniform magnetic field, *B* (perpendicular to the electron beam as it leaves the acceleration region). Predict the deflection *y* of the electron beam in terms of the field, *B*, and the accelerating voltage of the electron beam, V_{acc} .

Think about the shape of the path of an electron in a uniform magnetic field. Draw this path out. You will have to use some geometry to solve the problem. You can assume that the electrons are only deflected by the field after they leave the accelerating potential, V_{acc} .

From the Lorentz force equation, find an equation for the charge-to-mass ratio of an electron. Put it in terms of quantities you can measure, such as B, V_{acc} , the length D_{tot} , and the deflection y of the beam by the magnetic field as observed at the surface of the CRT.

Show that the field at the center of the Helmholtz coils (halfway between the coils) is given by

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 NI}{R}$$

Where N is the number of turns, I is the current and R is the radius, which is also the separation of the two coils. You can use this result in this lab.

Experiment

Equipment:

- a CRT tube
- a CRT power supply box
- a standard DC power supply
- a Digital Multi-Meter (DMM)
- Helmholtz 200 turn coils
- Hall probe
- labPro input box
- Digital Multi-Meter
- Compass
- Various wires and connectors

Set up the Helmholtz coils and connect them to a power supply. You will have a digital multimeter (DMM) and a Hall probe. You will need to measure the current and determine it's effect on the B-field.

For this experiment, quantitatively measure the deflection of the beam as a function of magnetic field applied (for a fixed accelerating potential, V_{acc} , which you should set to about 500 V, as measured with a voltmeter). Plot the results along with your prediction. Take a 5-8 data points for both signs of the field. The clever experimentalist can use this apparatus to measure the value of the Earth's magnetic field, at least the component along the axis of the Helmholtz coils! Can you verify that the beam in the CRT is formed by negatively charged particles? Make sure you measure the sign of the current in your coils correctly.

At this point, discuss whether the force you can infer from your measurements is proportional to the cross product of the magnetic field and velocity. Compare to your predicted equation.

From your results for deflection as a function of B at fixed V_{acc} , calculate e/m. This was an important early measurement made on electrons before individual charge and mass was accessible.

Problem 4.4: Hysteresis Curve

Goal: To explore the hysteresis related properties of nickel and iron.

Suggested Reading: Read about hysteresis curves and coercitivity.

Background

When we say that a material exhibits hysteresis, we mean that its properties are dependent on its history. In this case, you will show that the magnetic properties of a Ni rod depend on the manner in which the magnetic field has been increased or decreased. Read the relevant sections of your textbook on ferromagnetism and paramagnetism.

You will measure the hysteresis loop of a Ni rod. You will then explore the magnetic properties of "soft iron." The terminology is somewhat confusing. The word "soft" refers to a magnetic characteristic that you will investigate and not to the material's mechanical hardness.

You will be provided with a solenoid, power supply, magnetic field sensor, and the two types of rods. The goal is to plot the hysteresis that the metals exhibit in the presence of an external magnetic field. The solenoid will provide the magnetic field, and then you will have to measure an induced field due to the presence of the rod.

Experiment Equipment:

- Power su
 - Power supplyNickel and soft iron rod
 - Solenoid
 - Hall probe
 - Vernier voltage probe
 - labPro interface
 - compass
 - Digital MultiMeter
 - Wires and connectors

Part 1: The Solenoid

(This should only take 10 minutes of so. If you are having trouble with your field sensor, please consult your TA. **The key to this experiment is to avoid saturating the sensor.)**

The solenoid, like the Helmholtz coils, is another useful tool for making a uniform magnetic field. In this lab, however, we use it as a convenient way to magnetize a cylindrical rod.

You must first determine the 'background' magnetic field of your solenoid (i.e. the field it produces when it is empty). Call this field B_o , which will vary with the current through the solenoid and the distance along the solenoid's axis, or the z-axis. We will first determine how the background magnetic field changes with these two variables.

 Set the current through the solenoid at approximately 1 amp, making absolutely sure not to exceed the maximum magnetic field for your sensor, which is about 6 milliTesla. To be safe, do not exceed 4.5 mT at the center of the solenoid. If necessary, turn down the current. Measure the magnetic field every two centimeters or so along the z-axis, making sure that you continue 10 centimeters or so outside the solenoid. Make a suitable graph of the field as a function of the distance from the center of the solenoid. 2. Verify that the field at the center of the solenoid is proportional to the current and determine the constant of proportionality. Consider using the voltage probe in CH 2 of the input box to measure voltage over the solenoid and B-field simultaneously.

Write down the dimensions of your solenoid, including the number of turns of wire per inch. You will need this to answer one of the questions below.

Part 2: Hysteresis Curve of Ni

Take one of the Ni rods provided and insert it into the solenoid. The rod should stick out the end of the solenoid by a couple inches. Make sure that the rod does not move when the magnetic field is applied. Secure it if necessary. Place the probe close to the end of the rod (usually about 2-4 cm from the end is a good location).

Check the range of the hall probe and voltage probe using the power supply. Find out where you max out the probes readout. Make sure you do this BEFORE spending lots of time taking data. Once you find a good position for your field sensor, make sure that both the sensor and the rod do not move.).

By plotting the reading on the field sensor as a function of the magnetic field experienced by the rod (which was found in *part 1* to be proportional to the current in the solenoid) you can create a hysteresis curve for the nickel rod. You can take dozens (or hundreds) of data points by measuring B-field and voltage simultaneously using LoggerPro and varying the input over the sample time. This data can be used to make the needed plots.

- A. Start at zero field. Increase the current to a large positive value, sketching the output of the field sensor as a function of the current in the solenoid. What happens at large fields?
- B. Now decrease the current back to zero. Record the value of the field sensor.
- C. Reverse the leads of your current supply. **ONLY DO THIS WHEN THE CURRENT IS ZERO!** Now ramp the current to a large negative value. Record the value of the field sensor.
- D. Now ramp the current back to zero and again record the value of the field sensor. Compare with the value after step 2. If they are the same, then something is wrong.
- E. Reverse the leads and go back to high positive current. This should "close the loop."

Examine the shape of the loop you see. What does it mean? What does the 'width' of the loop imply?

Notice that if you simply turn down the magnetic field your Ni rod will be left magnetized. Come up with a strategy to leave the rod *demagnetized*. You may need to do a little research. Explain in your lab book how you demagnetized the rod.

Part 3: Hysteresis Curve of "Soft Iron"

Now acquire the soft iron rod. Once again, you should have the rod sticking out a couple inches from the end of the solenoid, and you will need to make sure that it does not move during the measurement. Iron has a larger magnetization than Ni, and so you may find that you will need to move the field sensor further away from the rod of limit your maximum input. **Make sure you do not saturate your field sensor.** Simultaneously take data for magnetic field and voltage for steps A-E as in part 2 and make the hysteresis curve.

Does the "soft iron" show significant hysteresis? Compare the results to that of nickel? Why might they differ? What is the relationship between the applied field and the magnetization of the rod?

Additional Questions for Problem 4.4

- 1. Determine the number of layers of wire in your solenoid.
- 2. Read in your textbook about coercivity and coercive field. A good permanent magnet needs to have a significant coercive field. Why? By this criterion, which of the two materials is a better permanent magnet? Estimate the coercive field. To do this you will have to plot the field sensor reading vs. the average applied field (determined from the current in the solenoid).
- 3. Explain how you demagnetized the Ni rod.
- 4. Soft iron is used in electromagnets and transformers, in which the experimenter needs a larger field than can be provided by the windings alone but still desires the magnetic field to be proportional to the current in the windings. In the language of magnetic materials: soft iron has a high permeability but a small coercive field. Explain how your data are (or are not) in agreement with this statement and how it relates to the desired property for making a good electromagnet.
- 5. As you pull one of the rods out of the magnet (with the magnet powered), at what point is the force exerted by the magnet largest? Why? You might want to investigate on the web some of the bizarre accidents that have occurred with MRI magnets.

Problem 4.5 : Gyroscope and Torque on a Magnetic Moment

Goal: To use a gyroscope to explore the effects of torque on a magnetic moment.

Suggested Reading: This uses the same apparatus as for the gyroscope lab in the Physics 1401 manual. The only difference is that in this lab we eliminate the gravitational torque and use only the torque exerted on a magnetic moment (a bar magnet) in a magnetic field. If you did not perform this previous lab, look up information on the precession of tops and gyroscopes.

Gyroscope

Before coming to lab, make sure that you have reviewed the section of your textbook on gyroscopes and that you understand how an object with angular momentum \vec{L} , subject to an external torque $\vec{\tau}$ will precess with a frequency Ω_P . We are considering only cases of reasonably high symmetry, in which the angular momentum $\vec{L} = I\vec{\omega}$, where I is the moment of inertia of the rotating mass that makes up the gyroscope. Make sure that you understand the distinction between the angular rotation frequency ω of spinning gyroscope and the precession frequency Ω_P .

Also, make sure you have read the section of your textbook on magnetic moment and that you understand that a magnetic dipole of dipole moment $\vec{\mu}$ experiences a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ in a magnetic field and that the potential energy of the magnetic dipole is $U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta$, where θ is the angle between the moment and the field. A key aspect of this lab is that the frequency of precession Ω_P of a magnetic moment in a magnetic field is proportional to the torque and hence to the magnetic moment. The precession of a magnetic moment in a magnetic field is the basis for magnetic resonance imaging as well as the characterization of molecules by nuclear magnetic resonance (NMR) spectroscopy. In this lab, the role of the nuclear magnetic moment is assumed by the bar magnet.

Background

There is a small magnet buried near the center of a cue ball with its "north" pole pointing towards the black post that is attached to the ball. The spinning cue ball has an angular momentum, and it has a magnetic moment due to the bar magnet. This is much like an elementary particle such as a proton or electron, both of which have an angular momentum and a magnetic moment. Unlike elementary particles, however, we can easily vary the angular momentum of our "particle" by spinning it using our fingers!

A. The cue ball has an obvious symmetry axis, which is the axis of the small black post that you can grab to spin it up. However, when you try to spin up the cue ball, you will usually see that the ball is not spinning around the axis of the post, and the axis of the post will therefore not be stationary in the lab frame. You can rectify this situation by using the tip of a pencil or pen. Once the ball is spinning, hold the pen tip gently against the side of the post, and you will see that you can force the ball to spin true about the axis of the post. This takes a little bit of practice. For this lab, it will suffice to spin the ball up with your fingers.



The line (which you should draw on the ball if it is not already there) is used in conjunction with the video camera to determine the rotation frequency of the ball.

B. Note that our gyroscope is (nearly) a sphere. As you found out last semester, it suffices to treat it as a sphere of mass *M* with a moment of inertia $I = 2MR^2/5$. Does the post add a torque? How much torque might it add?

C. The two coils provided create a magnetic field \vec{B} that points in the vertical direction \hat{z} . The magnet in the ball has a magnetic moment vector $\vec{\mu}$. The torque on the ball is given by the torque on the magnet, which is

$$\vec{\tau} = \vec{\mu} \times \vec{B} = -\mu B \sin\theta \widehat{\Phi}, \tag{0-1}$$

where θ is the angle between the polar axis of the sphere and the vertical (*z*) axis and $\hat{\phi}$ is the unit vector in the azimuthal direction (The negative sign in this equation appears because of the convention for the direction of the azimuthal unit vector $\hat{\phi}$).





Now, from Eq. 4-1 derive an equation for the precession frequency Ω_p for a given spin frequency ω of the ball. You will need to decompose the angular momentum vector \boldsymbol{L} into vertical and horizontal components, after which the remainder of the derivation is essentially identical to that for the gyroscope in your textbook. Complete this derivation before you come to lab. This is essentially identical to the calculation you did last semester, except that the gravitational torque has been replaced by a magnetic torque. Does θ appear in the result? It should not! When carrying out the experiment, use values of θ that are approximately 45 degrees in order to make data collection simplest.

Experiment

Equipment:

- Cue ball with embedded magnet
- Air bearing and pump
- A set of Helmholtz coils (diameter = 22 cm, 150 turns)
- Sorenson DC power supply and cables
- Video camera
- Stopwatch
- Hall magnetic field probe
- labPro input box
- Voltage probe (optional: for measuring B vs I)

The cue ball sits in an air bearing, which is simply a brass cup with a small hole in the bottom. Air (provided by a fish tank pump) passes through the hole at the bottom of the cup, preventing direct contact between the cue ball and the brass (provided the flow is set to the correct level). Verify that your air bearing works properly by spinning the ball. The cue ball has a mass of 140 g and a radius of 2.7 cm. The strength (dipole moment) of the magnet inside the ball is $\mu = 0.4 \text{ Am}^2$.



Gyroscope apparatus. The right-hand picture shows the cue ball sitting in the air bearing. The line on the ball is used to count rotations (with the video camera). The string is used to count precessional cycles (with a stop-watch). You do not use the rod and weight in this experiment.

Spin the ball by grabbing the black post while the ball is in the air bearing. Please make absolutely sure not to drop or otherwise abuse the ball. When spinning at several rotations per second, the ball should continue to spin freely for many minutes. If this is not the case, check that (a) the ball and brass cup are free of dirt and debris and (b) the air flow is set high enough. The air bearing is fixed in a plate that is in turn inside a set of coils.

Before starting, first make sure you practice spinning the ball up with your fingers and using a pen or your finger to get the ball to spin "true" about the rod axis without wobbling. Second, figure out how to measure the frequency of rotation of the ball with the camera. It will be sufficient to take 5 seconds of video in order to do this. At the usual frame rate of 1/30 of a second, you can easily count the number of rotations and divide by the total elapsed time. If there is not already a line drawn along a "longitude" of your cue ball with a Sharpie marker, put one on it. You will need to know the frequency of rotation for each trial. Do this as you are taking the data! One member of the group can take care of it. It is not necessary to store all of these videos.

You will also need to measure the precession period, which is the time for the rod to trace out one circular orbit. This is most easily done by using a fixed reference, such as a string hanging vertically, and a stopwatch. You might want to time a few cycles to get your error down to a few percent of the period. In principle, you could also measure the precession period by video, but you would end up with a lot of video, since, as you will see, the precession is rather slow.

There are two coils, which should be separated by a distance equal to their radius (11 cm). In other words, these are Helmholtz coils. The coils should be hooked up to the power supply so that the current in each flows in the same direction. Although the voltage drop across the coils is not very large, they can get quite hot, and so do not leave the current above 3 Amps for a prolonged period of time. Using your field probe, quickly determine the scale factor between the current flowing in the coils and the magnetic field at the center of the coils.

Precession due to magnetic torque

A. Verify that the ball does not precess when the magnetic field is off.

B. Turn on the power supply to the Helmholtz coils and set it to 2 A. If you find that you cannot increase the current enough, the reason is that the voltage limits (the two knobs on the right) are not set properly. You can turn those knobs fully clockwise. Also, make sure that the meter on the supply is reading current (not voltage). Finally, note that there are two knobs (coarse and fine) for adjusting the current. Your ball should now be precessing. If it is not, then it is likely that one of your coils is connected backwards.

C. Measure the precession frequency for approximately the same value of the spin frequency for several different values of the magnetic field (both positive and negative). To adjust the field, try changing the current in 0.5 A increments. What happens when you reverse the sign of the field? Graph your data. You will have to deal with the fact that ω was not exactly the same for each current, but you can cope with this by plotting the product $\Omega_p \omega$ vs. field, where Ω_p is the precession frequency. How does the precession frequency depend on magnetic field? Can you use your data to determine the magnetic moment μ of the bar magnet inside the ball? Does it agree with the value given above?

Oscillations about equilibrium

Without spinning the ball, nudge the post slightly from the vertical position. You will notice that for one sign of the magnetic field, the post will oscillate back and forth, while for the other, the post will try to invert and end up resting against the edge of the brass cup.

Using the stopwatch, measure the period of oscillation (the elapsed time for each cycle) for several different values of the current. It is best to count 10 or 20 cycles and divide by the total time to get the period. Do not worry if the plane of oscillation drifts (precesses) as you do the measurement. It will not affect your data. Do this for several currents between 0 and 3 A.

What is going on here? You have probably noticed that the ball is acting like an "upside down" pendulum, but unlike an ordinary pendulum we can "control" the magnitude of "gravity" by using the magnetic field:

$$\vec{\tau}_{mag} = \vec{\mu} \times \vec{B} = -\mu B \sin \theta \widehat{\Phi} \approx -\mu B \theta;$$

 $I \frac{d^2 \theta}{dt^2} = -\mu B \theta,$

where we have used Newton's second law for rotational motion and the small angle approximation $\sin \theta \approx \theta$.

Compare this with the equation of motion of a pendulum. We have a simple harmonic oscillator with a period

$$T = 2\pi \sqrt{\frac{I}{\mu B}}.$$

Note that the period should vary as the inverse square root of the field (or current in the coils). Is this what you observe? Use this to calculate the magnetic moment. Does it agree with the value you found above?

Laboratory 5: Faraday's Law

In this lab, you will explore Faraday's law of induction and its consequences.

Problem 5.1: Magnetic Induction

Goal: Explore the properties for the magnetic induction of an EMF and calculate the magnetic dipole of a magnet.

Suggested Reading: Read the section in your text about magnetic induction and magnetic flux.

Background

In this problem, you will take "single-shot" data of a bar magnet passing through a loop. This is simply a series of data points as a function of time collected for a single trial of an experiment.

Before you begin the experiment, sketch what you expect the induced EMF in a wire loop as a function of time would look like if a bar magnet with a velocity, *v*, passed through the loop along its axis. Assume that the north end of the magnet passes through the loop first. You will have to keep track of the helicity (clockwise or counterclockwise) of the coil in order to determine the overall sign correctly.

Describe the shape of the EMF vs. time graph you sketched. Also determine the sign of the EMF. How would your graph change if the orientation of the magnet were changed with respect to the loop? How might it change if the magnet passed outside of the loop?

What is the equation for a magnetically induced EMF in this situation? Your only unknown in the equation is the magnetic moment. Try to work through the math, keeping in mind that your variables of time, distance from the coil, and velocity of the cart are related through the kinematic equations. This calculation is fairly tricky, but a student of your level should be up to it. Some of the steps will be similar (math-wise) to other calculations you have done, such as the calculating the electric field along the axis of a charged loop. Express your final answer as a function of coil radius *R*, magnetic moment *m* (or μ , if you will not confuse it with μ_0), velocity of the cart *v*, and distance from the center of the coil *x*. Be prepared to convert this into a function of time as well.

Experiment

Equipment:

- Coil of wire encased in plastic, 200 turns,
- Aluminum Track,
- Bar Magnet, and bar magnet remagnetizer
- Cart,
- LabPro Interface,
- Vernier Voltage Probe.



To generate a change in the magnetic flux through the coil, you will use a somewhat crude setup. As shown in the diagram, set an aluminum track at a low angle, using wooden blocks, such that it passes through the wire coils. You can determine what direction the magnets should face.

Predict what a graph of the induced potential (EMF) as a function of time will look like from the moment that you release the cart to freely roll down the ramp until it is sufficiently far away from the coil.

NOTE: before using the bar magnets **ALWAYS** remagnetize them. These bar magnets are highly susceptible to losing their strength over time. It is simpler to assume they have gone bad and refresh them each time than to attempt to use them only to find out they are too weak.

Experiment

Position the coil so that the bar magnet travels through its center and as close to perpendicular as you can get. Combine three bar magnets and place them in one of the carts. Set the cart on the track and let it roll through the coil. To measure the induced EMF, connect the voltage probe to your Lab Pro[®] interface, and then connect the two clips to the terminals of the coil.

Open Logger Pro[®] on your computer. You will need to make sure that you can collect enough data and at a reasonable rate (a couple hundred points per second should do the trick).

Collect data (about 10 "single shots") of the bar passing through the coil, changing the initial height the cart is released from (and thus its velocity as it passes through the coil). Look at the induced potential vs. time graphs that you made in Logger Pro[®]. Notice how there is a part of the graph where the voltage dips down and another where the voltage dips up. Compare the areas underneath these curves. Are they the same? Describe what is happening, referring to these graphs, and explain why they look this way. Compare these plots to your prediction at the beginning of the problem. Does your prediction create a similar shape to your results? How can you use the results to measure the magnetic moment of the bar magnets? Using what you know about magnetic moments, figure out a second method to measure the magnetic moment of your magnets to verify your results.

Problem 5.2: The Generator

Goal: To explore the properties of a simple electric generator.

Suggested Reading: Read the sections on magnetically induced EMF.

Background

Your basic generator is a simple application of Faraday's Law. In general, to generate an alternating current, you only need a coil that can spin in a uniform magnetic field. The field is generated by permanent magnets. For this lab, we don't have magnets large enough for you to use, so we will be inefficient and create our own field using a set of Helmholtz coils (see Lab 4, Problem 3 if you need a refresher on Helmholtz coils). We will also be using a motor to turn the coil in the field.

Consider a diagram of the setup shown to the right. A small, square coil with 4,000 turns and sides of length *A* is mounted between Helmholtz coils. The coil is attached to a motor, which spins it about its axis parallel to the plane of the Helmholtz coils. A power supply is required to power the motor.

The Helmholtz coils are connected to a second power supply and generate a uniform magnetic field, B_o , in the region of the small coil.



What will affect the induced EMF in the small coil? Name and describe these variables.

Write an equation for the magnetic flux through the small coil when it is stationary and at some angle to the magnetic field. Next, assume that the motor rotates the small coil with an angular speed, ω . Write an expression for the change in magnetic flux through the small coil as it turns. What is the induced EMF in the small coil as a function of time?

Sketch a graph of the induced EMF of the small coil vs. time while it is in the uniform field, B_o . Make another graph of the angle, θ , the coil makes with the magnetic field vs. time. Compare these graphs in detail.

There are two ways to describe the amplitude of the induced EMF. One way is to look at the peak-to-peak voltage, Vpp. This is the potential difference between the highest and lowest values of the AC voltage. Another way to measure the amplitude of an AC voltage is to express it in terms of a reduced mean square (V_{RMS}) . The relationship between Vpp and V_{RMS} is given by

$$V_{RMS} = V_{pp} / 2\sqrt{2}.$$

In this experiment, you will be measuring both of these values on different parts of the apparatus. In order to compare the two, you will have to express your results in one of these forms.

Experiment

WARNING: You will be working with equipment that generates large electric currents. Improper use can cause painful burns. **To avoid danger, the power should be turned OFF and you should WAIT several seconds before disconnecting any wires.**

Equipment:

- Helmholtz 150 turn with motor and 4000 turn loop
- Two power supplies
- Hall probe field sensor
- Voltage probe
- labPro input box

Remember that B_0 will be proportional to the current flowing through the Helmholtz coils. Using a field sensor, determine the constant of proportionality.

Plug the voltage probe into the LabPro interface and attach the clips to the two ends of the small coil. You will need to set the probe to its most sensitive setting. The ends of the small coil are attached to terminals (small wires sticking up) located near the bearings on opposite ends of the axle about which the coil spins.

Now connect the motor to a power supply and note the appearance of the potential difference versus time display. Do not exceed 5 V on the DC motor! Please keep this in mind if you are using one of the lab supplies, which go up to 20 V. To achieve slower speeds, you can start the motor at higher speed and then reduce it.

WARNING: If you induce too high of a current through the 4000 turn loop you will melt the wires and destroy the equipment which is very expensive to replace. Keep the induced current low and **never** hook the loop directly up to a power supply!!

Try to measure the angular speed of the small coil. Use Logger Pro resolve the induced EMF vs. time graph for your generator. Then use a DMM to measure the RMS voltage, V_{RMS}, from the generator. Compare this to the result in Logger Pro[®]. Verify that the magnitude, frequency, and phase of the voltage signal measured in Logger Pro agree with your predictions.

Take about 10 data points to show how the EMF depends on: a) the magnetic field, B_o , and b) the rotation speed of the small coil. Compare these results to your predictions.

Problem 5.3: Time-Varying Magnetic Fields

Goal: To explore the properties of a variation of electric generator.

Suggested Reading: Read the sections on magnetically induced EMF.

Background

We will now explore another means by which we can induce an EMF in a coil. Consider a situation similar to that in Problem 3. Consider a diagram of the setup shown to the right. A small, square coil with 4,000 turns and sides of length *A* is mounted between Helmholtz coils. The coil can rotate about an axis perpendicular to the axis of the Helmholtz coils. Assume that at any given point, the small coil will be fixed at some angle, θ , with respect to the horizontal.

The Helmholtz coils are connected to a *function generator*, and generate a timevarying magnetic field, $B_o(t)$, in the region of the small coil. Write an equation for that $B_o(t)$ depends on current, I(t), through the coils supplied by the function generator.



Write an equation for the magnetic flux through the small coil when it is stationary and at some angle, θ , to the magnetic field.

Experiment

Equipment:

- Helmholtz 150 turn with motor and 4000 turn loop
- Pasco function generator
- Hall probe field sensor
- Voltage probe
- labPro input box
- a DMM

Use the sine function on the function generator. When the Helmholtz coils are connected to a function generator, an alternating current goes through the coils. Use only frequencies of less than 100 Hz.

- Set the small coil so that its area vector will be parallel to the magnetic field of the Helmholtz coils. Measure the magnetic field (with the Hall probe) and the induced EMF in the small coil simultaneously. Repeat this for several different frequencies and compare the results to your predictions. Do you observe what you expect (both qualitatively and quantitatively)? Compare the phases of the two graphs (voltage and B-field). Consider plotting the voltage over *the coils* vs. measured B-field. How does this relate to what you see?
- Rotate the coil a few degrees and make your measurements. Take data points over a full rotation to show how the induced EMF depends on the angle θ that the small coil makes with the magnetic field. Make an appropriate graph and compare to predictions.

Laboratory 6: RLC Circuits and AM Radio

In this lab we will explore the phenomenon of resonance in RLC circuits and its application to a practical problem: AM radio.

Read the section of your textbook on RLC circuits. Also, review the material on capacitance and dielectrics.

Problem 6.1: RLC Circuits and Resonance

Goal: To explore the properites of RLC circuits with alternating current.

Suggested Reading: Read the text on RLC circuits, paying special mind to information on the resonance (or 'natural') frequency of such circuits.

Background

In this part of the lab you will study a simple circuit with a resistor, inductor, and capacitor in series. You will build a resonant circuit and use it to make quite sensitive measurements of capacitance. Since capacitance depends on a dielectric constant, you can use this technique to measure the dielectric constant of materials.

Experimental Set-up



Figure 6.1.1: Schematic of the experiment: Note the special instructions. The resistor is used only in Part ID. Banana cables should be used for the connection from the plates to the oscilloscope to avoid additional capacitance in parallel with the plates. Note that the outer terminals of the BNC's on the scope are grounded. You can use a BNC coaxial cable to connect the TTL output of the function generator to the scope. See text for additional details.

3.

Equipement:

- BK precision function generator
- Oscilloscope
- Parallel plate capacitor set-up
- Various capacitors and resistors
- Coaxial adaptors
- Various wires and connectors
- Plexiglas, Pyrex, and Mylar dielectrics

The Circuit

Refer to Figure 6.1.1 above to guide you during your setup. Set up your experiment as a *series* RLC circuit. The capacitor in the experiment consists of two aluminum plates. A 0.01 Henry inductor (the precise value is printed on the plate) is connected permanently to the round plate, which is the upper plate of the capacitor. The inductor also has some resistance, which, along with the output impedance of the function generator, makes up the "R" in the RLC circuit.

At this point, you should leave out the $2 k\Omega$ resistor, and connect the output of the function generator directly to the inductor. Use *banana cables* (not a coaxial cable) to make the connections between the oscilloscope and the plates and between the function generator and the top plate. You can put a BNC to banana adapter on the scope input. *Make sure that the ridge on the banana adapter, which corresponds to the ground terminal of the BNC, is connected to the cable that goes to the <u>lower</u> plate of the capacitor.*

The experiment will work much better if you insert small capacitor (~ 20 pF or so) in series between the top plate and the banana cable (see Figure 6.1). Note that you can loosen the brass jack on the plate and tighten one lead of the capacitor underneath it. Furthermore, once you set up your wires, do not move them around during your measurements.

Meters and Instruments

Search for a resonance, which for the setup of Fig. 6.1.1 will correspond to a maximum the voltage output across the capacitor (make sure that you understand why). Note that you can measure resonant frequencies in this experiment to about 1 part in 1000. *Hence, make sure that you are reading the frequency to <u>at least 3 digits</u>. Your oscilloscopes can be used as frequency meters.*

Make sure that you connect the "TTL" output of the function generator to Channel 1 of the oscilloscope. The TTL signal is a square wave that is synchronized to the output of the function generator.

The signal output of the function generator should be connected ONLY to the RLC circuit (through a *banana cable*). The plates should be connected to Channel 2 of the scope as shown in the figure. There may be a phase shift of 90 or 180 degrees between the Signal and TTL outputs of the function generator, but this will be independent of frequency and will not matter to us.

Confirming the Setup

Before proceeding, confirm the following. Put the top plate of the capacitor on your work bench (i.e. physically apart from the bottom plate). Confirm that the resonant frequency of your circuit is greater than 200 kHz (it will probably be somewhere around 250 kHz) and that it is relatively stable.

If necessary, tie down your wires, keeping the two banana cables going to the scope as far apart as possible. Note that with the two plates completely separated, the capacitance would ideally be zero and the resonant frequency infinity. The fact that this is not the case is due to *stray capacitance*. One culprit is

capacitance to ground at the input of the oscilloscope, which is in parallel with our plate capacitor. The other stray capacitances are between the cables connected to your experiment. The effect of the stray capacitance is reduced by the small capacitor (20 pF) placed in *series* with the top plate. Think about how this works. Draw a circuit schematic that includes the parallel plates as a capacitor, the series capacitance (20 pF in the figure), and the stray capacitance between the two leads. Finally, note that YOU (or anything else connected to the plates) can be a significant source of stray capacitance if you are touching one of the plates. Keep this in mind while doing the experiment.

The bottom plate of the capacitor has spacers at the corners which are used to adjust the gap between the plates. Each of these spacers is 0.017 ± 0.001 inches thick, and the diameter of the top plate is 6.00 ± 0.05 inches. Note that the spacers occupy only a small fraction of the volume between the plates, so that the medium between the plates is effectively air for the first part of the experiment.

Part I: Resonance

- A. For this part of the experiment, set the spacing between the plates to about 2 mm. As discussed in your textbook the RLC circuit has a *resonant frequency* at which the voltage drop across the capacitor is a maximum. Your first task is to find the resonance by varying the drive frequency until you see the maximum response. The resonance will be a sharp peak. From the given inductance and other properties of your circuit, including the geometry of the plates, estimate the resonant frequency, and then find it experimentally. Record the resonant frequency as well as the amplitude of the resonance, but this will not happen in practice due to capacitive coupling between the leads. Minimize this by keeping the wire going from the function generator to the inductor far away from the cables going to oscilloscope.
- **B.** You should be looking at the voltage across the capacitor on Channel 2 of the oscilloscope and the TTL output of the frequency generator on Channel 1. Set the scope on alt or chop so that you can look at both channels simultaneously. As you sweep through the resonance, you will see a maximum response in Channel 2, but also pay attention to the *phase* of the output signal, i.e. the position of the sine wave showing on Channel 2 relative to the signal square wave showing on Channel 1. With the circuit tuned to resonance, sketch the two signals in your lab-book. Now, increase the frequency so that the circuit is no longer on resonance. Sketch the two signals in your lab-book for this frequency. (You may have to increase the Channel 2 sensitivity in order to see the output signal.) Compare the two sketches that you have made. How much (in degrees) does the phase of the output signal change relative to the drive signal as you tune the drive frequency from $f << f_0$ to $f >> f_0$, where f_0 is the resonant frequency?
- **C.** Measure the voltage across the capacitor as well as the phase shift as a function of the drive frequency. Record carefully at least 20 data points, including several close to the resonant frequency and others further away. Note that you will need to record data in very fine frequency steps (with a resolution of about 1 kHz) close to the resonance. Be patient! Plot your data (amplitude vs. frequency and phase shift vs. frequency) to see the full resonance curve. Recall that the quality factor *Q* of the circuit is defined as the ratio $f_0/\Delta f$, where Δf is the width of the resonance curve at the point $V = V_{max}/\sqrt{2}$. What is the quality factor *Q* of this RLC circuit? Estimate the equivalent series resistance of your circuit. Compare with the resistance of your inductor as measured with an ohmmeter. Where else does resistance lurk in your circuit?
- **D.** Place a $2 k\Omega$ resistor in series with the capacitor-inductor combination as shown in Figure 6.1. Repeat Part C, plotting your data on the same scale. How does the increase in resistance affect the resonance curve? Comment on the height and width of the resonance peak. Does the resistor change the resonant frequency? What is the Q of the circuit now? Suppose that you were building this RLC circuit into the tuner of your stereo. Would you want a high Q or a low Q for such an application?

Part II: Measurement of the Dielectric Constant

For this part of the lab, you will measure the dielectric constant of air and either Plexiglas, Pyrex, and PVC.

- **A.** REMOVE the 2 k Ω resistor. Verify that the resonance is again sharply defined as you tune the frequency. We will now use the RLC circuit to measure dielectric constants.
- B. Measure the resonant frequency as a function of the number of fiberglass spacers between the plates. Determine the capacitance for each data point. (Remember that you know the inductance of your circuit. It should be written on the top plate of the capacitor.) Determine the separation *d* of the plates for a given number of spacers. Plot the capacitance as a function of 1/*d*. From the slope of this graph and the geometry of the capacitor, determine the dielectric constant of air.
- **C.** What is the meaning of the *intercept* of the graph that you have just drawn? Note that the capacitance of an ideal parallel plate capacitor goes to zero as the separation *d* increases to infinity, so that in principle a plot of C vs. 1/*d* should pass through the point (0, 0). This does not occur due to the stray capacitance discussed above. Verify this statement by making the distance between the capacitor plates very large (several inches). Measure the resonant frequency and then determine the corresponding stray capacitance. How does this compare with the intercept of your C vs. 1/*d* plot?
- D. Repeat part B using plates of one of the dielectrics provided (Plexiglas, Pyrex, and PVC). (Do not use the fiberglass spacers for Parts D and E.) What is the dielectric constant? Warning: there may be different types of Plexiglas. Make sure all of the plates you are using come from the same batch. This will be apparent from the shape in which they were cut.

Problem 6.2: Construction of an AM Radio

Goal: Make an AM radio.

Suggested Reading: Research the construction of AM radios. Look up your local AM radio listings.

Background

You now know enough to construct an AM radio. Research how to do it. (There are numerous resources on the web. Cite the ones that you use.) Try searching under "AM Crystal Radio." You must do this research before coming to lab.

Experiment

Equipement:

- Oscilloscope
- BK function generator
- Diodes, resistor, and capacitors
- a tunable capacitor from about 10 - 350
- "tank circuit" inductor
- BNC cables and
- Various wires and
- Battery powered
- Antenna
- Braided grounding
- 1. Diodes We have

germanium diodes (1N34A's) that can be *rectifiers*. First, you might



Figure 6.2: Circuit for measuring the current vs. voltage characteristic of a diode.

used as want to

figure out how a diode works. Use a 1N914 diode from our supply to wire up the circuit in Figure 6.2. How does the current flowing through the diode depend on the voltage drop across the diode? Measure the current at 10 points covering *both* polarities of the voltage and make a graph.

Build a simple *half-wave rectifier*. Replace the DC power source with a function generator and hook up an oscilloscope across the resistor as shown. Sketch what happens when driven with an AC signal greater than 2 volts or so. Based on what you saw in the previous exercise, explain what is going on. Finally, replace the scope with a DC voltmeter. Explain why you can now measure the magnitude of the AC signal with a DC voltmeter.



Figure 6.3: Half-wave rectifier.

The germanium diodes work the same way, with a smaller *forward voltage drop*, allowing you to *rectify* the much smaller signals you will be detecting with your radio at frequencies of 600 – 1500 kHz. Make sure you understand how you can use such a diode to detect an *amplitude-modulated* (or AM) radio signal. You should do research on the diode receiver before coming to lab.

2. You will need to build a tunable *tank circuit*. A tank circuit is an LC circuit with the inductor and capacitor in parallel. Look it up and figure out how it works. The resonant frequency is determined by the same condition for the series LC circuit, but on resonance the currents flowing through the inductor and capacitor are equal and *opposite*. They can be much larger than the current flowing into and out of the tank circuit from the outside world. This is what allows the tank circuit to work as a sensitive radio receiver.



We have tunable air gap

Figure 6.4: Tank circuit (left) and a tapped coil right. The taps are simply sections of uninsulated where you can attach clip leads. Note that this allows you to vary the inductance in discrete increments.

capacitors and coils with "taps." The taps allow you to vary both the total inductance of the coil, as well as the "loading" of the tank circuit by the diode receiver. Keep in mind that you will need the tank circuit to tune over the appropriate frequency range for AM radio, and therefore you must choose an appropriate value of the inductance for your coil. Given that the range of your capacitor will be from about 10 - 400 pF, make sure that you choose your inductance intelligently. The insulation should be scraped off the wires at the taps. Make sure that this is the case! The banana clips must make contact with uninsulated wire at the point where you are attaching the clips. When attaching the

variable capacitor, note that the black terminal should be at ground. The black terminal is attached to the chassis of the capacitor and hence to the knob that you are turning.

To test your tank circuit: Hook up the variable capacitor across two of the taps (labeled A and D in the accompanying figure) of the inductor. Connect a long wire (the antenna) to another tap (labeled B). Connect a cable from another intermediate tap (labeled C) to Channel 2 of your oscilloscope. To trigger the oscilloscope, connect the TTL output of your function generator to Channel 1. Finally, take a single banana cable and connect it to the signal output of the function generator using a BNC to banana adapter. This wire should be brought into close proximity with your antenna wire, but do not connect them directly. Wrapping the insulated banana cable around the insulated antenna wire will be fine.

The connection to ground is extremely important, because the currents flowing from the antenna to ground are extremely small. Please consult with the TA on a suitable choice of ground.

Check your wiring carefully, particularly the grounds, and set your function generator to a frequency of 1.0×10^{6} Hz (1.0 MHz). Set the output to a maximum and look at Channel 2 of your oscilloscope as you tune the variable capacitor. If you have chosen the correct set of taps A and D (i.e. the correct inductance), you should see a strong resonance as the tank circuit is tuned through the 1 MHz frequency of your function generator, which is acting as a radio transmitter. If you have wired the circuit properly and chosen the correct taps A and D, you should be able to tune the circuit to frequencies from around 600 kHz to 1.5 MHz.

Make sure that the quality factor Q of your tank circuit is at least 50 and that you can tune it from about 600 – 1500 kHz before proceeding.

3.





Hook up the diode detection circuit using the germanium 1N34A diode and a series resistor of $80 - 100 \text{ k}\Omega$ as shown in the second figure. For the moment, hook up the scope (Channel 2) where the speakers are shown in the circuit. Do you see what you expect as you tune the "transmitter"? Because of the diode, you should see a dc voltage when your tank circuit is tuned through resonance. Why? This is an extremely important step, as it confirms that your "radio receiver" is working.

Before proceeding: turn off your function generator! You are now going to try and find a real radio station.

- 4. The output of your tuner/detector will have a relatively high output impedance, which is not suitable for driving a speaker or even modern headphones. We have several sets of amplified speakers (such as for an Ipod dock). These have an input impedance of a few kΩ, as opposed to the 8 ohm input of a typical speaker. Hook them up as shown in the figure.
- 5. Test your radio in the obvious way! How well does it tune? Explain how the Q of your tank circuit is related to your ability to isolate a single station. The easiest station to find around here is Radio K (770 KHz). When you find a station, how can you determine exactly at what frequency it is broadcasting? Here is a simple trick: while listening to the station, turn your function generator back on, put the banana cable from the function generator nearby, and vary the frequency. As the frequency approaches that of your radio station, you will hear a high-pitched whistle that will drop dramatically in frequency as you tune through the exact frequency of your station. This technique is known as heterodyning and relies on the phenomena of "beats." Can you explain it?
- **6.** Your lab-book should include a complete description of the important steps in the design and construction of your radio, including calculations and drawings. List the radio stations you could find along with their broadcast frequencies.



Figure 6.6: An AM radio. Note that the antenna must be strung out over 10 - 20 feet outside the building in order to get decent reception. The ground connection is critical. To debug the circuit, you can hook up an oscilloscope in place of the speakers. There will be a speaker jack on the Lucite boards that have the terminals for connecting the other components.

Appendix A: The Oscilloscope

In this course, you are using modern digital oscilloscopes. These are so easy to use (with built-in help menus) that it is not worth including a manual in these notes. In virtually all cases, you will be using the scope in two channel mode, for which connections are shown here.



Connections for an oscilloscope in two-channel mode. Make sure both channels are displayed on the scope. The "outer" connections of the coaxial BNC connectors, which are the rings around each of the solid dots, will automatically be connected IF you are using coaxial BNC cables. **The outer shell of the BNC connections on the scope is always grounded**. Note that in some cases, it is convenient to "split" the output of the function generator using a "tee" and send one signal to CH1 of the scope and the other to the experiment. In this case, you would not use the SYNC output. In some cases, you may also wish to use the external trigger connection for the scope.

One of the most important secondary features of the oscilloscope is the "Measure" button. By pressing this button you will have the choice of which channel and which variables of that channel you will see displayed on the screen. The most useful values to display are: Pk-Pk amplitude, phase (in degrees), and frequency.

There are two important things you must understand about oscilloscopes

1. Triggering: The scope provides what is essentially a running trace (voltage *vs.* time) of whatever is hooked up to the input. The scope must know, however, exactly at what point on the signal it should start plotting each trace. Otherwise, the signal that you see on the screen would keep drifting. If you have your circuit hooked up but either do not see a signal, the signal is drifting, or you simply see gibberish, then your scope is not triggering properly. On new scopes if you hit "auto-setup" after hooking up your circuit, the scope will probably find the right triggering conditions. Otherwise, look at the instruction manual or ask for assistance.

2. Grounding: The connections on the front of the scope are coaxial BNC's. The outer ring of a BNC is always grounded unless explicit steps are taken to isolate it. This means that when you are hooking up the inner and outer leads of a BNC to your experiment, the lead that is connected to the outer part of the BNC will also be grounded through the scope. If you are using a function generator, which probably has its own ground, then you must make sure that the scope ground and the function generator ground are at the same point in the circuit. A circuit can only have one ground point! For example:



This circuit will work. The scope and the function generator share a common ground point. Note that the ground connections are not always explicit. They may be built into the instruments.

This circuit w so that no cur case, you wou remove one oj

Appendix B: Other Equipment

ELECTROSTATIC PAPER AND ACCESSORIES:

To investigate electric fields with the electrostatic paper, you need to do the following:

- Lay the electrostatic paper flat. .
- Distribute the pieces of metal (called "electrodes") on the paper, in the configuration whose field you wish to examine. The tips of the long brass rods may also be used as electrodes, to create point-like charges.
- Connect the electrodes to a source of charge. This is done by connecting a wire from the positive ("+") side of the battery or power supply to one electrode and the wire from the negative ("-") side to the other as shown in Figure 1.
- You may wish to place a wooden block on top of the brass rods to increase contact pressure with the paper. This can increase the magnitude of the electric field created on the paper. It also helps to place an extra sheet of paper under the electrostatic paper.

Figure 1: Electrostatic paper Setup







To measure the electric field from the charged electrodes, you will use a probe connected to a digital Multimeter set to measure volts (see Figure 2). For best results, turn the DMM to measure in the two-volt DC range, as indicated in Figure 2.

THE DIGITAL MULTIMETER (DMM)

The DMM is a common piece of lab equipment that can be used to measure various electrical quantities, most often current, resistance, and potential. The DMM's you will be using are capable of measuring both "direct current" (DC) and "alternating current" (AC) circuits. Be careful about knowing which type of measurement you need to make, then set your DMM accordingly. Some DMM's might be slightly different from the one pictured to the right.



The DMM can measure currents anywhere from 10 amps to a microamp (10^{-6} amps) . This versatility makes the DMM fragile, since measuring a large current while the DMM is prepared to measure a small one will certainly harm the DMM. For example, measuring a 1 ampere current while the DMM is on the 2 milliamp scale will definitely blow a fuse! If this happens, your instructor can change the fuse. However, if you damage the DMM beyond repair, you will have to finish the lab without the DMM.

Measuring Current:

- 1. Set the selection dial of the DMM to the **highest** current measurement setting (10 amps). Insert one wire into the socket labeled '10A' and a second wire into the socket labeled 'COM'.
- 2. Attach the DMM into the circuit as shown below:



To measure current, the DMM must be placed in the circuit so that all the current you want to measure goes **through** the DMM.

- 3. If no number appears while the DMM is at the 10A setting, move the wire from the 10A socket to the 200mA socket and then turn the selection dial to the 200 milliamp (200m) setting. If there is still no reading, change the dial to the 20 milliamp setting, etc.
- 4. When you have taken your measurement, return the DMM selection dial to the highest current setting (10 amps) and move the wire back to the 10A socket.

Measuring Voltage:

- 1. Set the DMM selection dial to read DC volts (¥=). Insert one wire into the socket labeled 'V?' and a second wire into the socket labeled 'COM'.
- 2. Set the selection dial of the DMM to the **highest** voltage measurement setting. Connect the two wires from the DMM to the two points between which you want to measure the voltage, as shown below.



To measure voltage, the DMM must be placed in the circuit so that the potential difference across the circuit element you want to measure is **across** the DMM.

3. If no number appears, try a different measurement scale. Start at the highest voltage scale and work your way down the scales until you get a satisfactory reading.

Measuring Resistance:

The element whose resistance you are measuring **must** be free from all other currents (due to other batteries, power supplies, etc.) for the DMM to work. That means you must **remove** it from a circuit.

To measure resistance:

- 1. Set the DMM selection dial to measure ohms (Ω). Insert one wire into the socket labeled 'V Ω ' and a second wire into the socket labeled 'COM'.
- 2. Make sure that the circuit element whose resistance you wish to measure is free of any currents.
- 3. Attach the wires across the circuit element, as shown in the example below.



4. If no number appears, try a different measurement scale. Use a logical method that covers all scales, such as beginning at the largest scale (20 M Ω) and working your way down.

CATHODE RAY TUBE (CRT) AND ACCESSORIES:

Use of the cathode-ray tube and its relatives is widespread. It is the heart of many familiar devices, from your computer monitor to your television. The following is a sketch of the tube you will be using and its connections.



For CRT tubes (National Electronics): D_{tot} = 12.5 cm

Y deflection plates L = 2.5 cm S = 0.35 cm D= 7.25 cm

X deflection plates L = To be determined in lab 2.3 S = 0.35 cm D= To be determined in lab 2.3

The x-deflection plate *begins* 0.5 cm in front of the end of the y-deflection plate.

How the CRT works:

Within the electron gun:

- A thin filament (represented above as a coil of wire), similar to a light-bulb filament, is heated by a current. When the CRT is operating, this filament can be seen as an orange, glowing wire. This hot filament ejects slow-moving electrons.
- Some slow electrons drift toward the high-voltage "acceleration plates." These plates are labeled as Vacc in Figure 3. The electric field between the charged plates accelerates the electrons to high velocities in the direction of the fluorescent screen. The final velocity of an accelerated electron is much greater than its initial "drift" velocity, so the initial electron velocity can be ignored in calculations.

After the electron gun:

- Before hitting the screen, the high-velocity electrons may be deflected by charged plates along the length of the CRT. These charged plates are usually called the "x-deflection" and "y-deflection" plates.
- When the electrons reach the end of the tube, their energy causes the material that coats the end of the tube to glow. This material is similar to the material inside fluorescent light bulbs. The end of the CRT is called the fluorescent screen.

To supply the necessary electric potentials to the CRT you will use a power supply. The power supply provided has the proper potential differences to heat the CRT filament and to accelerate the electrons. The

power supplies we use also have built-in circuit breakers. Should you attempt to draw too much current from your power supply, it will shut itself off with an audible "click." If this happens, check to make sure all of your wires are connected properly, then press in the small white button on the side of the power supply.

Note that the CRT and power supply come as a set, and many of the connections are color-coordinated to avoid potentially damaging misconnections. Please verify that you have a correctly matched pair of supply and CRT.



WARNING: You will be working with equipment that generates large electric voltages. Improper use can cause painful burns. To avoid danger, the power should be turned OFF and you should WAIT at least one minute before any wires are disconnected from or connected to the power supply.

To properly connect the CRT's to the power supply:

- 1. Turn the power supply off.
- 2. Connect the power supply ports marked "AC 6.3V" (they are green; the voltage differs slightly from one supply to another, but should be clearly marked) to the ports marked "HEATER" or "FILAMENT" on the CRT (these are also green).
- 3. Connect the appropriate accelerating potential across the cathode and anode. For instance, if your experiment calls for a 500 volt accelerating potential, connect the cathode to the port marked "-250 V" (which may be black or white) and the anode to the port marked "+ 250 V" (which is red). This gives a total potential difference of 500 volts.
- 4. Turn the power supply on.

RESISTOR CODES

A resistor is a circuit element manufactured to have a constant resistance. The resistance is coded onto the side of the resistor in colored bands, where the color and position of the bands tell you what the resistance is.



To read the color bands on the resistor, begin by finding the gold or silver band on one end of the resistor; this is the back of the resistor. You begin reading from the other end. Most resistors (including those you will use in lab) are coded to two significant digits. The first two color bands correspond to these two significant digits.

The third color band is called the multiplier. The number coded by this band represents a power of ten which you multiply by the number from the first two bands to get the total resistance.

The fourth color band tells you the tolerance, or error bounds for the coded resistance: gold means $\pm 5\%$ tolerance, silver means $\pm 10\%$ tolerance and no fourth band means $\pm 20\%$.

Some resistors have a fifth color band, which represents the reliability of the resistor, and can just be ignored for the purposes of these labs.





SORENSEN POWER SUPPLIES

The Sorensen power supply is an all-purpose power supply for the production of constant currents and voltages.

At the top is the main display that reads either current in Amperes or voltage in Volts. There is a switch there that allows you to switch between them.

The current and voltage controls are located in the middle. In between the constant current and constant voltage knobs is a switch that allows you to toggle from high currents to low currents.

This power supply normally operates in the constant voltage mode. As such, you can only change the voltages by using the constant voltage knobs. In the event that too much is being pulled from the power supply (as in a short), it will automatically switch to the constant current mode, where the amount of current flowing is greatly reduced. This is a signal that something is amiss with your circuit.

There is a *mater-slave* switch on the back of the power supply. This should always be set to master for the DMM to function properly. If you experience any problems, this is the first place to check.

THE MAGNETIC FIELD SENSOR (HALL PROBE)

To measure magnetic field strength, you will need a measurement probe (the magnetic field sensor) that connects to a computer through the Vernier *LabPro* lab interface.



The tip of the measurement probe is embedded with a Hall Effect transducer chip (shown below as the white dot on the end of the probe). The chip produces a voltage that is linear with the magnetic field. The maximum output of the chip occurs when the plane of the white dot on the sensor is perpendicular to the direction of the magnetic field, as shown below:



The *LabPro* allows the computer to communicate with the probe. In order to measure magnetic fields, the wire leading out of the probe must be plugged into the LabPro port labeled "CH 1". The *LabPro* itself should be plugged into the modem port of the computer. The *LabPro* turns on automatically when its power supply is plugged in. A green light on the top of the *LabPro* indicates that it is on.

NOTE: The Hall probes have yellow (sometimes black) sleeves around them to keep them from bending at a joint. Bending the probe at that joint, even just the small amount the sleeve allows, will short out the probes ability to take data. Always hold the probe by its non-data-taking end and never apply pressure near the bend.


The reason for the sleeves; while the probes were meant to bend, they were often bent in the wrong direction, which broke the connection wires. Never try to bend the probes.

BREADBOARDS

The various holes on your breadboard are connected as shown below. The lines represent the electrical connections underneath the holes.



Top-view of a "breadboard" and its internal connections.

On these breadboards, the outer strips are usually used for ground and supply voltages. Note, these outer strips are disconnected at the center of the board. There are also three banana sockets on the breadboard that should be used to connect to the power supply. You can make connections to your measurement instruments using alligator clips, which can be plugged into banana cables or BNC cables.

FUNCTION GENERATORS

The function generator has a two-terminal output (one of the terminals is grounded). You can set the amplitude and frequency of the output signal as well as the type of waveform (sine, triangle, or square). There is also a sync output that can be used as a reference. This can be useful for triggering an oscilloscope. Once again, when working with an oscilloscope, it is essential that there be only one ground in the circuit. This means that the negative terminal of the function generator and the negative terminal of the oscilloscope must be connected to the same point in the circuit. There are two types of function generator we use. The Pasco function generator which is good for low f cases (Below 100 hz) and the BK Precision, which is good for high f cases (500 Hz and above).

Appendix C: Statistics and Error Propagation

C.1 Some Basic Statistics (adapted from Keith Ruddick)

If a quantity *x* is measured *N* times, the individual measurements x_i form a distribution from which the experimenter wishes to extract a "best" value for *x*. Usually, the "best" value is the mean \overline{x} of the distribution:

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$
(B.1)

The spread or width of the distribution is an indication of the precision of the measurement. As a measure of this width we consider the deviations of the individual measurements from the mean, i.e.

$$\boldsymbol{d}_i = \overline{\boldsymbol{x}} - \boldsymbol{x}_i \tag{B.2.}$$

From our definition of \bar{x} , the sum of the deviations must equal zero. The mean deviation, defined in terms of the magnitudes of the deviations:

$$\overline{d} = \frac{1}{N} \sum_{i=1}^{N} \left| \overline{x} - x_i \right|$$
(B.3.)

is often useful, but a more useful measure of the dispersion of a measurement is the *standard* deviation σ . The variance of the distribution σ^2 is the average of the squared deviation:

$$\sigma^{2} = \frac{1}{N} \sum_{i=1}^{N} (\bar{\mathbf{x}} - \mathbf{x}_{i})^{2}$$
(B.4.)

The standard deviation is then the square root of the variance, or the root mean square of the deviations.

To be more mathematically correct, we should recognize that the measured distribution is a "sample" distribution, which will be different every time we make a different series of measurements of x, rather than the "parent" distribution for which the mean is the exact, or "true" value. The best experimental estimate of the parent or "true" standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\bar{x} - x_i)^2}$$
(B.5.)

The quantity N -1 is the *number of degrees of freedom* in the problem, which equals the number of data points minus the number of fit parameters. If you ever have to worry about the difference between \sqrt{N} and $\sqrt{N-1}$ in your data, then you should probably take more data!

Note that this σ is a measure of the uncertainty in a *single* measurement; it measures the width of the distribution of all measurements and is independent of the number of measurements! If one knows σ and the distribution of errors is what is called *normal*, then there is a roughly 2/3 probability that a single measurement will fall within one standard deviation of the mean. On the

other hand, the more times we make a measurement, the better we expect to be able to determine the mean. It can be shown that the standard deviation of the mean is given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N} (\bar{x} - x_i)^2}{N(N-1)}}$$
(B.6.)

i.e. in order to improve the accuracy by a factor 2, you must make 4 times as many measurements. This quantity is also called the *standard error* or the *error in the mean*.

We emphasize that these definitions are general, and apply to any distribution of measurements. There also exist specific theoretical distributions which can be used as models of actual data. These will be introduced in detail in Physics 2605.

C.2 Propagation of Errors (Adapted from Keith Ruddick)

Note: This section provides a "derivation" of the rules for error propagation that are stated in the body of these notes. The proof requires a little multivariable calculus. It is included here only for completeness.

Suppose we measure several quantities *a*, *b*, *c*,... each with its own standard deviation, $\sigma_a, \sigma_b, \sigma_c$... and then use these values to determine a quantity y = f(a, b, c...). What is the standard deviation of the quantity *y*?

We can differentiate the function to find how changes Δa , Δb , Δc ... in each of the *a*, *b*, *c*... affect the value of *y*:

$$\Delta y = \Delta a \left(\frac{\partial y}{\partial a} \right)_{bc_{uv}} + \Delta b \left(\frac{\partial y}{\partial b} \right)_{ac_{uv}} + \dots$$
(B.7.)

This is, of course, really the first term in a Taylor expansion, and corresponds to assuming that the partial derivatives do not change over the ranges Δa , Δb ,... If the function y=ab, or y=a/b for example, then

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \qquad \qquad \frac{\Delta y}{y} = \frac{\Delta a}{a} - \frac{\Delta b}{b}$$

respectively.

In general, we do not know the absolute errors Δa , Δb , Δc , in the measurements of *a*,*b*,*c*, but rather a quantity such as their standard deviations $\sigma_a, \sigma_b, \sigma_c$, (or the probable errors). However, from the above equations it is intuitively likely that the variances should add in the form:

$$\sigma_{y}^{2} = \sigma_{a}^{2} \left(\frac{\partial y}{\partial a}\right)^{2} + \sigma_{b}^{2} \left(\frac{\partial y}{\partial b}\right)^{2} \dots$$
(B.8.)

That this is indeed true follows from:

$$\sigma_a^2 = \frac{1}{N} \sum_{i=1}^N (\overline{a} - a_i)^2 = \frac{1}{N} \sum_{i=1}^N \Delta a^2$$
(B.9.)

and then

$$\sigma_{y}^{2} = \frac{1}{N} \sum_{i=1}^{N} \Delta y^{2} = \frac{1}{N} \sum \left[\Delta_{a}^{2} \left(\frac{\partial y}{\partial a} \right)^{2} + \Delta_{b}^{2} \left(\frac{\partial y}{\partial b} \right)^{2} + \ldots + \Delta_{a} \Delta_{b} \left(\frac{\partial y}{\partial a} \right) \left(\frac{\partial y}{\partial b} \right) + \ldots \right]$$
(B.10.)

which leads to equation A.8 if we neglect the cross-terms: This will be valid if the *a*'s and *b*'s are independent, but if they are related this correlation must be accounted for.

Thus, for example, for a function *y*=*abc*, or *y*=*ab/c*, we obtain:

$$\frac{\sigma_y^2}{y^2} = \frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}$$
(B.11.)

and for y=a+b+c:

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 \ .$$

Appendix D: Excel Spreadsheets

D.1 How To Get Started

The spreadsheet currently used in the lab is Microsoft Excel. To start Excel either:

Select it from the START / PROGRAM / Microsoft Excel menu;

Or click double click on the Excel icon (shown on your right) on your desktop.



You should see the spreadsheet screen similar to the one shown below.

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1. Figure C.1.

You may enter a value in a cell by selecting the cell (click on it with the mouse pointer) and then start typing. To edit a cell, click on it, press F2 to activate it and then edit it.

D.2. Basic Concepts

Cell Addresses

The spreadsheet consists of cells each having its unique address consisting of the row and column position it occupies. Note that the row addresses use numbers while letters are used to specify the columns. For example, cell C4 refers to the cell in the spreadsheet in Figure D.1 that contains the value -0.95892.

Cell Contents: Constants and Equations

A cell may either contain a constant or an equation (sometimes also referred to as "formula"). The difference between the two is whether the object is preceded by an equal sign: anything preceded by an equal sign is considered an equation, anything else a constant.

Constants

Typical constants in a spreadsheet are text labels such as cell A5, "TOTAL." Any number entered that is not preceded by an equal sign is also considered a constant.

Equations (or Formulas)

Though the sample spreadsheet above contains numerous equations they can not be seen because equations are normally not displayed. Usually, the computer executes the equation and displays only the resulting value. Figure D.2 is a picture of the same spread sheet as shown in figure 1 but revealing the "real" contents of the cells:

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3		3	=SIN(<u>B3)</u>	=\$C3*B\$2		
4		=B3+\$B\$2	=SIN(B4)	=\$C4*B\$2		
5	TOTAL	=SUM(B2:B4)	=SUM(C2:C4)	=SUM(D2:D4)		
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2. Figure 2

As you can see now, the majority of the cells actually contain equations. For example, all cells in column D contain equations while cells B1, B2 and B3 contain constants.

If you want to see the content of a particular cell just move the cursor to that cell and the "real" content of that cell is displayed in the window above the column headers. For example, as you can see from figure 2, cell C3 contains the equation: =SIN(B3). This information is normally displayed in the window directly above the column header (see figure D.1).

More Cell Addresses

The majority of the cells in a typical spreadsheet usually contain equations. Though such equations do not have to relate to values stored in other cells (for example: =2*7 is perfectly OK) most equations do. When an equation refers to a value in another cell, it can do so using an

absolute, a relative or a mixed reference for the address. As long as you do not move or copy a cell referring to another cell, the way absolute and relative addresses function are identical.

Absolute Reference or Address

To make a reference absolute, it is preceded by a dollar sign. The address can be absolute with respect to the column (for example: =C4), the row (for example, =C4) or both, colon and row (for example = C4).

Relative Reference or Address

Any address not preceded by a dollar sign is relative. As previously mentioned, addresses may either be entirely relative, (for example: =C4) or they can contain both absolute and relative references.

Address Range

For certain mathematical functions, an entire range of cells can be specified. This is indicated by inserting a colon between the starting cell and ending cell, for example A5:C9 indicates all cells in the range from and including A5 to C9. As you can see, though most of the times a range only spans part of a column or row, it can span multiple columns and rows. Another example is shown in cell B5, figure2, which adds the contents of cells, B2, B3 and B4.

How to Enter the Address in an Equation

Two methods exist: First, you click and move to the appropriate cell and then type the equation with the necessary address. This is difficult because it's easy to mistype the address. A better method is, to move to the cell and enter the equation till you have to enter the address. At this point move the cursor to the actual cell that you are referring to and hit return; the computer will automatically insert the address into the equation. If you are referring to a range of cells you can select those directly by moving the cursor to the first cell, then while holding down the shift key, move to the last cell in the range.

Copying

As previously mentioned, the difference between absolute and relative addresses becomes apparent when copying the content of a cell to a different location.

Copying a Relative Address

Consider the following example: we want to multiply by 2 all constants in column A and store the results in column B. As you can see from the spreadsheet in table 1, we have already entered the corresponding equation in cell B5.

	Α	В	С
5	5	=2*A5	
6	6		
7	7		

3. Table 1 (Note: these and the following tables display what is stored in the cells, not what the computer would display on the screen!)

Instead of entering subsequent equations in cells B6 and B7 we decide to copy the content of cell B5 into B6 i.e. we use Excel's copy/paste feature. When we copy the content of cell B5 into cell B6 the following happens:

	Α	В	С
5	5	=2*A5	
6	6	=2*A6	
7	7		

4. Table 2

Though we expected cell B6 to be =2*A5, the computer automatically updated the referring address because it was a relative reference.

When copying cells with relative addresses to a different location one should read them like: this reference is in regard to the cell directly to the left; multiply its content by 2. In other words, when copying relative addresses the computer will do some thinking for us.

Copying an Absolute Address

Now we repeat the example shown in table 1 using an absolute address:

	Α	В	С
5	5	=2*\$A\$5	
6	6		
7	7		

5. Table 3

When the content of cell B5 is copied into B6, the address remains unchanged.

	Α	В	С
5	5	=2*\$A\$5	
6	6	=2*\$A\$5	
7	7		

6. Table 4

Warning About Copying Data to Different Places in a Spreadsheet!

Though these examples appear rather trivial they lead to a very important point. What is shown in the above tables is not what you would normally see on the computer screen because the computer displays only the results from your calculations and so you don't know if the results were obtained using constants, or formulas with either absolute or relative addresses. Therefore, if you copy a data to another place in the spreadsheet or to another spreadsheet using the copy/paste feature, the copied data may not be the same as the original. This is almost always the case if the data your copying has relative addresses.

You have two options to avoid this problem. Either make the addresses in the original data absolute; this can be annoying at times because you have to go back and change your equations. The second approach is to use the **copy/paste special** feature: **values**. If you use this feature, the program copies the equations, evaluates them and finally only stores the resulting values as constants in the new location.

Making Multiple Copies: Filling in

Filling in: Fill Down, Fill Right

So far our examples for copying cells have been more for illustration purposes and have not yet revealed the power of spreadsheets. Most of the times, we do not just want to copy one cell but a whole range of cells. For example, going back to table 1, if we could copy the first cell multiple times in one keystroke then we could multiply all values in the first column. The process of copying a cell multiple times is called filling and depending if it is done in a row or column, it's called **fill right** or **fill down**. The fill-process involves three steps:

1) enter the equation or constant in the first cell that you want to be duplicated

2) select an adjacent range of cells into which the equation or constant will be copied. (To highlight or select, move the cursor to the first cell, hold down the shift key while moving the cursor to the last cell in the range.)

3) select the "fill down or fill right" feature under the EDIT / FILL menu or as short-cut, press CTRL-R or CTRL-D, respectively.

Filling "in" Equations Containing Relative Addresses

Let's repeat the example in table 1. After having entered the equation in the first cell, (B5), we select or highlight cells B5:B7. After having selected fill down, the spread sheet looks like:

	Α	В	С
5	5	=2*A5	
6	6	=2*A6	
7	7	=2*A7	

7. Table 5

Note how all the relative addresses are automatically updated

Filling "in" Equations Containing Absolute Addresses

If we use a fill-down on the example in table 3, all cells would be filled with the equation: = 2*\$A\$5.

Filling "in" Equations With Containing Relative and Absolute Addresses

For a more advanced example consider what happens if we decide to fill down a "mixed" referenced address. For example we want to multiply the values in column A by a value at the top of column B.

	Α	В	С
5		2	
6	5	=\$B\$5*A6	
7	6		

8. Table 6

The equation in cell B6 of table 6 could be filled down and would produce the desired effect. What about if we want to multiply column A with various constants specified in row 5? Certainly we could enter in cell C6: =C = C = C and a similar equation in column D. This is cumbersome. A better way is shown in cell B6, Table 7:

	Α	В	С
5		2	
6	5	=B\$5*\$A6	
7	6		

9. Table 7

Now we can both fill the value in cell B6 down and right and with very little effort set up the following table:

	Α	В	С
5		2	
6	5	=B\$5*\$A6	=C\$5*\$A6
7	6	=B\$5*\$A7	=C\$5*\$A7

10. Table 8

D.3. Advanced Topics

Equations

Excel has an extensive library of advanced mathematical and statistical functions. The ones most often used are:

=if(condition, A, B) if condition is true, A will be executed, else B

=sum(RANGE) totals all values for cells specified in RANGE

=max(RANGE) finds the largest value for the cells specified in RANGE

=min(RANGE) finds the smallest value for the cells specified in RANGE

=count(RANGE) returns a number indicating how many cells in RANGE contain values

=average(RANGE) calculates the average for the cells specified in RANGE

These are just some of the many functions that are provided. For a listing of the functions and their use click on the "function wizard" button, f_{\ast} .

Graphics

To Create Graphs

It's very easy to make graphs: select or highlight the data and then use the charting options to plot it. If you should change your data, Excel automatically updates your graphs also.

There is a "chart wizard" button but it can be cumbersome. Instead, use the direct step by step approach to create charts:

1) Select the columns or ranges (hold down the CTRL key if selecting non adjacent columns) containing the data and any header that you want to plot. Do not select any data that you want to use for errors bars, i.e. your σ_i .

2) Select: Insert, Chart.

3) A menu will guide you through different charting options; select the ones you want. If you want to plot x vs. y, which is the case most of the time, then select **XY (Scatter)**. **Warning**: if you select **Line graph** instead, the x data will not be plotted but appears instead as evenly spaced labels along the x-axis! Be careful! Go to the next panel by clicking on NEXT.

4) The next window displays the data range selected for plotting and confirms that the data series are stored in columns rather than rows. Usually, you do not need make any changes in this window.

5) Go to the next panel, which will give you the option to enter chart titles and axis labels. Proceed to the next panel.

6) In this panel, you have to choose if you want the new graph to be inserted as a new sheet or as an object in the current spreadsheet. Always select *As New Sheet*. Do not *select As Object in...* because this generally leads to tiny charts that are difficult to see, annoying to resize, place and print. Select "FINISH."

7) You should see a new tab added to your worksheets, by default called CHART1, containing the new graph. To view your spreadsheet again click on the tab on the very bottom of the chart, most likely it will say *Sheet1*; to go back to the chart click on *Chart1*. You can change these labels by double clicking on them.

Adding Error Bars to Your Graph

To add error bars to your graphs, in your spreadsheet (ideally adjacent to the real data) add a column with the σ_i for each data point:

1) With the **right** mouse button click on any data point in your graph; once selected the color of the data points will change and a menu appears.

- 2) On that menu, select: **Format Data Series**.
- 3) Click on the Y-Error Bars tab. A window like the one shown below is displayed.



4) Click on the colored button that the arrow in the above picture points to. A new window called *Format Data Series Custom* + will open. See below.

Format Data Series - Custom +	? ×
=Sheet1!\$G\$10:\$G\$19	<u></u>

5) Select the worksheet containing the data with the σ_l : Select it, by clicking on its tab, at the bottom of the chart.

6) On that sheet, highlight the σ_{l} . Note that the address range is automatically filled in *the Format Data Series-Custom+* window as shown above.

7) In the *Format Data Series-Custom*+ window, click on the colored button to the right, directly below the "x" button. It will return you to Y-Error Bars window.

8) Click on the colored button in the *Y*-*Error Bars* window directly below the one you just clicked on, i.e. the one to the right in the *Custom - window*. A new window *called Format Data Series Custom -* will open. Repeat steps 4 through 7.

9) When done, click Enter in the *Y*-*Error Bars* window. Your graph should now display the appropriate error bars.

Note: make sure you match the y_i and the σ_i . If the addresses are not matched, strange stuff can appear on the graph.

Adding a Trendline to Your Data

Though Excel has some line fitting capabilities, you should use these with great care! They are only useful in that they give you a quick, first glance at the data. For more serious fitting, use your own program! Anyway, here it is:

- 1) Just like in step one for selecting the error bars, click on one of the data points in the chart.
- 2) Select in the Add Trendline menu and select the type you want, i.e. usually linear.
- 3) Click on the *Options* tab and select *Display Equation on Chart*.

4) Since the font size for the displayed equation is very small, click on the equation, click on *Font* and then Increase the *Size* to 20.

Basic Tips:

Do Not Start a Spreadsheet at Cell A1!

Always leave a few columns to the right and a few rows on top free. As you will work on your spreadsheet, they will come in very handy because you may want to add something. In case you do not need them, delete them when you are done.

Always List the Results in the Top Rows!

You will have to break with the habit of listing your results at the bottom of a column. Unless the columns are very short, always display your results on top. If you list them on the bottom, you or the user has to go searching for them; if they are on top, they are visible and easy to find.