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Physics 1401 Lab Notes

Fall 2017

PREFACE

The laboratory portion of Physics 1401/1501 requires considerable creative input. This resource manual outlines the basic expectations for each lab, the use of some (but not all) of the important equipment and software, and other miscellaneous information that may prove useful.

This collection of notes is NOT a lab manual! We are not attempting to provide you with a recipe for the completion of each lab.

<u>Basic Expectations</u>:

Laboratories will be held each week starting in the second week of the course. There will be five lab units of either two or three weeks duration: kinematics, collisions, rotational motion, simple harmonic motion, and gravitation. A detailed time-table will be provided at your first laboratory meeting. Attendance at all laboratory sessions is required. **YOU MUST COMPLETE ALL FIVE LABORATORY UNITS IN ORDER TO PASS THIS COURSE**. Your final lab grade will be based on your lab logbook (see the first chapter of these notes) as well as two written reports that will be completed over the course of the term (unless otherwise noted in your syllabus). Note that although you will be working on both data collection and analysis in groups, you will maintain your own logbook. Written reports will also be prepared individually. The expectations for reports will be introduced by your laboratory instructor.

These notes incorporate some material prepared by other instructors for other courses. In particularly, some of the introductory material and later chapters were prepared by Keith Ruddick and Kurt Wick for the sophomore and junior lab courses. Paul Barsic prepared some of the material, and I am particularly indebted to him for the Kepler simulation lab. The gyroscope lab is based on a commercial apparatus sold by Teachspin, Inc. Numerous students and TA's have tried out ideas and provided feedback.

In 2017, the driven harmonic motion lab has been replaced with a new version. The write-ups for the gyroscope and first simple harmonic motion labs have also been edited.

Paul Crowell
August 2017

STATEMENT ON ACADEMIC HONESTY

Note that the College of Science and Engineering policy on academic honesty applies to both the laboratory and lecture sections of this course. It can be found at the following web address:

http://cse.umn.edu/services/advising/CSE_CONTENT_188716.php

In particular, in submitting your logbook and written reports for grading, it is understood that all data collection and analysis have been carried out by you and your lab partners. You may not use data that was collected by some third party. Written reports may include data and analysis carried out in collaboration with lab partners but must otherwise be your own work. Note that *data fabrication*, which refers to making up data that were not actually collected by you using your experimental apparatus, is a serious academic offense. The minimum penalty for data fabrication is a failing grade (F) for the entire course. In preparing your written reports, please be aware of the usual standards for the use and citation of reference materials. In particular, the use of text or graphics from others without appropriate citation is *plagiarism,* for which the minimum penalty is a failing grade (F) for the entire course.

WARNING ON COMPUTERS AND SAVING DATA

The computers in the 1401 lab are maintained by the department's instructional staff. Individual accounts are not created, and all data are deleted when you log out. For this reason, you must either save your data on a memory stick, upload them to the web, or email them to yourself *before* logging out at the end of the lab period. Data should also be recorded into your logbook (either by hand or by printing out a spreadsheet) before leaving the laboratory. As discussed in Chapter 1, your lab book functions as an ongoing record of your activities and is the primary means for the staff to evaluate your work. DO NOT rely on storing data on media such as flash drives that can be lost or written over.

Table of Contents

Introduction

Laboratory Notebook

All raw data, calculations, and conclusions are to be recorded in a lab logbook. The lab book must be a bound notebook containing quadrille ruled paper and should be used only for this course. Lined logbooks, looseleaf binders, or scraps of paper are not allowed. You will paste in data, computer generated graphs, etc. when they are taken. Think of this lab book as a journal in which you will record all activities related to the lab, including calculations or analysis that are carried out at home. (This allowance for using a lab book at home is different from the standard practice in most companies and even some academic labs, in which it is often forbidden to remove a logbook from the laboratory.)

Each page of your logbook should be numbered and each entry should be dated. It is useful to keep a few pages at the beginning of the book blank in order to fill them in as a “table of contents”. For the purposes of organization, when we change from one unit to the next, skip a few pages, and start the unit with a title page such as “Kinematics.”

You should never tear out pages from your logbook. Whenever you make some sort of mistake (we all make them), just tidily cross out that section of your logbook. Make sure that it is still possible to read whatever is under the cross-out; it is quite common to find out later that what is there is not actually wrong, or that it includes some useful information.

This logbook is very important. It should contain enough information so that, after a period of, say, 6 months, you would be able to completely reconstruct all the steps you took during the experiment. In an academic laboratory, we regularly consult logbooks from over five years ago. All raw data must be included. All arrangements of equipment, settings of instruments, etc., must be included. All axes and tables should be carefully labeled with the correct units. Words of explanation should be present wherever necessary.

The technique of keeping a good lab notebook is a very useful skill to develop. Obviously, tidiness and clear writing are essential attributes; it is possible that others may also need to be able to decipher what you have done in your experiment. Note that a ruler and pens of a couple different colors will help you keep a well-organized lab book. A pair of scissors and a roll of tape or a glue stick will also be useful for cutting out graphs and tables from printouts for insertion into your lab book. A simple hand calculator is also useful.

Much of the data you collect in the course will be obtained with the assistance of computers. In most cases, the data should be printed out and pasted into your lab book, along with the corresponding graphs. **There may be an exceptional case in which the amount of data generated is so large that printing it out would obviously be a waste of space and paper.** In that case, you should print out the graphs only, but record in your lab-book where the original data are stored.

Things to include in your notebook:

1. A detailed set-up of the experiment with enough information that you could acquire the materials and re-create the set-up if you had to.

2. All relevant equations and useful derivations of the equations with the variables clearly defined. Make sure you can understand the equations if you look at them on a later date. **You must answer any questions in the lab manual.**

3. **All raw data taken**. Often you will end up creating a data table in a spreadsheet, but remember to also include constant values (like a mass or length that did not change over multiple trials).

4. Plots should be clearly labeled, and fully analyzed (data fit with equations, those equations compared to the relevant physics, etc.). **A plot is not a substitution for a data table**! Only when the data table is too large (over 50 data points per trial) should you not have a data table.

5. Uncertainties for all measured points, and propagation of uncertainty to final results should be clearly displayed.

6. A conclusion based on the numerical results. You should state a clear conclusion on the results of the experiment, but you must also clearly reference the data that quantitatively supports that conclusion.

DO NOT LOSE YOUR LAB BOOK! Put your name, course number, and email address on the cover just in case.

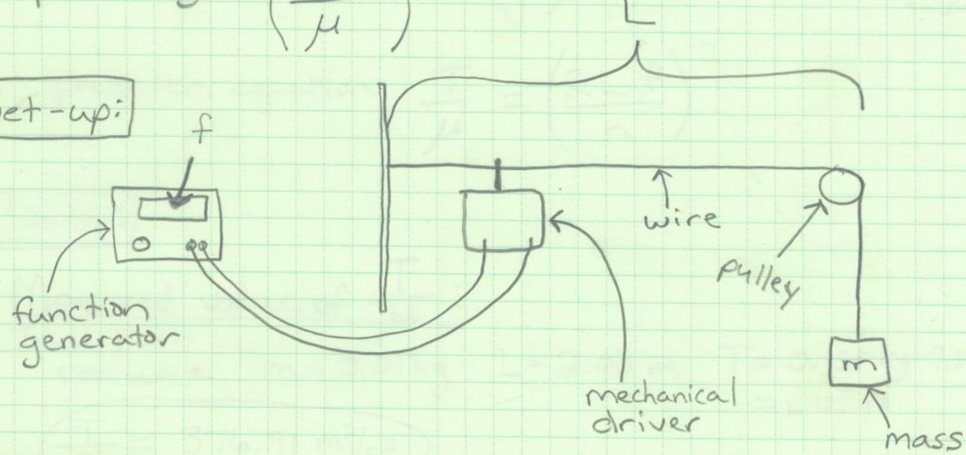
Sample Notebook Pages

There is no single "correct" way to keep a lab book. These examples illustrate some things to think about. Clear sections and labeling make a lab write up easy to read, and easy to write.

Title: Lab 1.1 Standing Waves on a String

Purpose: Use the speed of sound on a wire to determine the ratio of tension to mass density per length $\left(\frac{T}{\mu}\right)$

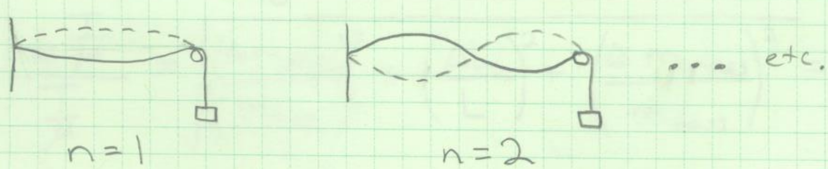
Set-up:



Procedure: Varied frequency (f) to find n^{th} harmonic of wire for as many values of n as possible. Tension was provided by a hanging mass. The measured frequencies were used to determine the ratio $\frac{T}{\mu}$, where T = tension and μ = mass density per length. This was then compared to the directly measured $\frac{T}{\mu}$ of the wire. We did three trials.

Trial 1: Red wire with 0.151 kg of hanging mass.
Trial 2: Red wire with 0.251 kg of hanging mass.
Trial 3: White wire with 0.254 kg of hanging mass.

Examples of modes (n):



Make sure to use units on all numbers that have them.

Equations:

speed of sound $v = \lambda f$, $\lambda = \frac{2L}{n}$

for a wire $v = \sqrt{\frac{T}{\mu}}$, $\mu_{measured} = \frac{m_{wire}}{L_{wire}}$

prediction equation $\frac{T}{\mu} = \left(\frac{2Lf}{n}\right)^2$

Data:

Uncertainty of Measured Value of $\frac{T}{\mu}$:

$\Delta m_{wire} = \pm 0.0005\ kg$ $\Delta L = \pm 0.005\ m$ $\Delta m_{mass} = \pm 0.0005\ kg$

$$\frac{\Delta \frac{T}{\mu}}{\frac{T}{\mu}} = \sqrt{\left(\frac{\Delta m_{wire}}{m_{wire}}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta m_{mass}}{m_{mass}}\right)^2} = 1.25\%$$

Measured Value of $\frac{T}{\mu}$:

Trial 1:
red wire $m = 0.01\ kg$ $L = 2.03\ m$ $T = 0.151\ kg \cdot 9.81\ m/s^2 = 1.485\ N$

$\frac{T}{\mu} = 377 \pm 5\ m^2/s^2$

Trial 2:
red wire $m = 0.01\ kg$ $L = 2.03\ m$ $T = 0.251\ kg \cdot 9.81\ m/s^2 = 2.466\ N$

$\frac{T}{\mu} = 626 \pm 8\ m^2/s^2$

Trial 3:
white wire $m = 0.02\ kg$ $L = 2.02\ m$ $T = 0.254\ kg \cdot 9.81\ m/s^2 = 2.492\ N$

$\frac{T}{\mu} = 222 \pm 3\ m^2/s^2$

For the sake of space only three trials were done for this lab notebook. In normal execution of this lab you would likely do one for each type of wire available or multiple tensions. Your write ups should include ALL data taken.

Experimental value of $\frac{T}{\mu}$:

$L = 1.540\ m$ (for all trials)

Trial 1:

n	f (Hz)	$\frac{4L^2f^2}{n^2}$
1	6.34	381.3
2	12.7	382.5
3	19.1	382.5
4	25.3	379.5
5	31.7	381.3
6	38.1	382.5
7	45.0	392.0
8	51.3	390.1
9	57.8	391.3
10	63.3	380.1
11	71.0	395.2

Avg. $\frac{4L^2f^2}{n^2} = 385 \pm 5\ \frac{m^2}{s^2}$

Trial 2:

n	f (Hz)	$\frac{4L^2f^2}{n^2}$
1	8.3	653.5
2	16.4	637.9
3	24.7	643.1
4	32.9	641.8
5	41.2	644.1
6	49.0	632.7
7	57.2	633.4
8	65.7	639.8
9	74.8	655.3

Avg. $\frac{4L^2f^2}{n^2} = 642 \pm 9\ m^2/s^2$

Trial 3:

n	f (Hz)	$\frac{4L^2f^2}{n^2}$
1	4.78	216.8
2	9.7	223.1
3	14.7	226.2
4	19.6	227.8
5	24.6	229.6
6	29.9	235.6

Avg. $\frac{4L^2f^2}{n^2} = 227 \pm 3\ m^2/s^2$

Observation:

As tension and μ increased, waves became more difficult to see due to decreased amplitude.

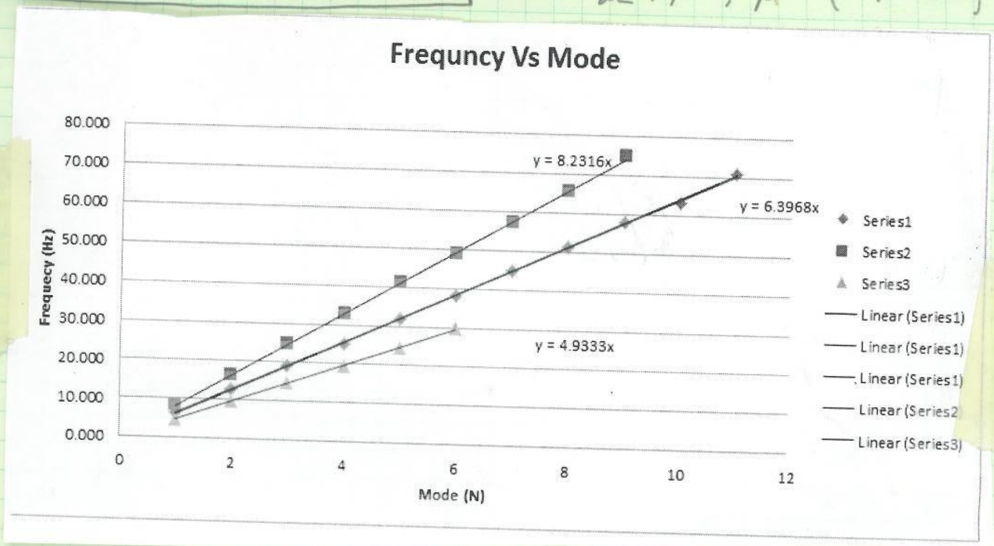
Uncertainty of $\frac{4L^2f^2}{n^2}$:

$\Delta L = \pm 0.001 m \quad \Delta f = \pm 0.1\ Hz$

$$\frac{\Delta \frac{4L^2f^2}{n^2}}{\frac{4L^2f^2}{n^2}} = \sqrt{2\left(\frac{\Delta L}{L}\right)^2 + 2\left(\frac{\Delta f}{f}\right)^2} = 1.4\%$$

Data Plot f vs. n:

slope $= \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, $\frac{T}{\mu} = (\text{slope} \cdot 2L)^2$



	Trial 1	Trial 2	Trial 3
$(\text{slope} \cdot 2L)^2$ (m^2/s^2)	385 ± 5	643 ± 9	231 ± 4

Final Comparison:

	$\frac{T}{\mu}$ $(\frac{m^2}{s^2})$	$\frac{4L^2f^2}{n^2}$ $(\frac{m^2}{s^2})$	within uncertainty?
Trial 1	377 ± 5	385 ± 5	yes
Trial 2	626 ± 8	642 ± 9	yes
Trial 3	222 ± 3	227 ± 3	yes

Conclusion:

Our prediction that $\frac{T}{\mu} = \frac{4L^2f^2}{n^2}$ was shown to be within uncertainty. Our calculation of $\frac{T}{\mu} = (\text{slope} \cdot 2L)^2$ agreed strongly with our measurement of $\frac{4L^2f^2}{n^2}$, except for trial 3, which was still within 2 uncertainties of the measured value.

Experimental Precision and Uncertainty

The experimentalist is responsible for communicating his or her results in a manner that conveys the appropriate experimental precision and uncertainty. For example, the apparatus used in this course is capable of measuring *g* with a precision of at best about 0.1 - 1%. If, at the end of my calculations, I write down a number in my lab book that appears on my calculator as g = 9.80451 m/s^2, the reader can only assume that all of the digits reported are *significant* and that I am claiming a precision of around 0.001%. This would obviously be absurd. If the precision were 1 %, a somewhat better choice would be to report the result as 9.8 m/s^2. Even this format, however, leaves some ambiguity about the precision of the result. By careful analysis of experimental uncertainty, let's assume that I have determined that the uncertainty in my result is 0.2 m/s^2. If I then write down my result as 9.8 ± 0.2 m/s^2, it is absolutely clear to the reader both *what* I have measured and *how well* I have measured it.

To summarize: an experimental measurement should always be recorded along with its uncertainty. For the purposes of this course, we will rely more on common sense rather than statistical techniques, but always take time to consider the sources of experimental error and discuss them in your lab book.

Uncertainty

Uncertainty is an important part of understanding a lab because it lets you quantify what your margin of error actually is. If you find yourself writing a conclusion and listing 'sources of error' you should also be including those sources into the presentation of your data and results.

Uncertainty is not a hard limit. If your uncertainty is ±1mm this does not mean that 0.9mm error from prediction is perfectly acceptable, but 1.1mm error is completely unacceptable. Uncertainty is not a wall that cannot be passed, it is more akin to the standard deviation of a bell-shaped curve. On a bell-shaped curve 68% of the curve is within one standard deviation, 95% is within two standard deviations, and 99.7% is within three standard deviations. Likewise, you should understand that in a correctly done experiment uncertainty will only be greater than difference from prediction "68%" of the time (the actual percent is not precise or relevant, but should be understood to represent "most" of the time), but two uncertainties should cover "95%" of the time. Error falling within one uncertainty is great, but error falling within two uncertainties should still be seen as possibly agreeing, but errors that fall outside of two uncertainties are when you begin to doubt that they are actually agreeing. Five uncertainties is right out.

There are many causes of uncertainty; here we will look at three common ones: Instrumental precision, human error, and systematics. In general, uncertainties should only have one significant figure.

Instrumental Precision

Instrumental precision is a fairly easy uncertainty to understand and a fairly simple one to determine. In general, the uncertainty from instrumental precision is half of the smallest marking or digit displayed on the measurement device. This in general leads to things like ±0.5 mm on a meter stick or ±0.005 sec on a stopwatch.

While easy to determine, it is often the smallest contribution. More often than not instrumental precision is overwhelmed by uncertainty from human error. DO NOT simply look at instrumental precision when determining uncertainty unless you are sure human error is not a factor.

Human Error

Human error is by far the most likely source of uncertainty for most measurements. While a ruler may have a precision of ±0.5 mm, the ability of humans to see lines that small, hold a ruler at the correct spot, look from the correct angle, deal with curved edges or difficult to reach locations means that the actual uncertainty of a measurement is noticeable higher. As a rule of thumb, the smallest markings are usually the extent of human ability to gauge so expect ±1 mm from a meter stick due to human error as a minimum (it may increase if the object measured is at all difficult).

A decent method of determining human error is to vary the measurement a set amount and ask yourself "could I believe this is the correct measurement". The range through which you can believably move the measurement will show you what human error adds.

Another type of human error is easily seen in the use of a stopwatch. While the instrumental precision may be 0.005 sec, human reaction time is more like 0.2 sec, and thus a stop watch far more likely has an uncertainty of ±0.2 sec.

Do not add instrumental precision and human error, simply choose the larger one.

Systematics

In the above examples, I have been discussing *random* errors. We are assuming that the measured values, although they may not be perfect, fluctuate about the correct value. Unfortunately, we often deal with apparatus that is not properly calibrated or an experimental approach that has some implicit bias which is not easily recognized. For example, if I measured falling objects with a stopwatch that was miscalibrated (so that it ran fast), I might find values for g coming out as 10.2, 10.3, 10.6, 10.4, 10.2, and 10.3 m/s^2. Averaging these, I would find g = 10.3 ± 0.1 m/s^2, which is clearly higher (by 5 error bars) than the expected value of 9.8 m/s^2. This very large discrepancy is an example of a *systematic* error, which can be eliminated only by fixing the apparatus (in this case the bad stopwatch). In other cases, systematic errors may result from a flaw in technique, as you may find out the first time you use a video camera to measure a falling object. **Note that in an extreme case, one can make a measurement with very small random errors, so that the result appears to be extremely *precise*, but the *accuracy*, which is determined by the systematic error, could be very poor.** If you notice this sort of behavior, it is your job to try and track down and quantify the likely source of the systematic in your data.

Propagation of Uncertainty

Once we have established the uncertainty of a measurement, we need to make sure that uncertainty is applied to all values derived from that measurement. Suppose that we make two independent measurements $A \pm \Delta A$ and $B \pm \Delta B$, where ΔA and ΔB are the uncertainties in each measurement. What is the error in the quantity $y=A+B$? What about for y=A/B?

Addition and subtraction:
In the cases of $y = A+B$ and $y=A-B$, the uncertainty of y (Δy) is:

$$\Delta y = \sqrt{(\Delta \mathrm{A})^2 + (\Delta \mathrm{B})^2}$$

(often one is told $\Delta \mathrm{y} = \Delta \mathrm{A} + \Delta \mathrm{B}$, but the above version is correct for the case when $\Delta \mathrm{A}$ and $\Delta \mathrm{B}$ are statistically random)

Multiplication and Division:
When y=AB or y=A/B

$$\frac{\Delta \mathrm{y}}{\mathrm{y}} = \sqrt{\left(\frac{\Delta \mathrm{A}}{\mathrm{A}}\right)^2 + \left(\frac{\Delta \mathrm{B}}{\mathrm{B}}\right)^2}$$

Note that this gives a fractional uncertainty, the result will be unitless and you will need to be multiplied by *y* to get $\Delta \mathrm{y}$. Sometimes it is instead written

$$\Delta \mathrm{y} = \mathrm{y} \times \sqrt{\left(\frac{\Delta \mathrm{A}}{\mathrm{A}}\right)^2 + \left(\frac{\Delta \mathrm{B}}{\mathrm{B}}\right)^2}$$

Let’s see what happens for the case of a rectangle, for which $L = 4.5 \pm 0.1$ cm and W = 7.5 ± 0.1 cm. Letting $y = LW$ be the area of the rectangle, we find

$$\frac{\Delta y}{y} = \sqrt{\left(\frac{0.1}{4.5}\right)^2 + \left(\frac{0.1}{7.5}\right)^2} = 0.13\,,$$

so that Δy = 0.026 x 33.75 cm^2= 0.878 mm^2 (which should be shortened to just 0.9 cm^2 to obey the limitations on significant figures in uncertainties).

A slightly more general form for multiplication and division is for the case of $y=A^nB^m$:

$$\frac{\Delta y}{y} = \sqrt{n^2\left(\frac{\Delta A}{A}\right)^2 + m^2\left(\frac{\Delta B}{B}\right)^2}$$

General Rule:
The above rules can be used to find the uncertainty for probably 90% of the equations you will face in these labs, but if you find yourself in need of a general formula, for any function y(A,B):

$$\Delta y = \sqrt{\Delta A^2\left(\frac{\partial y}{\partial A}\right)^2 + \Delta B^2\left(\frac{\partial y}{\partial B}\right)^2}$$

Which when applied to each of the special cases above will return the provided equations.

At this point, you might be completely confused or simply depressed at how tedious the propagation of experimental errors in a calculation can be. Do not worry! The goal of this course is for you to begin getting used to thinking about uncertainties and start you getting comfortable with including them in your lab work. To begin with, make sure you always note uncertainty on any measurement you make. Next, look at which uncertainties will propagate to the final results, and consider how big of an effect they should make (it's is often a good idea to think in terms of percentages). Then begin propagating uncertainties in a simple form you can get used to using. Perhaps start by using $\frac{\Delta y}{y} = \frac{\Delta A}{A}$, where A is your greatest source of uncertainty and see what you get. Then try blindly applying $\Delta y = y \times \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$ (in most situations this will at least get you the correct order of magnitude) until you become more familiar with the procedure and expectations.

See Appendix B for more information on uncertainty and propagation.

Significant Figures

You will be carrying out calculations based on your raw data. At times you will end up with a lot of digits (especially when using computer based timing or dividing numbers). It is important that you understand how many of those digits are relevant, and that you show that understanding by only ever reporting those digits.

As a rule of thumb, you should never write down more than 4 significant figures without being **<u>very</u>** sure that you can claim to be that precise. But let's consider the actual way you should determine significant figures.

Suppose that I am asked to calculate the area of a rectangle, based on my measurements of $L = 4.5 \pm 0.1$ cm and W = 7.5 ± 0.1 cm. It is trivial to calculate the product on a calculator, which will duly record 33.75 cm^2 as the answer. Is this what I should report?

Previously we calculated that the uncertainty of this area was 0.9 cm^2, and this limits how much importance we can place on any digit less than 0.1 cm^2. As such, I would write down in my lab book: "The area of the rectangle is 33.8 ± 0.9 cm^2." Note that the number of significant digits in the result is determined by the error, but that the error itself should be recorded with just a single significant figure. It would not accurate to state the result as "33.75 ± 0.87 cm^2".

Read the previous paragraph as many times as is necessary to understand the fundamental point: *there is no sense in reporting the results of a calculation with a precision better than that of any of the inputs into that calculation*.

Note that handling significant figures (and many other things) can be simpler if you use scientific notation. For example, imagine that you write down the number 1560 cm^3 in your lab-book. Is the last zero significant or not? All confusion would be avoided if the number were written 1.56×10^3 cm^3 , if you did not mean the zero to be significant, or 1.560×10^3 cm^3, if you did. Note that to record an experimental uncertainty in scientific notation, you should place it before the "x" sign. For example, 1560 ± 20 cm^3 would be written as $(1.56 \pm 0.02) \times 10^3$ cm^3. The uncertainty is <u>always</u> packaged with the root inside parentheses.

Analyzing Data Plots

Making plots is a natural part of most labs. Plots are useful for visualizing the behavior of data, and they allow us to easily fit the data with a line. In honors physics it is not enough to simply make a plot and slap a fit line on it, we expect that you will use plots to their full potential for not just displaying data, but for interpreting it as well.

Linearizing plots

A very common technique when making a plot is to linearize it, that is to adjust the data so that when it is plotted it makes a straight line. For example, if we are plotting distance (y) vs time fallen (t), the equation should be:

$$y = y_0 + \frac{g}{2}t^2$$

which when plotted as *y vs. t* will give a parabola. Parabolas are very hard to fit by hand. They are also hard to examine for how well a fit line matches the data. If instead you were to plot *y vs* t^2, you would find the data should make a straight line of the form y=mx+b, where m=g/2 and b=y_0 and x= t^2. You will find that linearizing the data will make the graphs you plot easier to fit, and easier to analyze. In the labs of this course you will likely have to fit data that is in the form y=1/x or y=e^x. You will find the data much easier to interpret if you plot them as *y vs 1/x* and *ln(y) vs x*.

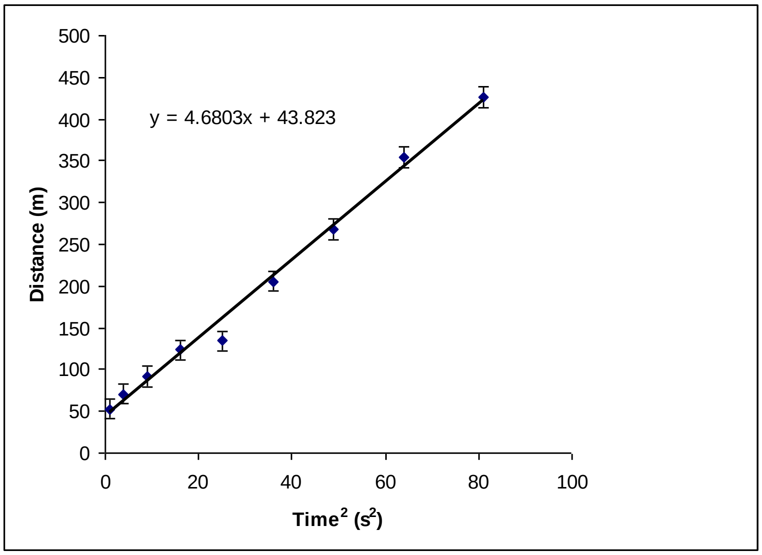


Figure 1.1: Data (symbols) and best-fit line (solid line) for the hypothetical experiment discussed in the text. The equation for the best-fit line is also indicated. This particular result was generated by Excel's "Trendline" function, which does not estimate errors in the slope and the intercept. Note that the program has spit out five digits, but not all of these are significant!

Goodness of Fit

When comparing a plot of data to a predicted equation, it is not immediately apparent how to quantify how well the measurements agree with predictions. The simplest way is to fit the measurement with its own equation and compare to the predicted equation, and this is a reasonable first step. The "Goodness of Fit" equation (also called a reduced χ^2 (chi-squared), though this is a very general term that applies to many similar calculations) is

$$\chi^2 = \frac{1}{N}\Sigma_i \frac{(M_i - P_i)^2}{\Delta^2}$$

where N is the number of data points, M are the measured values, P is the predicted value and Δ is the total uncertainty at that point. This provides an average comparison between the uncertainty and the distance from prediction for every point along a line. While χ^2 values of 0 to 1 imply good agreement ('0' implies a perfect match, a value of '1' or lower implies that the prediction is within the uncertainty of the measurement), values of 1.1 to 4 are still believable (a value of ‘4’ implies you were within two uncertainties of the prediction on average), and it is really only values of 4+ that should be immediately discarded.

A note on standard deviation: In general, standard deviation is not as good a gauge of uncertainty as other methods described earlier. Either it will be about the same size as your estimated uncertainty (because the standard deviation will be due to that uncertainty) or it will be notably smaller due to consistency of measurement. If standard deviation is notably larger than your estimated uncertainty, it likely means you neglected a major source of uncertainty in your estimation. Bottom line: most of the time standard deviation will be smaller than your actual uncertainties.

A note on R^2: If you use a trend-line fit, R^2 is never a useful number. Never ever. It is a measure of correlation, and in physics your data will always be well correlated, even if it is otherwise not very good. This value does not tell you how correct your data is, and is not remotely a gauge of uncertainty. Never use R^2. **Never.**

Fitting a Line by “Hand”

Imagine taking a ruler and putting it down on the plot, shifting and rotating the ruler in order to find the “best” line that goes through all of the data points. Can we quantify what you are trying to do visually? Choosing the best line is equivalent to minimizing the average distance between the line determined by the ruler and each of the data points. Let us consider the case in which the value of *x* for each data point is known exactly. Then the best fit line $y = a+bx$ will have an intercept *a* and slope *b* that *minimize* the quantity

$$D = \sum_{i=1}^{N} (y - y_i)^2 = \sum_{i=1}^{N} (a + bx_i - y_i)^2 \quad ,$$

where the sum is over all N data points. The quantity *D* is the sum of the (squares) of the deviations of each data point y_i from the fitting function $y = a + bx_i$, and the problem of selecting the best choice of intercept *a* and slope *b* is equivalent to *minimizing D*. Note that if the line passed through every data point exactly, then *D* would be zero. Since all of the data points (x_i, y_i) are known (they are your data!), *D* is just a mathematical function of the two parameters *a* and *b.* Minimizing *D* is merely a problem in differential calculus (evaluating the derivatives with respect to *a* and *b* and then setting them equal to zero). There is little value in reproducing the calculation here. Be aware, however, that Excel’s “Trendline” function carries out this minimization automatically. The result is shown in Fig. 1.1. Excel will provide you with values for *a* and *b*, often with many insignificant digits, and no estimation of the errors! Think about a way to estimate the error in the slope of the line shown in Fig. 1. Hint: given the error bars shown, draw the lines of largest and smallest slope that a “reasonable” person would draw through the data.

Better routines for fitting lines will estimate the errors Δa and Δb as well as account for the fact that the errors Δy_i in each data point are not necessarily the same. Accounting for the effects of unequal errors is carried out by *weighting* each term in the sum that makes up D.

Before leaving this issue, note that the general concept of *fitting* data to a function can be extended to other functions besides lines. The fitting procedure is always a matter of minimizing the average deviation of the fitted function from the data. For polynomials (quadratics, cubics, etc.), it turns out that this problem can always be solved in closed form, and the minimization procedure goes by the somewhat confusing name of *linear least squares.* An extension, known as *non-linear least squares,* applies to arbitrary functions.

Laboratory 1: Kinematics

Lab 1.1: Galileo's Experiment: Measuring Acceleration due to Gravity

Goal: To familiarize yourself with the methods and tools used in a physics lab by performing an introductory lab.
Suggested Reading: Read the section in your text pertaining to acceleration and gravity. Read the "Experimental Precision and Uncertainty" section at the beginning of this manual.

Background

How does the acceleration of a freely falling body depend upon its mass? How can this be measured in our lab?

In this lab, you will measure the distance fallen vs. time for balls with the same diameter but different masses using a video camera. For each ball, you will measure the position as a function of time, and then you will determine the velocity and acceleration by differentiation of your data.

You will be using computers to take and analyze data. This lab will help you through an example of recording a ball drop, analyzing the video to get position vs. time data, and using a spreadsheet to analyze that data. At the end of the lab period, each member of your lab group should have a copy of the spreadsheet and all relevant graphs. These should be included in your lab notebook along with any relevant calculations and notes about your experiment. You should also keep an electronic copy of the spreadsheet in case you need to use the data again.

Experiment

We will give you:

- set of 1" (2.5 cm) diameter balls, including
    - tungsten carbide (~128 g)
    - steel (~66 g)
    - aluminum (~23 g)
    - white plastic (~11 g)
    - polyethylene plastic (~8.8 g)
    - wood (~4.9-6.0 g)
    - hollow plastic (~2.6 g)
- meter sticks
- stopwatch
- video camera
- computer
- foam pad to catch the balls. Do not let them bounce on the floor.

For a first approach, use stopwatches and rulers to measure the acceleration of the falling balls. Remember the basic kinematics equation,

$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2, \qquad (1\text{-}1)$$

where y is position, y_0 is the initial position, v_0 is the initial velocity, t is the time, and a is the acceleration. What are the dimensions of these quantities? How can you measure a? Can you measure it directly, or do you to calculate a from other values you *can* measure? Take several trials with a stopwatch using the same ball and calculate its acceleration for each trial. How much variation is there from trial to trial? What does this mean your uncertainty is? Now perform **one** trial on another ball of different mass and calculate its acceleration. Report the accelerations you calculate for the two balls, including the uncertainty you believe you have on both, in your notebook. Do they answer the question "How does the acceleration of a freely falling body depend upon its mass"? Is the method precise enough to have confidence in this answer?

Using the Video Camera

Read the section of **Appendix A: Making Videos – Using ProCam**. Open the program called ProCam on the video camera (iPod). Explore how it works a bit and see what effects adjusting the exposure and ISO have. Determine what settings you should set in the program for taking clear videos of moving objects.

The video camera is a great tool, but it has its limitations. Once aware of these limitations, we can use the camera to our full advantage. Hold a meter stick in the camera frame of view. Now look at it on the camera screen. Is it a wonderfully crisp image of a straight meter stick? Use a small ruler to judge. Where is the image good? Where is it bad? How can you minimize the bad effects? In this case, make sure the image of the ball in each frame is not blurry. This requires you to set the exposure and ISO correctly.

If you move the camera closer to or further away from the meter stick, its apparent size changes. Take two meter sticks, hold one near the camera and hold the other farther away. You know that they are the same size, but what does it look like on the screen? Is there something that you can do to make sure that you measure lengths consistently?

Now hold the two rulers horizontally, one above the other. Separate them so that one is near the top of the frame and the other is near the bottom. Tilt the camera up (turn the tripod handle counterclockwise to loosen this degree of motion). Now use your small ruler to measure their lengths on the screen. Are they the same length? Do you think that a tilt in the camera angle will have some effect on your data?

Now that you have an idea of how you want to use the camera, try recording a video. Save your video to the "LabData" folder. Remember that your video will be erased when you log out, so save it to a flash drive or e-mail it to yourself if you think that you may want it at a later date.

Digitization with Tracker

Tracker is a very nice piece of software which lets you measure positions of various objects in each frame of a video. It then gives you the positions as functions of time. Open the 'Tracker' program (answer 'yes' to any dialogue boxes regarding re-launching). Choose the 'import' option from the 'Video' menu in the Tracker window. Select the file that contains the relevant video. You need to set up an axis and define your units. There are two buttons in the button bar that will allow you to do this.

a) *Axes*: Click the 'Axes' button to put a coordinate system on your picture. You can move the origin and rotate. Try to make it so that your motion is in one direction only.

b) *Calibration Tool*: Click the 'Calibration Tape' button to define your units of length. A blue line with arrows on both ends and a number above it will appear on your screen. Drag the arrows onto an object of known length. Double-click on the number above the arrow, and type in the length of the object. What are your units of length?

The third button on that bar is the '*Create *'* button. This is what allows you to measure the position of your object in the video. Click this button. In the small window which appears, select 'New' and choose 'point mass'. You are now ready to record the position of your object.

Move the video slider to a time when the motion starts. Hold down the shift key, carefully position the cursor over the object you wish to track, and click. Note that the video has now advanced one frame. Also, note that the data points are now shown on the right portion of the screen. Continue with this procedure until you have taken all of your data. What happens when the object moves faster than the camera can capture it? If the axes and previous points are getting in your way, click the 'Axes' button to make them invisible. Once you have all of your data, you can still rotate the axes. Click the 'mass A' button and de-select 'visible' to make the 'mass A' data trail invisible.

When you are done taking the data, you need to analyze them. There are many ways you could do it. We will walk you through the analysis using a spreadsheet program.

Take data for at least one massive ball (e.g. steel) and one light one (plastic or wood).

Data Analysis

Select the columns of numerical data in Tracker. Hit 'Ctrl-c' to copy the data to the clipboard. Choose 'Excel' from the 'Microsoft Office' folder in the 'Start' menu. Appendix B provides a more detailed introduction to Excel. When a blank spreadsheet appears, put the cursor in the spreadsheet and hit 'Ctrl-v' to paste the data. You should leave a few blank rows above the data so that you can add column headings (***with units***) and make some notes about the data represented in the worksheet (mass of the ball, etc.).

Your data is position as a function of time. You should calculate the velocity from your position data. Recall that velocity is defined as

$$v(t) = \frac{dy(t)}{dt}, \tag{1-2}$$

where y is position and t is time. As you know from your calculus class, a derivative is defined as

$$y'(t) = \frac{dy(t)}{dt} = \lim_{h \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}. \tag{1-3}$$

In the math department, they tell you to make Δt infinitesimal. However, we cannot do that in real life, since the camera captures only 30 or 60 frames per second. What is the value of Δt that you will use?

To enter a formula in 'Excel', put your cursor in an empty cell and type the '=' sign. This tells the computer that it needs to perform a calculation based upon what follows. Suppose that you want to take the difference between cells 'B4' and 'B5', divide them by 0.03333, and put the result in cell 'D5'. Put the cursor in cell 'D5', type:

```
= (B5-B4)/0.03333
```

and press 'Enter'. What appears in cell D5? Make sure it is correct. If you want to do this for a bunch of cells, you could type this formula into each cell individually, but is easier to copy and paste. Select this cell and hit 'Ctrl-c' to copy it. Highlight the cells in which you want to paste this formula and hit 'Ctrl-v' to paste

It is hard to look at a column of numbers and understand what it means. It is best to make a plot. We will make a graph of velocity versus time. Highlight the velocity and time columns (hold down the 'Ctrl' key to select non-continuous groups of cells). Create an XY-scatter plot.

Print the graph to fit a line to the data. Select the chart, and pick the 'Print' menu item from the 'File' menu. Choose our room number from the list of available printers. Print one copy for each group member.

When your graph is printed, use a ruler to draw a straight line that goes as close to all of the points as possible and determine the equation of the line. What is the y-intercept? What is the slope? What does the slope of this graph tell you about the acceleration due to gravity?

Compare your line to those of your colleagues. By comparing the slopes of the different lines that you and your colleagues have drawn, and by considering the maximum and minimum slopes that you could believe fit this data, come up with an estimate of the uncertainty in a.

Report the acceleration for each ball with uncertainty clearly labeled in your lab notebook.

Questions

1. What are the uncertainties on the measures values of your accelerations for each ball (both from using the stopwatch and the video analysis)?

2. What should you predict the slope and y-intercept of your plots to be? Are these predictions within uncertainties of your measured values on the plot?

3. Does *acceleration* depend on the density of the ball? Do your results agree with what physics tells us?

Lab 1.2: Gravity and Air Resistance

Goals: To see how air resistance influences the acceleration of falling objects. To get an approximate measurement of the drag of a falling ball. To experimentally measure g in a manner that removes the effects of air resistance.

Suggested Reading: Read the sections of your text that discuss the kinematics of acceleration and the effects of air resistance.

Background

You know that it is difficult to completely isolate a projectile from the effects of air resistance. For example, if you were to drop a ball from a large height, you would find that its acceleration is less than g. In fact, after a time, you would find that its acceleration becomes zero. Since the ball is moving through the air, there is a force on the ball due to air resistance. For small velocities, this force is of the form

$$F_{air} = -\frac{1}{2}C\rho A v^2 = -kv^2, \tag{1-4}$$

where ρ is the density of air, A is the frontal area (cross-section) of the body perpendicular to its velocity v, C is the drag coefficient of the object, and $k = C\rho A/2$ to simplify writing future equations. The negative sign reminds us that the direction of the force is opposite to the direction of the velocity. Since the acceleration is no longer constant in time, Eq. 1-1 is no longer valid. We can re-write Newton's second law for a falling object as:

$$m\frac{d^2y}{dt^2} = mg - k\left(\frac{dy}{dt}\right)^2, \tag{1-5}$$

This equation can be integrated to yield:

$$y(t) = \frac{V_t^2}{g}\ln\left(\cosh\frac{gt}{V_t} + \frac{v_0}{V_t}\sinh\frac{gt}{V_t}\right), \tag{1-6}$$

where V_t is the terminal velocity, which is the velocity at which the force due to gravity is exactly equal to the force due to air resistance, yielding a net force (and therefore acceleration) of zero. This is something which you can work out for yourself from Eq. 1-5. Derive an expression for V_t.

If $y(t)=Y$ (the distance fallen) and $v_0=0$, then Eq. 1-6 can be solved for t:

$$t = \frac{V_t}{g}\cosh^{-1}\left(e^{gY/V_t^2}\right), \tag{1-7}$$

which can be simplified by taking the first few terms of a series expansion:

$$t = \sqrt{2\frac{Y}{g}}\left(1 + \frac{gY}{3!V_t^2} + \frac{(gY)^2}{5!V_t^4} + \dots\right). \tag{1-8}$$

We really only need the first two terms. To make it useful, substitute for the terminal velocity to get a function of the mass and the initial height:

$$t(Y,m) = \sqrt{2\frac{Y}{g}\left(1+\frac{kY}{6m}\right)}. \tag{1-9}$$

This makes a lot of sense. Imagine dropping an object of infinite mass: you recover the simple case of no air resistance. A similar thing happens when you set k to zero (equivalent to dropping the object in a vacuum).

It his helpful to recast Eq. 1-9 in the following *approximate* form, valid in the limit $kY/6m << 1$:

$$Y = \frac{1}{2}g_{eff}t^2, \tag{1-10}$$

thus

$$g_{eff} = g\left(1-\frac{kY}{3m}\right). \tag{1-11}$$

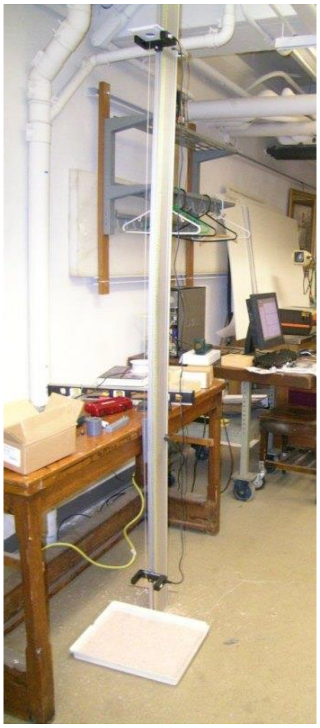
Note that the acceleration is no longer constant, since it depends on the distance fallen, but for our purposes, we can say that the balls will fall with an *effective* acceleration that is slightly smaller than g. While it is not vital that you understand every step of this derivation, make sure you understand the meaning and implications of Eqs. 1-4, 1-10, and 1-11, as they are the key physics that will be used in this lab.

Experiment

Material Required:

- set of 1" (2.5 cm) diameter balls
    - tungsten carbide (~128 g)
    - steel (~66 g)
    - aluminum (~23 g)
    - white plastic (~11 g)
    - polyethylene plastic (~8.8 g)
    - wood (~4.9-6.0 g)
    - hollow plastic (~2.6 g)
- 2 photogates and clamps
- Vernier LabPro computer interface
- desktop computer
- Aluminum track
- Basket with bubble wrap or cloth
- Plastic plate, cylinder, and plumb bob

We can verify the form of Eq. 1-11 by using Eq. 1-10 to measure g_{eff} for several different masses of identical shape and size. This will allow us to a) verify that the drag depends on the square of velocity, b) measure the drag coefficient C, and c) by an appropriate analysis, get an experimental value of g. To accomplish this, think about how you should graph your data for g_{eff} *vs.* mass (Hint: What happens if you use *1/m* instead of *m* as your *x*-coordinate?) and what equation this graph will represent.



Photogate set-up: Note the separation of the gates and the plumb-bob to assure that the balls pass through the center of both gates. The tray at the bottom is replaced with bubble wrap or cloth in your version of the experiment.

Photogates consist of a light source and a light detector. All these gates do is report to the computer whether the light beam is blocked or not. You will use your gates in the internal mode (in which you are using the light source built into the photogate.) To help you align your system, a red LED on the gate shines when the gate is blocked.

You will now use the photogates to carry out a much more precise measurement of g. The precision will be good enough for you to examine the effects of air resistance by comparing the falling time of balls with identical diameters but different masses.

The gates interface with the computer using a USB device called a Vernier LabPro, which is a teal box that is about the size of a graphing calculator. The LabPro box has two gate inputs (DIG/SONIC 1 and 2).

The program to take the data is called LoggerPro. This software is pretty good at discovering the gates automatically. If it does not find your gates, close the program and re-start it. The most useful features of the program are accessible through the button bar. The first one to use is the button to set data collection parameters. This button has a drawing of a stopwatch on it. You can determine how often the computer checks the photogates (sample rate) and how long it takes data (Duration). **You must configure the photogates correctly in order for this experiment to work. In particular, set the sample rate as high as possible (1500 samples/second at least).**

Once you have set the collection parameters, you need to take the data. There is a green button on the button bar to start taking data. As soon as you start collecting data, this green button turns red and becomes a button to stop collecting data. While you are taking data, the computer updates the display: there are graphs in the largest part of the window which show your data as a function of time, and there are columns of numbers to the left which show the raw numerical data. When you have performed an experiment, these columns of data are what you will copy and paste into a spreadsheet. You will find a long (2 meter) aluminum track clamped to the side of a table. Wooden blocks are provided for this purpose. *Please do not use excessive force when clamping the tracks. As long as the track does not move accidentally, it will be fine.* Two photogates should be mounted on the track. There are nuts that slide in the groove into which you can thread a screw and tighten it.

Make sure that the photogates are level (check this with a level) and *tight,* so that they do not rotate when hit accidentally by a falling ball. Set the two photogates so that they are about 2 meters apart. You will need to measure the distance between the photogates precisely. Note that the relevant distance is that between the photodiodes. Your TA will provide you with a plastic plate that mounts in the top photogate as well as a plumb bob (a weighted string) that fits snugly into the plate. Let the plumb bob settle down and make sure that the string passes through the centers of both photogates.

It is very important that the two gates are precisely level and that the string passes exactly through the center of the top gate. You can afford to be slightly off at the bottom gate. This procedure ensures that a falling ball will pass through both gates. There should be enough friction between the bottom of the track and the rubberized floor so that you can move the track around to align the photogates with the vertical axis determined by the plumb bob. Because each gate records two times (when the gate is blocked and then unblocked), you can use the known diameter of the ball to determine its velocity *provided that the exact center of the ball passes through the photogate.* **Make sure you take data at the fastest rate possible, at least 1500 readings per second, so that your timing error is less than 1 millisecond. Make sure that no other software is running.**

Given that you can measure the times at which each of the two gates are blocked and unblocked as well as the separation between the gates very precisely (quantify your error estimates), convince yourself that you should be able to measure an acceleration with a precision of better than 1%. Drop a stainless steel ball using your apparatus, taking at least 10 trials. From the mean find an experimental value for the effective acceleration, g_{eff}. What is the uncertainty on this measurement? The standard deviation will give part of this answer, but does not address the possibility of systematics.

Repeat for all the other types of balls, to cover a wide range of masses. Make sure that you include tungsten carbide (the densest material). Analyze your data by plotting g_{eff} vs. $1/m$. What does your data show the 'true' value of g to be? How does this compare with both the expected value as well as the values you obtained last week using the cameras?

You should also use your data to infer the value of the dimensionless inertial drag coefficient *C* in Equation 1-4 (Hint: what is the slope of the line in your plot of g_{eff} *vs. 1/m).* In fact *C* is not some known universal constant but should be around 0.2 to 0.8. What do you find?

Lab 1.3: Projectile Motion in Two Dimensions

Goal: To predict the two-dimensional path of a ball launched at various angles.

Suggested Reading: Read the section of your text that covers two-dimensional kinematics. Also look up the effects of air resistance on projectile motion.

Background

We start with the two-dimensional kinematics equation for constant acceleration:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0(t) + \frac{1}{2}\vec{a}t^2, \tag{1-12}$$

where ***a****(t)* is acceleration, ***r****(t)* is position., $\vec{r_0}$ is the initial position, and $\vec{v}_0$ is the initial velocity. Note that these are now full-fledged vectors, as we are allowing the projectile to follow some trajectory in space.

Consider an object moving only under the influence of gravity. In that case, acceleration is a constant pointing to the center of the earth (vertically "down"). If we give an object an initial velocity in both the horizontal and vertical directions, which we will call x and y, respectively, Eq. 1-13 can be re-written as two simultaneous one-dimensional equations:

$$x(t) = x_0 + v_{0x}t, \tag{1-13}$$

and

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}gt^2, \tag{1-14}$$

where we have assumed that gravity acts in the positive *y* direction (i.e., down is positive). Note that the only thing that the above equations have in common is time, *t*. Thus, given only the initial position, (x_0, y_0), and velocity, (v_{0x}, v_{0y}), of an object subject only to the force of gravity, we can predict exactly where it will be at any time.

Experiment

2-D Projectile Motion equipment:

- Pasco short range projectile launcher and balls
- 1 photogate
- 1 ring stand
- Vernier LabPro computer interface
- 1 desktop computer
- 2 wooden blocks
- Cork board wrapped in a fresh (unwrinkled) piece of aluminum foil.
- measuring tape
- 1 table clamp
- 1 stopwatch

We will now carry this out using spring-loaded projectile launchers. When using your launcher, remember that the muzzle velocity will depend on how far the spring is pressed. (a certain number of clicks). Use the photogates to determine the muzzle velocity for each click possible on the launcher. How is the muzzle velocity related to v_0 and v_{0y}?

You should now be able to write an equation that predicts the x location of a ball given its muzzle velocity, initial height from its target, and angle of launch (a "range equation"). Pick a muzzle velocity (start with the lowest given the limited space of a lab room) and an angle and determine where the ball should land. Mark that spot with a magic marker on a piece of aluminum foil wrapped around cork board that you place on the floor. When the ball hits the corkboard, it will leave a dimple on the foil, showing where it landed. Can you get it right on the first try?

Test the entire range of possible launch angles, and plot your results in a manner that will allow you to test your range equation. Take data for the range as a function of the launch angle. At what angle do you expect to find the maximum range? Is this what you observe?

If time permits, vary the change in height between your launcher and the target. You may fire from a lab table to the floor or from the floor to a target on a table or chair. How does the range vary with angle as you change the height?

Bonus Exercise: Air Resistance

Air resistance is commonly blamed by students for errors in labs such as this. Before attributing disagreement between measurement and prediction to air resistance, it is important to explore whether air resistance has a great enough effect to account for the observed error, or if other factors, such as standard uncertainty is the more likely culprit.

Consider the simple situation where the ball is fired completely horizontally. The equation for motion in the y-direction is essentially the same as Eqs. 1-10 and 1-11 from Lab 1.2. What is the difference between the time it takes to fall with and without air resistance in the calculation? Calculate the uncertainty of how long it takes to fall without air resistance (using your uncertainty of the height measurement) and compare to the previous difference. Now look at the x-direction. Use the equation

$$v = v_0 - a_{air}t$$

where $a_{air} = \frac{F_{air}}{m}$ is a function of v as shown in Eq. 1-4, and solve for the final velocity in the x-direction due to air resistance. Compare the uncertainty of your muzzle velocity to change air resistance has on final x-velocity.

For further investigation, research the generalized effects air resistance would have in this situation and quantitatively apply its results to your analysis of error.

Are the effects of air resistance a significant fraction of the calculated uncertainties or the measured difference from prediction?

Lab 2: Momentum and Energy

Velocity and (constant) acceleration are not enough to describe motion. We need the concepts of momentum and Energy to truly understand objects in motion.

Momentum is defined as

$$\vec{p} = m\vec{v}, \tag{2-1}$$

where *m* is the mass of the object and ***v*** is the velocity. If we have many objects in our system, we write the total momentum as

$$\vec{p}_{net} = \sum_i \vec{p}_i = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots, \tag{2-2}$$

where $\boldsymbol{p}_i$ represents the momentum of each object in our system. Newton's second law can be re-written as

$$\sum \vec{F}_{Ext} = \frac{d\vec{p}_{net}}{dt}, \tag{2-3}$$

where $\boldsymbol{F}_{ext}$ represents forces which are external to the system. If there is no external force on the system, there will be no change in momentum. In these cases, collisions for example, momentum is conserved such that:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f \tag{2-4}$$

Kinetic energy (T), the energy of motion, is defined as:

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{2-5}$$

Note that this is a scalar quantity.

Energy is related to forces through work. We define work by

$$W = \int_{\vec{x}_i}^{\vec{x}_f} \vec{F} \bullet d\vec{x}, \tag{2-6}$$

The work is related to the kinetic energy through the work-energy theorem

$$W = \Delta T. \tag{2-4}$$

Lab 2.1: 1-D Collisions: Colliding Carts

Goal: Predict and measure the final momentum cart after a collision. Examine energy loss from carts during a collision.

Suggested Reading: Read the sections of your text concerning momentum, collisions, and energy conservation.

Background

Collisions are a very common physical process and usually the first example used when discussing conservation of momentum. Collisions are used because in a perfect environment there are no external forces (thus momentum is conserved), but an obvious physical change is observed. There are two types of collisions, elastic and inelastic. Elastic collisions are when the colliding objects bounce off each other, inelastic collisions are when the colliding objects stick together. In this lab we will set up both types of collisions and observe how well we can measure their conservation of momentum.

Experiment

Below is a list of equipment that you may find helpful. You may not need to use all of it.

- 2 Pasco carts and mass set
- 1 Pasco aluminum track
- 2 Photogates
- Vernier LabPro computer interface
- a desktop computer
- a meter stick
- a level
- Ultrasonic position sensor (optional)

Setting up collisions using carts is fairly simple. Begin by setting up carts on the track and doing a few test collisions without worrying about recording data. Explore fast and slow collisions, try different ends of the carts hitting, play with masses a bit. How can you set up an elastic collision? How can you set up an inelastic one? It is okay (and all-around easier) to do these collisions with one cart starting at rest, but experiment with collisions with both carts moving (both in opposite directions and the same direction) to see if you may want to use those methods.

You can use either photogates or sonic position sensors to determine velocities. Devise a set-up that will let you measure the momenta of the carts for both types of collision as accurately as possible. How will the data you get be different for the two different types of collisions?

What types of trials will you take? You will need to take multiple trials for both elastic and inelastic collisions by varying mass and velocity. Determine what you can change reliably and what changes can only be done qualitatively, and plan out your experiment. You should also record the distances between the collision and the points where velocity is measured for every trial, as you will use this in your energy calculations.

How will you determine if momentum is conserved? What comparisons can be made between the two types of collisions given your plan? How will you show if your measured results agree with what physics predicts? You are unlikely to get precisely $p_f - p_i = 0$, so decide how you will accurately determine experimental uncertainty.

Collisions and Energy

We can also use this lab to explore the concept of energy conservation. Ideally there are no external forces doing work (otherwise momentum would not be conserved) but it is possible for *internal* forces to do work, specifically the forces that bind the objects together in an inelastic collision. How will this effect kinetic energy?

Calculate the measured initial and final kinetic energy for each trial. Determine when kinetic energy is and isn't conserved in your trials and see if that agrees with predictions. When energy isn't conserved, calculated the efficiency of the collision ($\epsilon = \frac{T_f}{T_i}$) from you data and compare to what you predict that efficiency should be based on momentum conservation.

Now consider the effect that friction could have on energy conservation or efficiency. In order to account for energy lost due to friction, you will need the distance between the collision and where you measure the carts velocity for use in equation 2-6. Determine the amount of work done by friction for each of the trials, leaving the coefficient of friction as an unknown (but constant between all trials). If you assume that the error between your predicted final energies and measured final energies is from friction, what do you calculate the coefficient of friction to be? Determine if this is a reasonable coefficient by looking up other coefficients of similar value. Determine how to apply the effect of friction to momentum conservation (consider Eq. 2-3). Does this improve your momentum results?

Lab 2.2: Two-Dimensional Collisions

Goal: To observe conservation of momentum in two dimensions.

Suggested Reading: Read the sections of your text concerning momentum and energy conservation.

Experiment

Equipment:

- An air table
- Several plastic pucks
- Video camera
- a desktop computer
- a meter stick
- a level
- shims or pieces of card board used for leveling the air table

Make sure the air table is setup in a suitable location. Make sure that it is as level as you can reasonably get it. To test this, turn on the table and set a puck in the center and see which direction it moves. Using a level and shims, try and get the table so that the puck has little natural acceleration in any given direction. You may wish to take a video of the natural acceleration of the puck to determine how much of a systematic error this will add to your experiment.

Begin by simply setting up several type of 2D collisions. You should try head on collisions, but also set up some collisions where the pucks approach eachother at perpendicular, obtuse, and acute angles. You may also try collisions with one puck not moving or moving in the same direction as the. Some pucks have velcro on them. These can be use to make inelastic collisions. Practice a few inelastic collisions, making sure you can set up the collisions so the final pucks are not rotating noticeably.

Chose several interesting trials for elastic and inelastic puck collisions and write them down. These will be the ones you take videos of, so they should explore the possibilities of two-dimmensional collisions.

Set up the video camera so it can see as much of the table as possible, while being close enough to see as much detail as possible. Make sure your camera is focused so you can see the puck and calibration object clearly. Make sure the exposure is fast enough that the motion is not blurry. The edges of the video will be slightly distorted, so make sure the action takes place mostly in the center of the cameras view. Take videos of each of your trials and save them.

Open up the ”Tracker” program (search for it from the start menu and answer ’yes’ to relaunching it. Refer back to lab 1.1 for more information on using Tracker), and load your video. You will be able to determine the orientation of the axes. What is the best way to orient the axes? Be careful not to change the axes between data taking in the same video. Take data for the motion of both the pucks before and after the collision in both the x and y axis.

From the distance vs time information, determine the velocities for each puck in the x and y axis. Is there any accelaration? What are your uncertainties on the velocities? Consider carefully how these uncertainties will propogate to the final values. Using velocity calculate the initial and final momenta (and thier uncertainties) in the x- and y-direction and see if momentum is consevered.

Lab 2.3: Energy and Explosions of Spring-Loaded Carts

Goal: Calculate the final kinetic energy of 'exploding' carts. Investigate the work done by springs.

Suggested Reading: Read the sections of your text concerning momentum and energy conservation, as well as sections discussing springs and friction.

Background

If you have two objects at rest with a compressed spring between them and then you release the spring, what will happen? Will momentum be conserved? Will energy be conserved? How is this like a collision? How is this like an explosion?

In this lab work will come from the extending springs. At this point in the course, we may have not yet learned about springs, but we can write that the work done by a spring that is either extending or compressed is:

$$W_k = \frac{1}{2}kx^2, \tag{2-9}$$

where k is the spring constant (a large k indicates a stiff spring) and x is the displacement of the spring from its equilibrium position. Note that an ideal spring stores energy whether it is stretched or compressed. Springs provide conservative forces, which means that all energy put into them can be taken out again. Air resistance and friction are famous examples of non-conservative forces. Any work you do against them is energy lost. If we write the total energy of a system as

$$E = T + W \tag{2-10}$$

then we say that the energy is conserved if there are no external or non-conservative forces. We write this as

$$\Delta E = W_{Ext} - W_{nc}. \tag{2-11}$$

There are many cases in which we can say that the work from external forces and the work from non-conservative forces are zero, and we are left with the useful idea of energy conservation, $\Delta E=0$.

Experiment

Equipment:

- 2 Pasco carts and mass set
- 1 Pasco aluminum track
- 4 photogates
- 1 Vernier LabPro computer interface
- 1 desktop computer
- Various masses
- 1 meter stick

The Pasco carts have spring loaded plungers at one end. You can compress the spring by different amounts ('x') and measure the final velocities of the carts after the carts are released (using either photogates or sonic position sensor). Set up several trials for several different mass configurations and spring compressions and measure the final kinetic energy after the explosions.

Make sure you measure the distance the carts travel before their energy is measured for each trial so you can estimate the work done by friction.

The k of the spring can be found by plotting Force applied vs. x. Often the springs are slightly compressed even when the plunger is at full extension, resulting in a non-zero y-intercept, so the slope of the line will give a more accurate value of k than by computing it at each individual point.

How does the final total energy depend on the compression of the spring? Is momentum conserved?

Lab 2.4: Momentum and Energy: The Ballistic Pendulum

Goal: Use momentum and energy conservation to measure the muzzle velocity of a ball launcher.

Suggested Reading: Read the sections of your text concerning momentum and energy conservation.

Background

A very reliable way to measure the muzzle velocity of a gun is to shoot it at a bucket of clay hanging from ropes. The final height of this bucket tells you about the initial velocity of the bullet. We cannot give you rifles, but spring-loaded projectile launchers are almost as exciting. The long-range launchers can be hazardous, so be sure to take proper precautions and aim the launcher away from your laboratory colleagues.

Experiment

Equipment:

- 1 Pasco long- range projectile launcher
- 1 Pasco short-range projectile launcher
- Steel or tungsten carbide balls
- 1 photogate
- 1 Vernier LabPro computer interface
- 1 desktop computer
- 1 Ballistic Pendulum set-up
- 1 meter stick
- stopwatch

Chose a launcher and steel ball and set up the ballistic pendulum. Fire the ball into the pendulum and consider what happens in terms of momentum and energy conservation. What are the initial properties of the ball (velocity, momentum, energy, etc.)? What are the final properties of the ball? What processes happened in between that may have conserved (or not conserved) momentum and energy? Was there a collision? Was there work done? In the end, you should be able to determine the muzzle velocity of the launcher based on the maximum change in height of the pendulum.

The ballistic pendulum lets you quickly measure the maximum angle the pendulum swings. How does this angle relate to the change in height of the pendulum? What point on the pendulum should you be measuring the change in height at?

Take as measurements for all compressions of both short-range and long-range launcher type. You may also change the mass by using a tungsten carbide ball. Calculate the muzzle velocity for each trial. Compare your results to measurements of muzzle velocity using a photogate. **Be very careful when launching the metal balls! Make sure they are always caught either by the pendulum or a padded basket. You are responsible for your safety and the safety of others in the room.**

Lab 3: Rotational Motion

The equations that define rotational motion are very similar to linear equations of motion.

$$x = x_0 + v_0 t + \frac{a}{2}t^2 \text{ is visibly similar to } \theta = \theta_0 + \omega_0 t + \frac{\alpha}{2}t^2 \tag{3-1a}$$

Likewise

$$\vec{p} = m\vec{v} \text{ is comparable to } \vec{\mathrm{L}} = \mathrm{I}\vec{\omega} \tag{3-1b}$$

and

$$\vec{F} = m\vec{a} \text{ is comparable to } \vec{\tau} = \mathrm{I}\vec{\alpha} \tag{3-1c}$$

For angle (θ), angular velocity (ω)(or 'angular frequency', they are effectively the same quantity), angular acceleration (α), moment of inertia(I), angular momentum (L), and torque (τ). In these labs, you will test the basic laws of rotational dynamics, including the relationship between torque and angular acceleration, the concept of rotational kinetic energy, and conservation of angular momentum.

Lab 3.1: Torque and Angular Acceleration

Goal: To explore the various rotational kinematics and their relationships and similarities to linear quantities.

Suggested Reading: Read the sections in your text related to angular motion and torque. Find a table of moments of inertia for various common shapes (especially disks and rings) and learn how to read it. Familiarize yourself with the Parallel Axis Theorem.

Background

The rotational equivalent to force is torque. Torque and force are related by the equation

$$\tau = F \cdot r \cdot \sin(\phi) \tag{3-2}$$

where *r* is the distance from the force (*F*) to the axis of rotation and ϕ is the angle between the *F* and *r*.

The rotational equivalent to mass is moment of inertia. Moment of inertia is related to mass by the equation

$$I = C_{shape} m r^2 \tag{3-3}$$

Where C_{shape} is a coefficient that depends on shape (and can be looked up in tables) and r is the radius or length of the object (usually also defined in the table). When an object is off-center (that is, not rotating around its center of mass) moment of inertia can be recalculated the Parallel Axis Theorem.

For a point radius *r* from the center of rotation, the linear distance traveled (*d*) and velocity and acceleration perpendicular to the rotating surface ($v_\perp, a_\perp$, respectively) are

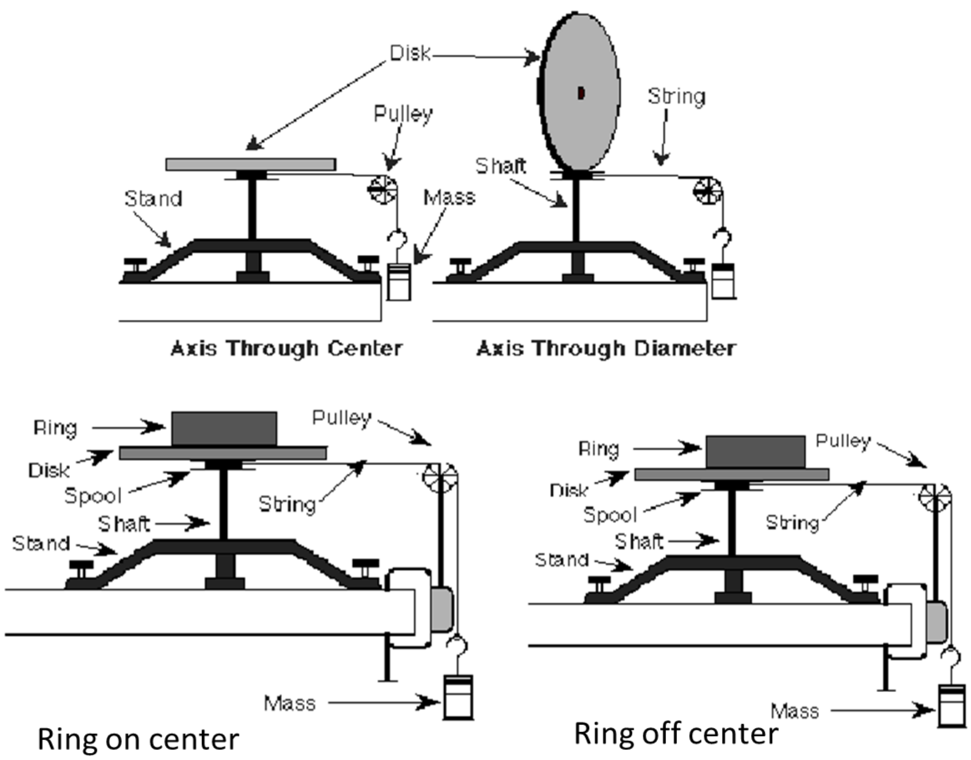
$$d = \theta r,\ v_\perp = \omega r,\ a_\perp = \alpha r \tag{3-4}$$

Experiment

Equipment:

- Rotational dynamics apparatus
- Meter stick
- Video camera
- Pulley
- String
- Set of weights
- Stopwatch

You will be given an apparatus consisting of a string wrapped around a spool that is attached to a disk. The string is run over a pulley and the other end is attached to a mass. You can measure all of the distances and masses. Note that your analysis will depend on your system satisfying the “no-slip” condition, under which you can relate the tangential acceleration and velocity of the point at which the string is unwinding to the acceleration and velocity of the falling mass. By considering the force on the string and the torque on the spool, derive an equation for the acceleration of the falling mass as a function of mass, moment of inertia, and the radius of the spool where the string is attached.



Four different configurations of the disk for Problem 3.1

Note that you can mount the large disk horizontally, as shown in the above figure, or with the disk rotating around an axis passing through its diameter, as shown in the figure below. When the disk is mounted horizontally, you also have the option of adding a massive ring, which can either be a) centered on the disk or b) placed off center. Note that these four set-ups provide FOUR different moments of inertia of the rotating disk. Given the masses of the disk and the ring, which you can measure, what are the moments of inertia for each of the four cases?

For each of the four cases described in the previous paragraph, make a prediction for the acceleration of the falling mass. Now measure the acceleration of the falling mass. **Make sure you choose a mass that is large enough so that**

the effects of friction (acting on the bearings of the rotating platform) are negligible. Make sure that this condition is satisfied for the largest of the four moments of inertia. Keep both the mass of the weight and the diameter at which you attach the string fixed. Use the video camera to measure the acceleration of the falling mass as you did in the first week of lab. Remember to put the meter stick right next to the mass so that the apparatus is properly calibrated.

Plot the acceleration of the mass as a function of the total moment of inertia of the platform for the four cases and determine whether your results agree with your predicted equation.

Lab 3.2: Conservation of Angular Momentum

Goal: To observe the conservation of angular momentum

Suggested Reading: Read the section in your text about angular momentum.

Background

Angular momentum (refer to Eq. 3-1b) functions very similarly to linear momentum in terms of conservation. While an elastic rotational collision is hard to set up, a rotational inelastic collision is as easy as dropping a ring onto a spinning disk.

Experiment

Equipment:

- Rotational dynamics apparatus
- Meter stick
- Video camera
- Pulley
- String
- Set of weights
- Stopwatch

Set up a very simple test of angular momentum conservation, noting that you can drop the ring onto the spinning disk. You can do this in a way that does not exert any torques (why is this important?) on the disk, although it may take some practice. You can use the video camera to measure the angular velocity of the disk before and after the ring is dropped onto it.

Compare the angular momentum before and after the ring is dropped onto the desk for several different initial angular velocities of the disk. Is angular momentum conserved? Is energy conserved or is the energy loss what you would predict?

Lab 3.3: Work and Energy Conservation

Goal: To investigate rotational kinetic energy and how it relates to work done and linear kinetic energy.

Suggested Reading: Read the sections of your text pertaining to rotational kinematics and rotational energy.

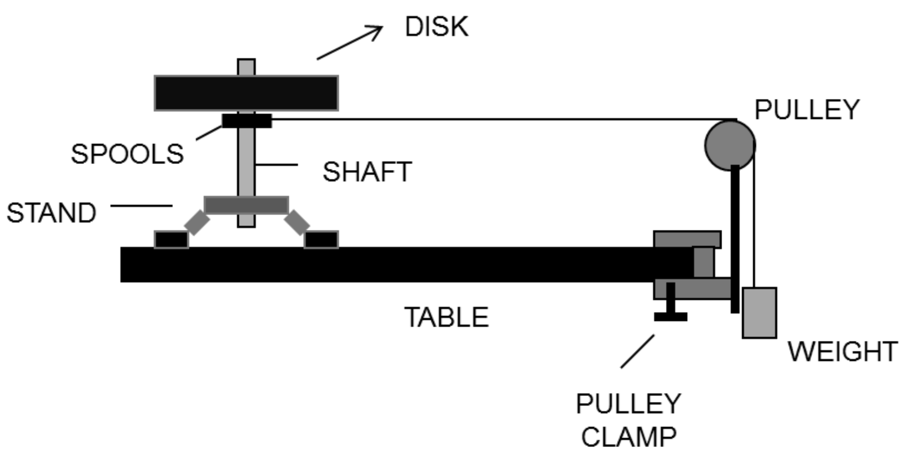
Background

Energy itself is neither linear nor rotational, though linear kinetic energy and rotational kinetic energy are different properties. Thus the energy of a rotating system obeys all the behaviors of energy when relating to work done, etc.

$$T_{rot} = \frac{1}{2} I\omega^2 = \frac{L^2}{2I} \tag{3-5}$$

This can be combined with linear kinetic energy and work done to find the total energy of the system.

Experiment



Problem 3.2: Rotational dynamics apparatus. The string attached to the weight can be wrapped around either of two spools or to the perimeter of the disk.

Make a quantitative test of energy conservation for your system as follows: Measure the velocity of the mass after it has fallen a fixed distance. Determine the kinetic energy of the mass, the kinetic energy of the rotating disk, and the total work done on the system. Compare with your predictions.

Note that there are three ways to attach the string to the platform: (a) inner spool, (b) outer spool, or (c) attached to the perimeter of the disk itself. Repeat the measurement for each of these cases, making sure that the mass falls the same distance each time. For each case, determine the total work done on the system and the kinetic energies of the falling mass and the rotating platform.

Now use your knowledge of the forces and torques, as well as the measured acceleration of the mass to compute the work done on the mass and (separately) the work done on the rotating platform for each of the three cases. Discuss in the context of your calculations of the kinetic energies of the mass and the platform. Does all of the accounting make sense? Represent your results in a suitable table. Is the work-energy theorem satisfied?

Lab 3.4: Gyroscope

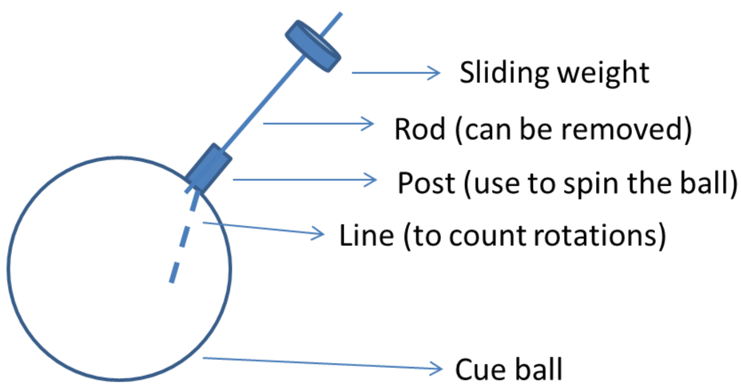
Goal: To compare the observed precession of a gyroscope with predictions based on rotational dynamics of a rigid body.

Suggested Reading: Read the relevant sections on precession of angular momentum (the gyroscope) in your textbook.

Background

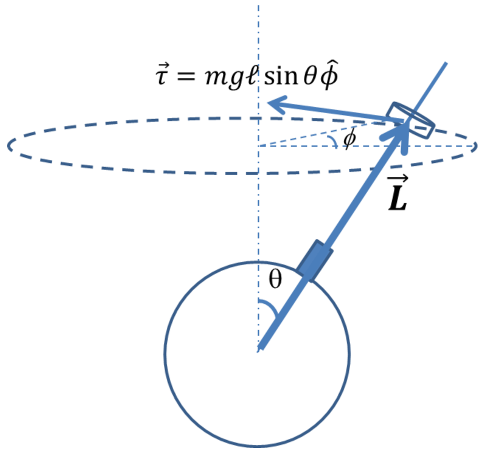
There is probably no more interesting aspect of the physics of rotations than the gyroscope, which is often one's first direct encounter with the necessity of treating angular momentum as a vector. Before coming to lab, make sure that you have read the section of your textbook on gyroscopes and that you understand how an object with angular momentum $\vec{L}$, subject to an external torque $\vec{\tau}$ will precess with a frequency Ω_P. We are considering only cases of reasonably high symmetry, in which the angular momentum $\vec{L} = I\vec{\omega}$, where I is the moment of inertia of the rotating mass that makes up the gyroscope. Make sure that you understand the distinction between the angular rotation frequency (ω) of a spinning gyroscope and the precession frequency (Ω_P).

Our gyroscope consists of a billiard cue ball with a small post that you can use to spin the ball. In the language of rotational dynamics, this particular version of a gyroscope is also called a "simple symmetric top." A graphite rod with a weight can be inserted into the post. The rod and weight provide gravitational torque on the ball, which is otherwise completely balanced.



The gyroscope. Note that the rod with the sliding weight can be removed. The line (which you should draw on the ball if it is not already there) is used in conjunction with the video camera to determine the rotation frequency of the ball.

What is the moment of inertia of the sphere when spinning about its symmetry axis, which passes through the post? Convince yourself that it suffices to use the usual result $I = 2MR^2/5$, where M and R are the given mass and radius of the ball, ignoring the complications created by the magnet and post. In other words, the deviations from the exact result for a sphere are negligible. Do the rod and weight, when added, change the moment of inertia by any measurable amount, assuming the sphere is spinning about an axis passing through the rod?



Vector diagram showing the angular momentum $\vec{L}$ of the spinning ball and the torque $\vec{\tau}$ acting on the ball.

Now, derive an equation for the precession frequency Ω_p for a given spin frequency ω of the ball. You will need to decompose the angular momentum vector $\boldsymbol{L}$ into vertical and horizontal components, after which the remainder of the derivation is essentially identical to that for the gyroscope in your textbook. Complete this derivation before you come to lab. Some steps are outlined here.

The torque acting on the ball due to the mass on the rod is

$$\vec{\tau} = mg\ell \, sin\,\theta\hat{\phi}\,, \tag{3-6}$$

where m is the mass and l is the distance of the weight from the center of mass of the sphere. Note that this torque is a vector pointing tangent to the dashed circle drawn in the figure. Now consider that

$$\Delta\phi \approx \frac{\Delta L}{L sin\theta} \tag{3-7}$$

where ϕ is the angular displacement due to precession. Note that Eq. 3.7 follows from geometry alone, where $\Delta L$is the differential arclength corresponding to the small angular displacement $\Delta\phi$ of the angular momentum $\vec{L}$along the dashed circle of radius $L\sin\theta$ shown in the figure. The precession frequency Ω_p is simply the rate at which ϕ changes:

$$\Omega_p = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}L}{\mathrm{d}t \, L sin\theta}. \tag{3-8}$$

Now, given that $\tau = \frac{dL}{dt}$, derive an equation for the precession frequency Ω_p for a given spin frequency ω of the ball. Does θ appear in the result?

Prelab Exercise 1: Make sure that you show your complete derivation in your lab book. How does the precession frequency Ω_p depend on ω? Will the ball precess faster or slower as ω increases? How should $\Omega_p$vary with the position of the weight?

Prelab Exercise 2: Modify your derivation so that it accounts for the mass of the rod.

Prelab Exercise 3: The rod and weight together have a mass that is a few grams, compared to the cue ball's mass of 140 g. Explain why the effect of these on the moment of inertia of the sphere is negligible. Be careful: a small mass by itself does not guarantee a small moment of inertia!

Experiment

Equipment:

- Cue ball
- Graphite rod (~1.0g)
- sliding weight (1.5g) that can be inserted into the post on the ball
- Air bearing and pump
- A plastic frame with copper coils. In this lab, you only need the frame to hold the camera.
- Video camera
- Ruler
- Stopwatch

The cue ball has a mass of 140 g and a radius of 2.7 cm.

You can easily measure the length of the rod. You can come up with a scheme to measure the distance of the weight from the center of the ball. Note that the end of the rod that is inserted into the ball will slide all the way to the center of the ball.

The cue ball sits in an air bearing, which is simply a brass cup with a small hole in the bottom. Air (provided by a fish tank pump) passes through the hole at the bottom of the cup, preventing direct contact between the cue ball and the brass (provided the flow is set to the correct level). Verify that your air bearing works properly by spinning the ball. **Please make absolutely sure not to drop or otherwise abuse the ball.** When spinning at several rotations per second, the ball should continue to spin freely for a few minutes. If this is not the case, check that (a) the ball and brass cup are free of dirt and debris and (b) the air flow is set high enough. Your TA will have some steel wool to polish the cup of the air bearing if necessary.

Before starting, first make sure you practice spinning the ball up. You can get away with grabbing the black post and spinning it with your fingers. Use a pen or your finger to get the ball to spin true, without wobbling. This takes some practice, particularly when the graphite rod is in place. Ask your TA for assistance if necessary.

Second, figure out how to measure the frequency of rotation of the ball with the camera. It will be sufficient to take a few seconds of video in order to do this. **Make sure that the camera is centered over the top of the ball**. At a frame rate of 30 or 60 frames/second, you can easily count the number of rotations and divide by the total elapsed time. If there is not already a line drawn along a "longitude" of your cue ball with a Sharpie marker, put one on it. For better or worse, you will be doing a lot of trials, and you will need to know the frequency of rotation for each trial. Determine the frequency as you are taking the data! One member of the group can take care of it. It is not necessary to store all of these videos.

You will also need to measure the precession period Ω_p, which is the time for the rod to trace out one circular orbit. This is most easily done by using a fixed reference, such as a string hanging vertically, and a stopwatch. You might be tempted to average many precession cycles in order to make your data more *precise,* but note that this will introduce a *systematic* error, because the spin rate is slowing down over time. Timing two or three precession cycles is a good compromise. You can also measure the precession period by video, but you will end up with a lot of video, since, as you will see, the precession is rather slow. You may use whichever approach you wish.

Precession due to gravitational torque

Turn on the air pump. Spin up the ball (without the post inserted) with your fingers. As noted above, the spinning ball will tend to wobble, and you can remove the wobble by nudging the post with your finger or the tip of a pen. Does the ball precess? Now add the rod and weight, and repeat. Make sure you can measure the precession frequency Ω_p and the spin frequency ω. While the angle of the post *(θ)* may not appear in the final equation you calculate, it is safest to stay fairly consistent with which angle spin the ball at. Experiment with a few different angles and pick one that is good for measurements. You will not have to keep track of the exact value of θ. Include the following steps in your investigation of precession:

A. Measure Ω_p for several different values of the spin frequency. Do this for a single position of the weight. Plot your results in a way that will allow you to verify the expected relationship between the precession frequency Ω_p and the spin frequency ω. Of course you should pick variables so that you can fit your data to a line. You should be able to compare the slope of the line with your quantitative prediction. Remember to include the mass of the rod when calculating the total torque. In practice it is difficult to obtain a large range of spin frequencies. Do your best to measure three or four.

B. Reverse the direction you are spinning the ball. What happens to the precession? Plot this data point appropriately on your plot from part A. Remember that ω and Ω_p have signs. Add a couple more data points for this spin direction.

   By fitting the data you obtained in (A) and (B) to a line, compare the slope of that line with your pre-lab prediction. Your lab-book should include a graph of the raw data, the fit, and a comparison with your prediction.

C. By varying the position of the weight, determine whether the relationship between Ω_p and ω depends on the gravitational torque as you expect. Try at least four different locations of the weight. You will not be able to keep ω exactly fixed during these trials. Instead, look at the relationship between the product $\Omega_p \cdot \omega$ and the gravitational torque. Does this behave as predicted?

   Using a plot of $\Omega_p \cdot \omega$ vs. ℓ (the position of the weight), compare the result with your prediction.

Nutation of a top

You have certainly noticed that in most cases the precession is not simply pure rotation. Unless you nudge it, the tip of the post does not simply move in a horizontal plane but will "nutate" up and down. The actual path traced out by the tip of the post can be quite complicated. Look up "nutation of a top" and discuss qualitatively the types of nutating motion that are typically found for symmetric tops. Our cue ball is an example of a symmetric top. Can you observe any of these?

Make a qualitative sketch of the nutation by plotting the polar angle θ of the graphite rod (with respect to the vertical) as a function of time. Although there are several types of possible motion, you only need to sketch one of them.

Precession of the equinoxes

Like our cue ball, the earth is almost, but not quite, spherical. The equatorial bulge of the earth, along with the fact that the equatorial plane is tilted relative to the ecliptic plane, results in a torque in the presence of the gravitational pull of the sun and the moon. This torque causes the Earth's rotational axis to precess with a period of about 26,000 years. It is a bit of a stretch, but this is essentially the same effect that you are observing in this lab!

Lab 4: Gravitation Simulation

In this lab, you will write a computer simulation of Newtonian gravity. You will use this simulation to verify Kepler's laws. Treat the computer simulation like an experiment. Do not just see that it works in one case and declare it a success. Vary the parameters of the system. See that the results are consistent.

Newton's Law of Universal Gravitation

The idea is simple: two masses, m_1 and m_2, feel an attractive force between their centers of mass, which are separated by a displacement $\boldsymbol{r}$. This force is written:

$$\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r}, \tag{4-1}$$

where G=6.673×10^{-11}Nm^2/kg^2. The masses attract each other.

Kepler's Three Laws of Planetary Motion

Just knowing equation 4-1 will not help us understand how gravity really works. Just by observing astronomical data, Johannes Kepler deduced his laws of planetary motion. Kepler pre-dated Newton, and by years of painstaking observation he was able to deduce three simple rules (copied directly from your textbook):

I. *The law of orbits.* All planets move in elliptical orbits having the sun as one focus.

II. *The law of areas.* A line joining any planet to the sun sweeps out equal areas in equal times.

III. *The law of periods.* The square of the period of any planet about the sun is proportional to the cube of the semi-major axis of the elliptical orbit..

Of course, armed with Newton's law of Universal Gravitation and a little analytic geometry, we can deduce Kepler's laws in an afternoon. We can also verify them by simulation, which is the point of this lab. We will solve Newton's second law for the case of a gravitational force (Eq. 5.1) with the aid of a computer.

Lab 4.1: Introduction to VPython

Goal: To become familiar with VPython. To write a simple simulation in VPython that will set-up further simulations in lab 4.2

Suggested Reading: Look up addition information on VPython at vpython.org

Background

Python is fully-featured modern programming language that is very easy to learn. If you are interested in learning more about python, go to http:// www.python.org. You can download the interpreter onto your home computer for free if you really decide that programming is fun. VPython is a nice set of graphical libraries on top of the Python interpreter. For more information about VPython, go to http:// vpython.org.

We will start with a simple programming exercise. It is best if you work in pairs. Run the program called 'VIDLE for Python'. This opens an editor that you can write your program in.

```
# This program draws a ball and allows it to go in a line
from visual import *
from visual.graph import *

#
# Define the attributes of the mass m1
m1=sphere(pos=(-5,0,0),radius=0.1,color=color.blue,mass=1)
m1.vel=vector(0.5,1,0)   # initial velocity of mass
# define trails for the motion of m1
m1.trail=curve(color=m1.color)
dt=0.5 # time of each step
maxtsteps=20 # maximum number of time steps
print "simulated time will be ",maxtsteps*dt," seconds."
# set-up the graph (colors, axis titles, etc.)
trajplot=gdisplay(x=0,y=0,width=800,height=400,
                  title="y vs x", xtitle="x position",
                  ytitle="y position",
                  xmin=-5.5, xmax=5, ymin=0, ymax=25,
                  foreground=color.black, background=color.white)
xypoints=[] # initialize the list of (x,y) data
for tstep in range(1,maxtsteps):
    rate(1000)  # slow rate of computation so we can visualize it
    # update the position
    m1.pos=m1.pos+m1.vel*dt
    # update the arrows and trails
    m1.trail.append(pos=m1.pos)
    # record the x and y value of this new point in xypoints
xypoints.append((m1.pos.x,m1.pos.y))
# show the (x,y) data in case you need it later
print xypoints
# plot the graph of (x,y) data
xyplot=gdots(pos=xypoints,color=color.blue)
```

Now select the "Run Module" option from the "Run" menu (or just hit 'F5'). What happened? Make sure you save often and as you change your code, save a copy of each iteration.

Here are some important facts about VPython:

- Comments: everything on a line following the # symbol is ignored by the python interpreter. These are comments, and they make it easy for you to remember why you programmed a certain thing in a certain way. To a novice programmer, these might seem superfluous, but they are crucial. Imagine trying to interpret the above code without the comments.

- Indentation is important. You can continue a long line (such as the "trajplot=gdisplay" line) by indenting the following lines.

- The for loop ("for tstep in..") assigns 1 to the variable `tstep` and executes all of the lines of indented code below the loop. It then changes the value of `tstep` to 2 and repeats. It does this until `tstep=maxsteps`. We ask the computer to execute the line `xypoints.append…` twenty times, but to print the list called `xypoints` only one time. How many points are in the list `xypoints`? If you change `maxsteps` to 10, does this change? What changes if you make `dt=0.05`?

- The = sign is not an equality symbol but an assignment operator. The line "`m1.pos=m1.pos+m1.vel*dt`" is not a mathematical equation, it is an order to the computer to replace the current contents of `m1.pos` with something new. The computer takes the value of `m1.pos` on the right side to be the old value.

- Python knows about vectors. It adds and subtracts them by components, it can find magnitudes, and it can take dot and cross products.

Now you need to start modifying this program. Perform each of the following tasks.

1. Introduce a constant force acting on this mass in the y direction. You can set $a = \frac{F}{m} = 1$ for simplicity.

2. Introduce a second mass, m_2. Make sure that this mass is plotted on the graph, too.

3. Remove the constant force introduced in the first task. Introduce a constant attractive force between m_1 and m_2. Be sure that your force obeys Newton's third law! Plot the position of each mass.

4. Modify this force to depend inversely on the distance squared.

Now you have a system of two objects that obey an inverse square law. Make one of the masses very large to simulate the sun. You may want to reduce the size of each time step to increase the accuracy of your simulation, but if you make them too small, your program will never finish. You can comment out the "`rate`" line if you are tired of waiting for your results. You can print the graphs using the print screen feature or with some software package which will allow you to do a screen capture.

Lab 4.2: Verifying Kepler's Laws

Goal: To show how simulations can help verify and illustrate Kepler's laws.

Suggested Reading: Read the section in your text about Kepler's laws.

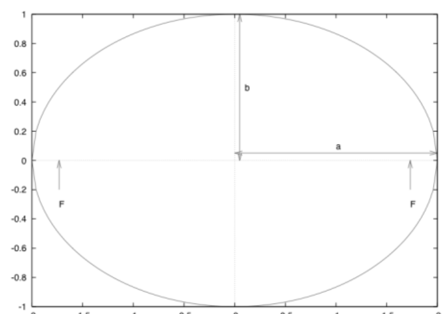
Background

In the previous section, you wrote a program that has two masses that interact via an inverse square law. In this section, you will use and modify that program to verify Kepler's laws. Be sure to save a copy of the source code for EVERY program you write for your lab journal, and make notes on the program so that the logic of the program would be clear to somebody who did not write it with you. Treat this like a real experiment (with virtual apparatus). Keep a copy of all calculated data and graphs in your lab book.

Save a back-up of each program separately and often!

I. The Law of Orbits

You are probably able to qualitatively verify the law of elliptical orbits visually very quickly. However, you should now do it quantitatively. The equation for this ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The distance from the center to either focus is $\sqrt{a^2-b^2}$. The figure below is an ellipse with semimajor axis *a=2* and semiminor axis *b=1*, with *F* representing the foci and the center at (0,0).



For your orbit, find the values of *'a'* and *'b'* and show that they satisfy the equation of an ellipse. Do this for several masses and initial positions. Each group member should have a record of all of the cases that you verify in their lab book.

There are a couple different methods that can be used to do this.

I. Since the sun is at one focus of the ellipse, you should be able to find the major axis (2*a*) by finding the distance between the nearest point to the sun on your orbit and the furthest point from the sun. You can then find the center of the ellipse and use its distance from the sun to find *b*.

II. The sum of the distances from any point on the ellipse to each of the foci is equal to *2a*. You could modify your program to calculate this sum for each point on your ellipse to determine the semimajor axis or as additional proof that it is an ellipse.

III. You may do this with a print-out and a ruler and just manually measure the axes. As the point of this lab is to show how simulations can save us the trouble of doing things with rulers and such, this violates some of the spirit of the lab, but is a good compromise if the above methods are proving troublesome.

II. The Law of Areas

You can calculate the area swept out in a single time step by approximating the area as a triangle. If we say that we are at time step number n, then $\boldsymbol{r}_n$ is the distance between the sun and the satellite at the current time step $\boldsymbol{r}_{n-1}$ is the distance between the sun and the satellite at the previous time step. This triangle has sides formed by the vectors $\boldsymbol{r}_n$, $\boldsymbol{r}_{n-1}$ and $\boldsymbol{r}_n$-$\boldsymbol{r}_{n-1}$. Modify your program to calculate the area of this triangle at each time step, and keep track of it as a list for plotting. Is this number constant? Does it get worse as your simulation goes on? Does it get worse if you make your time steps bigger?

To show that this works correctly, plot the areal velocity (i.e. $\frac{dA}{dt}$, where A is the area of the triangle as a function of time. Do this for all of the cases that you did for the law of orbits. Calculate the average for each of the cases. Is there a trend? These graphs also belong in your lab book.

III. The Law of Periods

For this one, you need to calculate not only the period but also the semimajor axis of the orbit. Since you have a vector which describes the position of the sun and one which describes the position of your satellite, the distance between the two is the magnitude of the difference of these two vectors. Vpython knows how to subtract vectors, and the `abs()` function will give you the magnitude of a vector. Recalling a little bit about the geometry of ellipses, you should now be able to complete the task.

Since the numerical error of each time step accumulates, the orbits will not close exactly. You can visually inspect the data to see when the orbit is approximately closed, but it would be better to make the computer do it. The programming feature of value will be the `if` statement. For example, if you wanted the computer to print the period when the current position is within .00001 of the current position, you would type (inside the for loop) :

```
if abs(m1.pos-initial.pos) < 1e-5:
      print "the time of this period was:",tstep*dt
```

This assumes that you have defined the vector `initial.pos` to be the initial position. You may adjust the distance of '`1e-5`' to work porperly within the framework of your own program.

Once again, you should do this for the same cases you tried previously. You may make the program print the numbers out and then copy-and-paste them into a spreadsheet. You should make a graph of period vs. the length of the semimajor axis. How would you transform your results to test the statement quantitatively? You should have enough data to fit the equation given by Kepler's third law. These results need to be in your lab book.

Further simulations

Now that we've show the basics of Kepler's laws in a simulation, let's explore some other aspects of an orbit that our code can simulate.

IV. **Angular momentum:** Compute the angular momentum of the satellite about the center of mass. Display it graphically with an arrow on your figure. Is the angular momentum constant?

V. **Precession of the Perihelion:** If you run your simulation for several orbits, does the position of the perihelion (the point in the orbit when the two masses are closest) change? What if you change the force to be an inverse cube force? What about just 1/r? How about $1/r^{2.5}$? You may want to look at improved integration algorithms to help answer these questions.

VI. **Binary Stars:** Consider two stars of roughly equal mass. Taking the center of mass of this two-body system as the origin, determine the orbits of both stars.

Optional Advanced Simulation: The Tides

The tides are caused by the gravitational attraction between the moon and the water in the oceans. A bulge occurs in the water nearest the moon and the water exactly on the opposite side of the earth. Adjust the ratio of the masses to approximate the earth /moon ratios. You also need to choose a sensible set of initial positions. The time between high tide and low tide at a particular place depends on the rate of rotation of the earth and the orbit of the moon. This is tricky to calculate, but it is possible. The first step is to get the moon's orbit correctly. This is hard to do, and you will need to do some additional research to determine whether your program is correct or incorrect. You will have to be very careful about units if you are using to make this work correctly.

Once you have the moon's orbit correct, you need to introduce a point on the surface of the earth to represent a particular shore. You can represent this point as an offset to the position of the center of the earth. This offset will have to change in time to represent the rotation of the earth. You now need to calculate the vector between the center of the earth and your point, and between your point and the moon. If these vectors are collinear (parallel or antiparallel), then the tide is high. If they are perpendicular, then it is low tide. How often do the tides occur in your simulation? Does this align with actual tide behaviors?

Lab 5: Simple Harmonic Motion

Simple harmonic motion begins with the idea of a restoring force. A good example is an ideal spring, which obeys Hooke's law:

$$\vec{F} = -k\vec{x}, \tag{5-1}$$

where k is the spring constant and x is the displacement from equilibrium. You must apply a greater amount of force in order to displace the spring further from equilibrium. The spring will always try to return to its equilibrium length. Now, if we fix one end of the spring and attach a mass m to the other end, Newton's second law gives

$$-kx = m\frac{d^2x}{dt^2}. \tag{5-2}$$

Solutions of this equation are of the form

$$x(t) = A\cos(\omega t + \delta). \tag{5-3}$$

The amplitude of the oscillation, or maximum displacement, is A. The angular frequency is ω and has units of radians per second. This is related to frequency f in Hertz (cycles/second) by $\omega = 2\pi f$. The angle δ is called the phase. Motion which can be described by Eq. 5-3 is called simple harmonic motion.

Lab 5.1: Damped Simple Harmonic Motion

Goal: To observe and quantify the effects of dampening on simple harmonic motion.

Suggested Reading: Read the sections in your text about simple harmonic motion.

Background

Let's consider how damping affects simple harmonic motion. A common example of a damping force is viscous drag. If we want to account for damping in our oscillator, we need to introduce a new term into Eq. 5-2:

$$F = -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}, \tag{5-4}$$

where b represents the damping force, which we have assumed is proportional (but opposite in sign) to the velocity. As you can verify by direct substitution, the time-dependent solution of this equation is an oscillating function with a decaying exponential envelope:

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \delta), \tag{5-5a}$$

where

$$\omega = 2\pi f = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \tag{5-5b}$$

is the observed angular frequency and the natural frequency, $\omega_0 = \sqrt{k/m}$, is characteristic of the *undamped* oscillator. Notice that the frequency of a damped oscillator is smaller than the undamped value.

Prelab Exercise 1: *Graph Eq. 5-5a (as a function of time). For simplicity, set* $A = 1, \delta = 0,$ *and* $\omega_0 = 2\pi$ *rad/sec. Graph a few different cases:* $\frac{b}{2m} = 0,\ \frac{b}{2m} = 0.1\omega_0,$ *and* $\frac{b}{2m} = \omega_0$.

Assuming weak damping ($b < 2m\omega_0$), the time for the amplitude of the oscillatory signal to decay by a factor of 1/*e* is $\frac{2m}{b}$, but it is more convenient to think in terms of the decay of energy stored in the system, which is proportional to x^2:

$$x^2(t) = A^2 e^{-\frac{bt}{m}} \cos^2(\omega t + \delta), \tag{5-5c}$$

and so we define the characteristic *damping time*

$$\tau = \frac{m}{b}$$

or *time constant* of the oscillator. Some textbooks define a characteristic damping *rate* $\gamma = \frac{1}{\tau} = \frac{b}{m}$.

In practice, the amplitude A and the phase δ are determined by the initial conditions. For example, if the mass on the spring is pulled back a distance L from equilibrium and released from rest, then the amplitude $A = L$ and the phase $\delta = 0$. We will not worry about the phase for this part of the experiment.

Prelab Exercise 2: *Setting* $\delta = 0, A = 1,$ *and* $\omega = 2\pi$ *rad/sec in Eq. 5-5c, plot* $x^2(t)$ *for the case* $\frac{b}{m} = 0.2\pi$ *sec*$^{-1}$. *Now plot only those points which correspond to maxima, i.e.* $\omega t = 0, \pi, 2\pi, 3\pi, etc.$ *vs. time. These points should fall on an exponential curve. Now plot these points on a logarithmic scale. Is the slope of the resulting curve –b/m?*

Experiment

Equipment:

- Pasco cart and mass set
- Piece of sheet metal to use as a reflector
- Vernier motion sensor
- Magnets for increasing damping
- Aluminum track
- Two end-stops
- Two springs
- Stopwatch

To observe simple harmonic motion, attach a cart to the two end-stops using springs. To measure the position of the cart as a function of time, we will use motion detectors that are compatible with the Vernier boxes. The motion sensors work by bouncing sound waves off of the object under test and measuring the time delay between when a pulse is launched and when it returns to the detector. Insert a piece of sheet metal into the slots on top of the cart to serve as a sound reflector. There should be a bracket behind the sheet metal so that it does not wobble. Set the data collection rate so that you obtain many points (30 per second) during each cycle of simple harmonic motion. **Like other apparatus, the motion detectors must be used correctly in order to obtain meaningful data. Make sure the data collection rate is high enough (use the maximum value), and make sure the sound waves being measured are bouncing off the sheet metal attached to your cart and not something else nearby.** Sometimes the motion sensors pick up sound waves that bounce off the table. This can be avoided by tilting the head of the sensor that it points slightly upward. If you hit the "Collect" button in the LoggerPro, you should be able to verify that the motion sensor works correctly by moving the cart back and forth with your finger and observing that the program measures the correct displacement.

Pull the cart back several centimeters and release it. Measure the motion the cart with the motion sensor, and determine the angular frequency ω from the resulting plot. You should see at least 15 oscillations (make sure that you set LoggerPro to take 10 – 20 seconds worth of data), and the amplitude should diminish gradually. If not, ask your TA for help, as it is likely that the cart should be replaced. This is a case of low damping. Does the plot agree with the prediction of Eq. 5-5a?

The data output by LoggerPro will include the time, displacement, velocity, and acceleration in three columns. Plot the position, velocity, and acceleration vs. time. Do they look as you would predict?

Now plot the natural logarithm of the square of the displacement vs. time. Since you should have observed 10 – 15 cycles of oscillation, you should see 20 – 30 *maxima* on this plot. Plot the value of each maximum vs. time. You should see a line, the slope of which is $1/\tau$, and which can therefore be used to determine the damping constant *b*.

Make sure the logarithmic plot is in your lab book. Discuss the linear fit and compare with your prediction. What is happening to the energy stored in the oscillator as a function of time?

Try several different values of the mass. Do you observe the expected relationship between the frequency of oscillation and the mass?

Answer this question by making a plot of frequency vs. $\sqrt{m}$.

It turns out that there is a simple and very reliable way to produce a damping force that is proportional to velocity (as is the case in Eq. 5-4) by attaching small disk magnets to the bottom of the cart. The magnets will stick to the screws on the bottom of the cart. The actual damping force is generated by the magnets moving over the (metallic) aluminum track.

Increase the damping by adding magnets to the bottom of the cart. Add the magnets one by one at each of the screws on the bottom. Make sure each magnet is mounted at the same height above the track.

Graph the data for each run, and calculate the values of b and ω for each trial. How does damping constant b depend on number of magnets? Does ω depend on damping as predicted (Eq. 5-5b)?

Lab 5.2: Driven Simple Harmonic Motion

Goal: To observe the resonant response of a driven simple harmonic oscillator. You will measure the amplitude and phase of the response as a function of frequency and verify qualitatively the form of the resonance curve. You will also observe the effect of damping on the resonance.

Suggested Reading: Read the section in your text about driven harmonic motion.

Note: Because the physics of this lab is reasonably sophisticated, we are going to focus on estimates and qualitative interpretation of the data. Although (as always) you should record your data with the correct number of significant figures, there is otherwise no error analysis required.

Background

Now suppose a driving force of the form $F=F_m\cos(\omega t)$ is applied to the oscillator. Then equation (5-2) becomes:

$$F(t) = -kx - b\frac{dx}{dt} + F_m\cos(\omega t) = m\frac{d^2x}{dt^2}, \tag{5-6}$$

which has solutions of the form

$$x(t) = \frac{F_m}{G(\omega)}\cos(\omega t - \delta), \tag{5-7}$$

where

$$G(\omega) = \sqrt{m^2({\omega_0}^2 - \omega^2)^2 + b^2\omega^2} \tag{5-8}$$

and

$$\delta(\omega) = \tan^{-1}\frac{b\omega}{m(\omega_0^2 - \omega^2)} \tag{5-9}$$

Not surprisingly, the mass will oscillate back and forth at the driving frequency, but the amplitude $A = \frac{F_m}{G}$ and the phase δ are functions of the drive frequency and depend on the damping constant. Note that the amplitude of the motion is given by

$$A = \frac{F_m}{G} = \frac{F_m}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{b^2\omega^2}{m^2}}}. \tag{5-10}$$

We are only going to be interested in the case where $\frac{b}{m} < \omega_0$, i.e. weak damping. To get a feel for what is going on here, we can change variables so that $x = \omega/\omega_0$ and $y = Am\omega_0^2/F$. Define the dimensionless constant $\epsilon = \frac{b}{m\omega_0}$. Then Eq. (5-10) can be rewritten in the simple form

$$y = \frac{1}{\sqrt{(x^2-1)^2 + \epsilon^2 x^2}}. \tag{5-11}$$

Pre-lab Exercise 1: Graph this function for three different values of ϵ: 0.5, 0.1, and 0.01. You will find that for $\epsilon \ll 1$ (very small damping), the function is extremely sharply peaked at $x = 1$, which corresponds to $\omega = \omega_0$. If the oscillator is driven at $\omega = \omega_0$ and the damping is small, the amplitude becomes extremely large. This is known as resonance, and the frequency $\omega = \omega_0$ is the resonant frequency. What happens as ϵ, which is proportional to the damping constant b, increases?

The experiment we are about to do will be done in the limit of very weak damping. Note that the amplitude is then significant only for frequencies very close to the resonant frequency, and so we can therefore make the approximation $\omega \approx \omega_0$. This allows for the simplification

$$\omega_0^2 - \omega^2 = (\omega_0 - \omega)(\omega_0 + \omega) \approx 2\omega_0(\omega_0 - \omega), \tag{5-12}$$

and then the amplitude and phase can be written in the remarkably compact form:

$$A(\omega) = \frac{F_m}{2m\omega_0}\frac{1}{\sqrt{(\omega_0 - \omega)^2 + \frac{1}{4\tau^2}}}; \tag{5-13}$$

$$\delta(\omega) = \tan^{-1}\frac{1}{2\tau(\omega_0 - \omega)},$$

where $\tau = \frac{m}{b}$ is the damping time of the undriven oscillator (see the previous lab). The assumption of small damping amounts to saying that $\omega_0\tau \gg 1$.

Let's talk this through. If the damping is small, then the response $A(\omega)$ is sharply peaked at ω_0. It is typical to quantify the sharpness of the peak by finding the frequency ω at which $A(\omega) = A(\omega_0)/\sqrt{2}$. We can see that this occurs when $\omega = \omega_0 \pm \frac{\tau^{-1}}{2}$). These frequencies are also those at which the *energy* stored in the oscillator, which is proportional to A^2, is ½ of its maximum value. A measure of the width of the peak is therefore $\Delta\omega = \frac{1}{\tau} = \frac{b}{m}$. This is often called the "full-width at half maximum" or FWHM. A measure of sharpness is the resonant frequency divided by this width. This is called the *quality factor Q* of the oscillator*:*

$$Q = \frac{\omega_0}{\Delta\omega} = \omega_0\tau = \frac{\omega_0 m}{b}. \tag{5-14}$$

A brief word on the phase. When the driving frequency is much smaller than the resonant frequency, then $\delta(\omega) \approx 0$. This means that the response and the drive are in phase. This is exactly what you will find if you push as mass on a spring very slowly with your finger: the mass simply moves along with your finger. At resonance, however, $\omega = \omega_0$, $\tan\delta \to \infty$, and so $\delta(\omega_0) = \pi/2$. The response on resonance is therefore 90 degrees out of phase with the drive, and so if the drive is a cosine function, then the response should be a sine. As the frequency increases further, so that ω is eventually much larger than ω_0, then $\tan\delta$ becomes small and negative, and $\delta \to \pi$. This means that at high frequencies, the mass moves 180 degrees out of phase with the driving force.

One of the beautiful things about the driven simple harmonic oscillator is that the frequency dependence of the amplitude and phase depend only on the resonant frequency ω_0 and the damping time τ. The fact that our analysis has focused on a mass on a spring is irrelevant! The oscillator in question might be a mass on a spring, but it could also be a pendulum, a torsional oscillator, a vibrating thin ruler (see below), an electronic circuit, the excited state of an atom or molecule, light in the optical cavity of a laser, etc… As long as the equation of motion looks like Eq. 5-6, the response can be described by Eq. 5-10 or (if the damping is small) Eq. 5-13. The details of the system really do not matter! This is what makes this lab so important. The simple harmonic oscillator pops up in some form in just about every discipline of science and engineering.

Pre-Lab Exercise 2: Remember that you typically measure frequencies f in the laboratory in Hz, and $f = \omega/2\pi$. In this week's lab, the resonant frequency $f_0 \approx 10$ Hz. Letting $\frac{F_m}{m\omega_0} = 1$, graph the amplitude and phase (Eq. 5-14) as a *function of frequency f for damping times $\tau = 1, 2,$ and 10 seconds. What are the corresponding quality factors? What is the characteristic width Δf of the resonance peak in each case? Be careful with factors of 2π!*

Experiment

It turns out to be quite difficult to implement a quantitative experiment demonstrating driven simple harmonic motion using only masses and springs. The problem is that the resonant frequency $f = \omega_0/2\pi$ is usually small (~ 1 Hz), and in order to observe a well-defined and sharp resonance (high Q), the time constant τ must be very long. As a result, it takes a long time to collect the necessary data, and our lab periods are only two hours long!

We will use instead what is sometimes called a "saw-blade" oscillator, although the apparatus employs a steel ruler instead of a saw-blade. If a 30 cm (1 foot) thin ruler is clamped at one end and then struck lightly at the opposite end, it will vibrate back and forth for several seconds (like a diving board after you have jumped off) before coming to rest. In this case, the frequency scale is about 10 Hz, which makes it convenient for this experiment. The natural frequency of the vibrating ruler can be found from elementary (but slightly tedious) theory, which we will not replicate here. It matters only that the vibrating ruler behaves like a simple harmonic oscillator, with a natural frequency

$$\omega_0 = \frac{C_1 t}{2L^2}\sqrt{\frac{E}{3\rho}}, \tag{5-15}$$

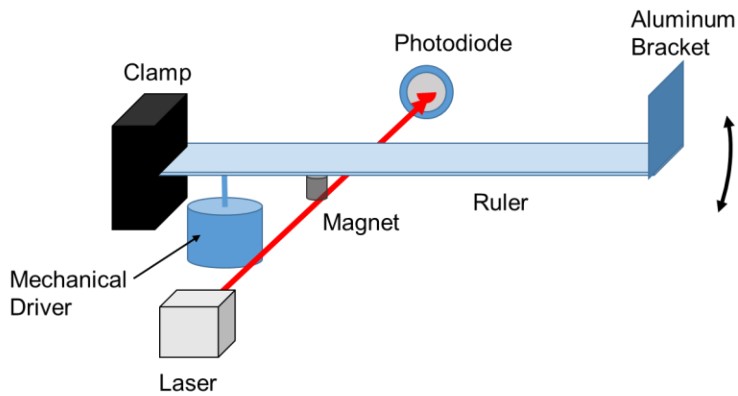
where L is the free length of the ruler (the part that is not clamped), t is the thickness, E is the Young's modulus (elastic constant) of the ruler material, and ρ is the density. The constant $C_1 = 3.5$ for the resonance that we care about. Note that although this is not a simple mass on a spring, the frequency involves the square root of the ratio of an elastic constant divided by a mass (in the form of the density ρ), and so a higher frequency is achieved by using a stiffer (higher E) or lighter (lower ρ) material. In this sense the ruler is "like" a mass-spring system. For stainless steel, $E = 2 \times 10^{11}$ N/m^2 and $\rho = 8 \times 10^3$ kg/m^3. In the set-up that you will use, the unclamped length of the ruler is about 27 cm and the thickness of the ruler is 1.0 mm.

<u>Pre-Lab Exercise 3</u>: *Using the above information, estimate the resonant frequency $f_0 = \omega_0/2\pi$ of the vibrating ruler. Given that the time constant τ is a few seconds, what is the expected width (in Hz) of the resonance curve? Keep this in mind when doing the experiment.*

Equipment:

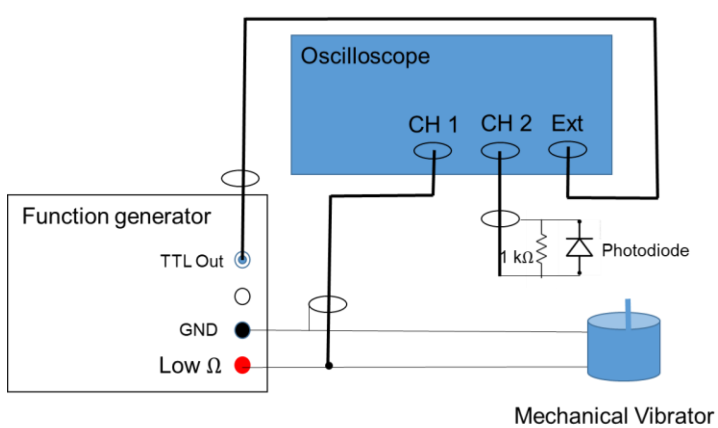
- 30 cm stainless steel ruler clamped on one end. A small magnet is stuck to the bottom and a piece of aluminum is attached to the end.
- Pasco Scientific mechanical vibrator
- Sine wave generator to drive the vibrator.
- Oscilloscope (to view the signals)
- Diode laser with clamp to attach it to the table
- Photodiode with clamp to attach it to the table
- A small magnet on a post for controlling damping
- Various cables
- Stopwatch

A diagram of the apparatus is shown in the figure. A sine wave generator is connected to a mechanical vibrator, which consists of a short post connected to the cone of a loud-speaker. The post presses against the bottom of the ruler about 1.5 cm in front of the point where it is clamped. This provides the driving force. A small magnet is stuck to the bottom of the ruler about 3 cm beyond the post. This magnet partially blocks a laser beam that is incident on a photodiode. As the ruler vibrates, the intensity of the light oscillates, allowing us to detect the motion. The electrical signal out of the photodiode, which is proportional to the displacement of the ruler, is read by the oscilloscope. To take data, you will vary the frequency of the sine wave generator and record the magnitude of the drive voltage (proportional to the driving force) and the photodiode voltage (proportional to the displacement of the ruler). You can also measure the phase shift δ between the two signals, as discussed below.



Schematic diagram of the experiment.

You will find the apparatus assembled. The oscilloscope should be hooked up and turned on. If not, turn it on and wait for it to go through its self-test. Turn on the function generator and set the frequency to about 8.0 Hz. You should see a sine wave on the oscilloscope. If you see nothing, turn up the amplitude so that the indicator on the dial points to the "9:00" position. For the TDS-1001 scopes used in 2017, the sine wave will be yellow. **If you still do not see a signal, ask your TA for help.** Once you have a signal, verify that the frequency and amplitude of the sine wave change as you adjust the relevant knobs on the function generator. The knob to change frequency is labeled "Adjust". On the right hand side of the oscilloscope screen, you should see a menu with some readings. You only need to worry about two of these: CH 1 PK-PK and CH 2 PK-PK. These are the peak to peak amplitudes of the drive and displacement signals. The drive signal that you are looking at is being recorded by CH 1. The displacement signal (in blue) is CH 2.



Electrical connections for the experiment. The apparatus should already be set up. If not, ask the TA for assistance.

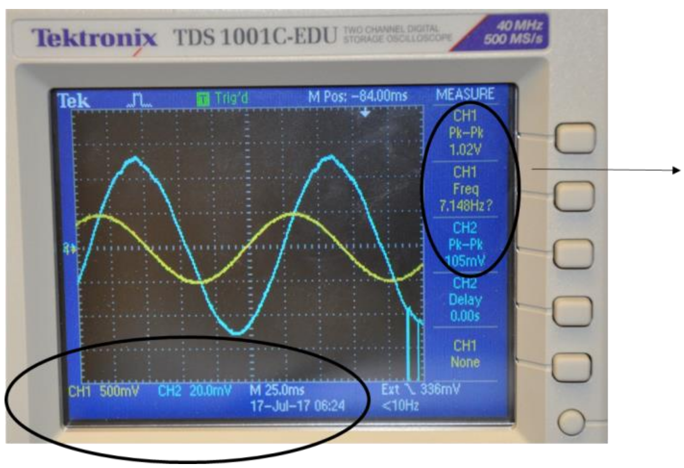
Once you have verified that the drive signal behaves properly, turn the amplitude on the function generator all the way down (but do not turn the function generator off!). Tap the ruler gently so that it vibrates. You should see the blue signal on the oscilloscope jump and then settle down into a sine wave that decays over several seconds. If you do not see a signal at all, ask your TA for assistance. If the signal decays too quickly, verify that nothing is touching the ruler. There is a larger magnet on a ruler attached to a wooden block. Make sure that it is not close to the vibrating ruler.

Using the stopwatch (or a method of your choice), measure the decay time for the vibrating ruler after it is tapped by simply looking at the magnitude of the blue sine wave on the oscilloscope. Keep in mind that the time for amplitude to decay by $\frac{1}{e}\sim 0.37$ is $\frac{2m}{b} = 2\tau$ (see previous lab). This does not need to be an accurate measurement. An uncertainty of 1 second or so is fine. You should find a value for τ around 2 - 3 seconds. Record the time constant.

On the bottom of the oscilloscope, you should see the following:

CH 1 500 mV CH 20.0 mV M 25.0 msec

The first two numbers are the calibrations of the y scales for CH 1 and CH 2, meaning that each vertical division of the oscilloscope (the large squares forming a grid) corresponds to 500 mV and 20.0 mV for the yellow and blue sine waves respectively. Note that mV means "millivolts," but for the purposes of this lab we can treat it as an arbitrary unit of amplitude. The third number is the calibration of the time base, meaning that each horizontal division corresponds to 25.0 milliseconds. **If different numbers appear, ask your TA for assistance**.



Use the CH1 Pk-Pk and CH2 Pk-Pk readings. Ignore the others. The frequency measurement here is not particularly accurate. Use the reading on the function generator instead.

This is how the front of the oscilloscope should look. Note the settings in the lower left corner. The amplitude of the blue sine wave will be significant only near resonance. Note the meaning of the time base: 1 division on the oscilloscope is equal to 25.0 msec. Each small tickmark is 0.2 divisions. For example, the period of the sine wave is approximately 5.6 divisions X 25.0 msec/division = 0.140 sec. If your screen does not look like this, ask the TA for assistance.

Given the 25.0 millisecond/division calibration, estimate (to within 5% or so) the period of free oscillations of the ruler. Note that you can hit the "Single" button on the upper right of the oscilloscope panel to take a "snapshot" of the oscilloscope screen and freeze it before the amplitude decays away. Hit the "Run/Stop" button to restore normal operation. Convert the measured period to a frequency. The answer should be between 6 and 10 Hz. (Because of the additional mass of the magnet and the aluminum bracket at the end of the ruler, the value you measure will probably be smaller than the frequency you calculated in the prelab exercise).

Answer the following questions before proceeding:

1. Given your rough measurements of the time constant τ and the natural frequency f_0 of the ruler, estimate the quality factor Q.

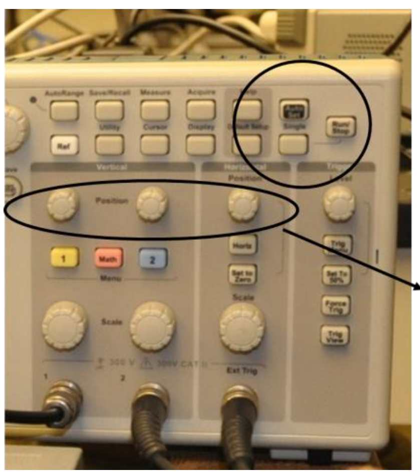
2. Given the quality factor, what is the width of the resonance peak that you would expect for the driven oscillator? You will need this information to develop a sensible data-collection strategy.

Resonance Curve

You will now drive the oscillator, measuring the amplitude and phase of the response as a function of the frequency. Turn the drive amplitude back up and set it so that it is about 1.5 V Pk-Pk on the oscilloscope (the yellow sine wave). Now adjust the frequency and watch the ruler as well as the blue trace (CH 2) of the oscilloscope. Try setting the frequency to the value f_0 you found above. You should now see a blue sine wave on the scope. Now, try adjusting the frequency to maximize the response (amplitude of the blue sine wave). You should notice a visible vibration of the end of the ruler. **Be patient! When you are near the resonance, it will take several seconds (a few time constants) for the ruler to settle in at a new amplitude when you change the frequency. You will end up making very small adjustments to the frequency in order to maximize the amplitude.**

3. Find the frequency for which the amplitude of the ruler is a maximum. Do not worry about uncertainty too much, but how precisely can you determine this frequency? How does the peak frequency compare with your estimate of f_0?

4. When the response is a maximum, what is the phase of the response (the blue sine wave) relative to the drive (yellow sine wave)? As you tune away from the resonance, what happens to the relative phase of the two signals? Based on what you are seeing, come up with a strategy for measuring the phase shift between the two signals (in either radians or degrees). Hint: Note that you can measure the time delay between the zero crossings of the two signals, assuming that they are both centered vertically around $y = 0$ on the oscilloscope screen. You can measure the delay in divisions and use the timebase calibration (25.0 milliseconds/division) to convert to a time delay. If you know the period of oscillation, it is then possible to compute the phase shift. There is a knob on the scope that lets you shift the time coordinate so that one of the curves crosses zero at the center of the screen. This makes it easy to measure the shift (in divisions) between the two zero crossings. Your TA can show you how to do this.



Some useful controls on the oscilloscope.

Now take data as a function of frequency, sweeping from well below (1 Hz below) the resonance to well above it (1 Hz above). At each frequency, record the frequency, the drive amplitude (CH 1 PK-PK), the displacement amplitude (CH2 PK-PK), and the delay (in divisions) between the two zero crossings. You should take 25 – 30 data points covering the entire resonance curve, but note that these will not be uniformly distributed in frequency. **Make sure to acquire more data near the peak.**

5. Graph the displacement amplitude as a function of the frequency. If the drive amplitude (CH 1) was not stable, normalize the displacement amplitude by the drive amplitude. If the drive amplitude was stable, you do not have to worry about normalization.

6. Graph the phase shift as a function of the frequency. You will need to do a simple spreadsheet calculation to convert the measured time delay to a phase shift.

7. Do your graphs look as you expected based on the pre-lab exercises? What is the width Δf of the resonance peak, as determined by the distance between the points at which the amplitude is about $A/\sqrt{2}$, where A is the value of the signal at the peak? Determine the quality factor $Q = f/\Delta f$ and compare with the value you estimated in response to Question 1.

8. Does the phase shift depend on frequency as you would expect based on Eq. 5-14? What is the phase shift on resonance? What is the difference in the phase shift between the lowest and highest frequency points that you measured? How does it compare with the expected value of π radians or 180 degrees?

Now set the frequency to the resonant frequency f_0. Monitoring the amplitude on the oscilloscope, move in the wooden block with the magnet on it until the magnet is about 5 – 6 mm away from the aluminum plate attached to the end of the ruler. You will notice that the amplitude decreases due to damping. Position the magnet so that the amplitude is a factor of 2 – 3 smaller than it was without the magnet. Now repeat the data collection using the same steps as above, mapping out the resonance curve.

9. Repeat steps 5 and 6 above for the data taken with the higher damping.

10. Compare the data with and without damping. What happened to the amplitude and width of the resonance peak when damping was added? Does this agree with your expectations?

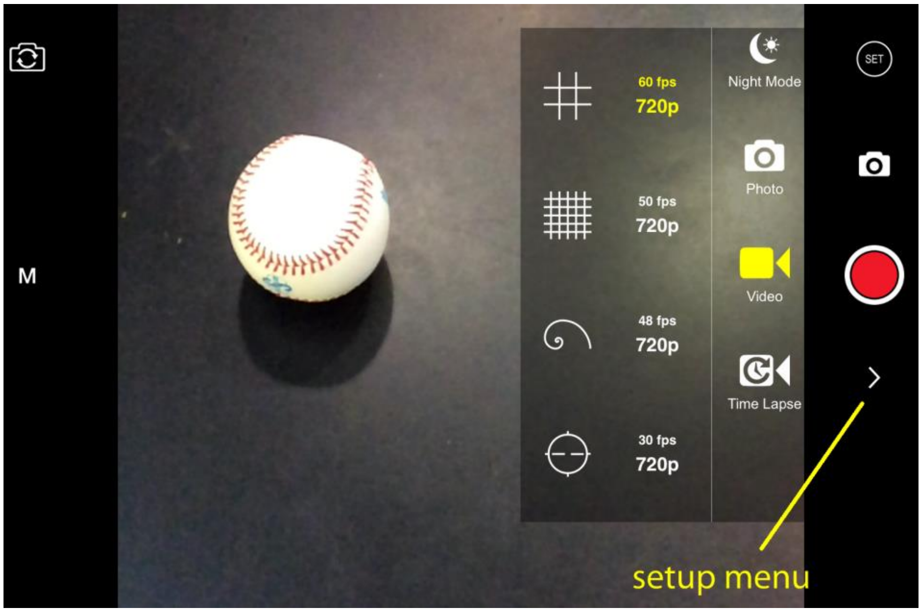
11. *Very large skyscrapers are not unlike the ruler in this lab. They are "clamped" at their base and are otherwise free to vibrate. As you can imagine, this can be unpleasant (or worse!) for the occupants of the building. Look up how engineers address this problem and explain in a few sentences how the solution is related to this lab.*

Appendix A: Special Equipment

The section that follows comprises information that has been cut-and-pasted from manuals for equipment that you will use in the lab. We have attempted to fix typographical errors wherever possible.

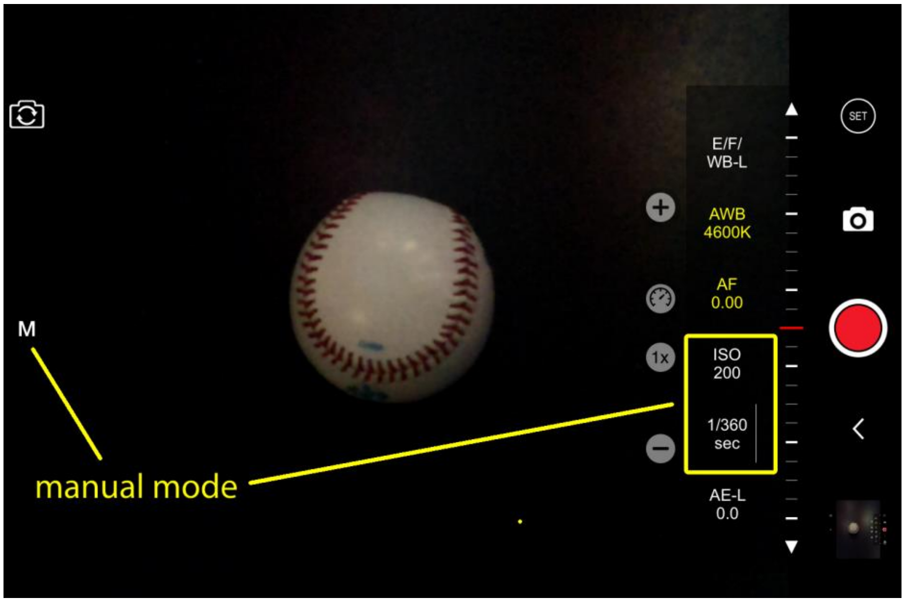
MAKING VIDEOS – USING PROCAM

Press the home button (the circle button on the front) in order to unlock the iPod. The application to use is called ProCam. Press the home button to reach the home screen where all the apps are displayed; the ProCam icon is at the bottom of the screen. After you have opened ProCam, you should see a "live" video image of whatever is in front of the camera. You can open the setup menu on the right side of the screen using the "<" button located below the record button. If the menu is already open, the button will appear as ">". When the menu is open, your screen should look like the screenshot below.



Make sure that the program is set to "video mode" (and not "camera mode") by selecting the "video" button. It should be highlighted yellow. The desired format is 60 fps or 120 fps - 720p; you can scroll vertically to find the appropriate format if it is not already selected.

Once in video mode, use the "M" button on the left side of the screen to manually set the exposure. The screenshot below shows the options for the manual settings. The settings you will adjust are the ISO and the shutter speed; click on the setting and adjust its value using the vertical scroll bar. The ISO determines how sensitive the camera is to light (so higher ISO means more noise), and the shutter speed controls how long the shutter is open. An ISO of 200 and shutter speed around 1/360 should work for most purposes, but you can adjust the values as necessary to get a clear image where the moving object is discrete in each frame. It is okay if the video appears dark because you can adjust the contrast later, after you import the video.



To import your videos to the computer, connect the iPod via the white cable. Click "allow" (or "trust") to the message asking for access to the device. A pop-up window will appear on the computer. Select "Open device to view files" and then browse to your videos. The path is Internal Storage > DCIM > 100APPLE.

Your videos need to be stored within the **C:\LabData** folder. Make sure that you are using **C:\LabData** otherwise an error will occur as you try to open the videos in MotionLab. Within the LabData folder, you can make a folder with your section number for you to store your own files. Your folder name cannot include spaces. Copy the files from the iPod into your folder and rename them so you can identify them. Once you have copied your videos to the computer, you should delete them off the iPod to avoid cluttering. If there are multiple videos on the iPod you can sort them by date created to find yours.

Vernier Photogates

The Vernier Photogate can be used as a traditional photogate for objects traveling between the arms of the gate, and also as a laser gate for objects passing outside of the arms of the gate. A mechanical shutter is used to block the internal gate, switching the device to laser gate mode. The laser gate mode requires a visible pen laser (not supplied). You can expect good results with a typical Class Illa type laser pointer, with a power of less than 5 mW.

The Vernier Photogate can be connected directly to an interface, or in a daisy-chain configuration. In the daisy-chain mode, up to four photogates can be connected to a single channel of the interface by connecting one photogate to another photogate, connecting the last one directly to the interface. This method is useful when timing between the gates is important.

The Photogate is designed for use with the Vernier LabPro .

The Vernier Photogate includes a cable for connection to one of the interfaces listed above. An accessory rod is included for attachment to a ring stand.

Internal Gate Mode and Laser Gate Mode

The Vernier Photogate operates in two modes. A shutter over the internal gate detector determines the operating mode. The shutter is on the inside of the thinner gate arm. Open the shutter to use the internal gate, and close the shutter to use the external laser gate. A red LED is on when the gate is blocked in either mode.

To use the internal gate mode, open the shutter and position the photogate. When the gate is blocked the red LED will be illuminated.

To use the external laser gate mode, close the shutter for the internal gate. The laser port is on the outside edge of the gate adjacent to the captive bolt. Align your laser so the beam enters the port and turns off the LED. Blocking the laser beam at any point in its path will then turn the LED back on. The path of the laser need not be a straight line. You may want to use mirrors to create a complex path that is crossed by the moving object multiple times. This mode is not currently used in these labs.

Laser Safety Note: Do not align the external laser gate by sighting by eye. Follow all safety precautions indicated by the laser manufacturer.

This sensor is equipped with circuitry that supports auto-ID. When used with LabPro, the data-collection software can identify the sensor and uses pre-defined parameters to configure an experiment appropriate to the recognized sensor. The auto-ID feature is not supported when using calculators, computer software older than Logger Pro 3, or LabPro OS older than 6.26.

Use of the Vernier Photogate with a Computer

This sensor can be used with either a Vernier LabPro or a Universal Lab Interface.

1. Connect the photogate or photogates to the appropriate port or ports on the interface.
    - LabPro-Connect the VPG-BTD photogate to DIG/SONIC I on the interface.
    - A second VPG-BTD photogate can be connected to DIG/SONIC 2

2. Start Logger Pro, the data-collection software, on the computer.

3. Open an experiment file in the Logger Pro of ULI Timer folder, and you are ready to collect data.

Here are some brief examples of things you can do with a photogate.

1. If you know the diameter of a ball rolling through a photogate, you can determine the speed of the ball from the ratio of the diameter to the time the gate is blocked by the ball. This requires only one gate, but the gate has to be positioned carefully so the light beam intersects the middle of the ball.

2. Using two photogates positioned at a known separation, you can determine the speed of an object from the time interval between the breaking of the first beam to the breaking of the second. This mode is known as pulse timing.

3. Set up a pendulum so that the bob swings through the photogate. The time interval from the first block to the third block yields the pendulum period.

4. Use the laser gate at floor level to measure the "hang time" of a jumper. The jumper's shoes will block the beam while on the floor. The time interval of interest is the time between blocks.

5. Use a super pulley to construct an Atwood's machine, consisting of two masses connected by a flexible string. The string passes over the pulley, causing it to rotate as the masses move. Use motion timing to measure the position, velocity, and acceleration as a function of time.

6. Measure the free fall acceleration of a picket fence using either the internal gate or the laser gate. Motion timing will give you the position, velocity, and acceleration as a function of time. Do the two modes give different results?

Vernier Motion Sensors

The motion sensors combine a source of sound and a detector and are very easy to use. The sensor emits a string a sound pulses and measures the time it takes for each pulse to return to the detector. Since the speed of sound in air is known, the position of the object can then be inferred. There are some tricks. There is a minimum distance (about 15 cm) below which the sensors do not work, and they can be fooled when the acceleration is too big or when there are spurious reflections from other objects in motion. Test your sensor by ensuring that it correctly records simple harmonic motion of a cart on two springs. **Make sure to set the collection rate fast enough so that you have approximately 30 points per cycle of motion.**

Appendix B: Statistics and Error Propagation

Some Basic Statistics (adapted from Keith Ruddick)

If a quantity x is measured N times, the individual measurements x_i form a distribution from which the experimenter wishes to extract a "best" value for x. Usually, the "best" value is the mean $\bar{x}$ of the distribution:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i . \tag{0-1}$$

The spread or width of the distribution is an indication of the precision of the measurement. As a measure of this width we consider the deviations of the individual measurements from the mean, i.e.

$$d_i = \bar{x} - x_i . \tag{0-2}$$

From our definition of $\bar{x}$, the sum of the deviations must equal zero. The mean deviation, defined in terms of the magnitudes of the deviations:

$$\bar{d} = \frac{1}{N}\sum_{i=1}^{N} \left| \bar{x} - x_i \right| \tag{0-3}$$

is often useful, but a more useful measure of the dispersion of a measurement is the *standard deviation* σ. The variance of the distribution σ^2 is the average of the squared deviation:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} \left(\bar{x} - x_i \right)^2 . \tag{0-4}$$

The standard deviation is then the square root of the variance, or the root mean square of the deviations.

To be more mathematically correct, we should recognize that the measured distribution is a "sample" distribution, which will be different every time we make a different series of measurements of x, rather than the "parent" distribution for which the mean is the exact, or "true" value. The best experimental estimate of the parent or "true" standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} \left(\bar{x} - x_i \right)^2} . \tag{0-5}$$

The quantity N -1 is the *number of degrees of freedom* in the problem, which equals the number of data points minus the number of fit parameters. If you ever have to worry about the difference between $\sqrt{N}$ and $\sqrt{N-1}$ in your data, then you should probably take more data!

Note that this σ is a measure of the uncertainty in a *single* measurement; it measures the width of the distribution of all measurements and is independent of the number of measurements! If one knows σ and the distribution of errors is what is called *normal*, then there is a roughly 2/3 probability that a single measurement will fall within one standard deviation of the mean. On the other hand, the more times we make a measurement, the better we expect to be able to determine the mean. It can be shown that the standard deviation of the mean is given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(\bar{x}-x_i)^2}{N(N-1)}}; \qquad \text{(0-6)}$$

i.e. in order to improve the accuracy by a factor 2, you must make 4 times as many measurements. This quantity is also called the *standard error* or the *error in the mean*.

We emphasize that these definitions are general, and apply to any distribution of measurements. There also exist specific theoretical distributions which can be used as models of actual data. These will be introduced in detail in Physics 2605.

Propagation of Errors (Adapted from Keith Ruddick)

Note: This section provides a “derivation” of the rules for error propagation that are stated in the body of these notes. The proof requires a little multivariable calculus. It is included here only for completeness.

Suppose we measure several quantities *a, b, c*,... each with its own standard deviation, $\sigma_a, \sigma_b, \sigma_c$ ... and then use these values to determine a quantity *y = f(a,b,c...).* What is the standard deviation of the quantity *y*?

We can differentiate the function to find how changes Δa, Δb, Δc... in each of the *a, b, c*... affect the value of *y*:

$$\Delta y = \Delta a\left(\frac{\partial y}{\partial a}\right)\bigg|_{bc...} + \Delta b\left(\frac{\partial y}{\partial b}\right)\bigg|_{ac...} + \dots \qquad \text{(0-7)}$$

This is, of course, really the first term in a Taylor expansion, and corresponds to assuming that the partial derivatives do not change over the ranges Δa, Δb,... If the function *y=ab*, or *y=a/b* for example, then

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \qquad\qquad \frac{\Delta y}{y} = \frac{\Delta a}{a} - \frac{\Delta b}{b}$$

respectively.

In general, we do not know the absolute errors Δa, Δb, Δc, in the measurements of *a,b,c,* but rather a quantity such as their standard deviations $\sigma_a, \sigma_b, \sigma_c$, (or the probable errors). However, from the above equations it is intuitively likely that the variances should add in the form:

$$\sigma_y^2 = \sigma_a^2\left(\frac{\partial y}{\partial a}\right)^2 + \sigma_b^2\left(\frac{\partial y}{\partial b}\right)^2 + \dots \tag{0-8}$$

That this is indeed true follows from:

$$\sigma_a^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\bar{a} - a_i\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\Delta a^2 \tag{0-9}$$

and then

$$\sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\Delta y^2 = \frac{1}{N}\sum\left[\Delta_a^2\left(\frac{\partial y}{\partial a}\right)^2 + \Delta_b^2\left(\frac{\partial y}{\partial b}\right)^2 + \dots + \Delta_a\Delta_b\left(\frac{\partial y}{\partial a}\right)\left(\frac{\partial y}{\partial b}\right) + \dots\right], \tag{0-10}$$

which leads to equation 9.8 if we neglect the cross-terms: This will be valid if the errors in *a* and *b* are uncorrelated.

Thus, for example, for a function *y=abc*, or *y=ab/c*, we obtain:

$$\frac{\sigma_y^2}{y^2} = \frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2} + \frac{\sigma_c^2}{c^2}, \tag{0-11}$$

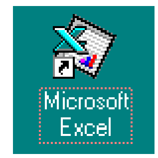
and for *y=a+b+c*:

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 \ . \tag{0-12}$$

Appendix C: Excel Spreadsheets

How to Get Started

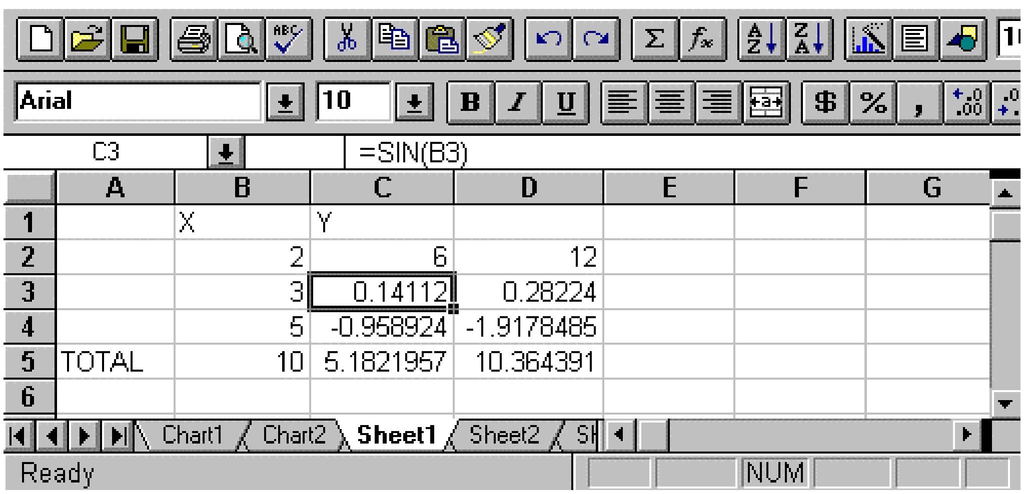
The spreadsheet currently used in the lab is Microsoft Excel. To start Excel either:



Select it from the START / PROGRAM / Microsoft Excel menu;

Or click double click on the Excel icon (shown on your right) on your desktop.

You should see the spreadsheet screen similar to the one shown below.



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1. Figure C.1.

You may enter a value in a cell by selecting the cell (click on it with the mouse pointer) and then start typing. To edit a cell, click on it, press F2 to activate it and then edit it.

Basic Concepts

Cell Addresses

The spreadsheet consists of cells each having its unique address consisting of the row and column position it occupies. Note that the row addresses use numbers while letters are used to specify the columns. For example, cell C4 refers to the cell in the spreadsheet in Figure D.1 that contains the value -0.95892.

Cell Contents: Constants and Equations

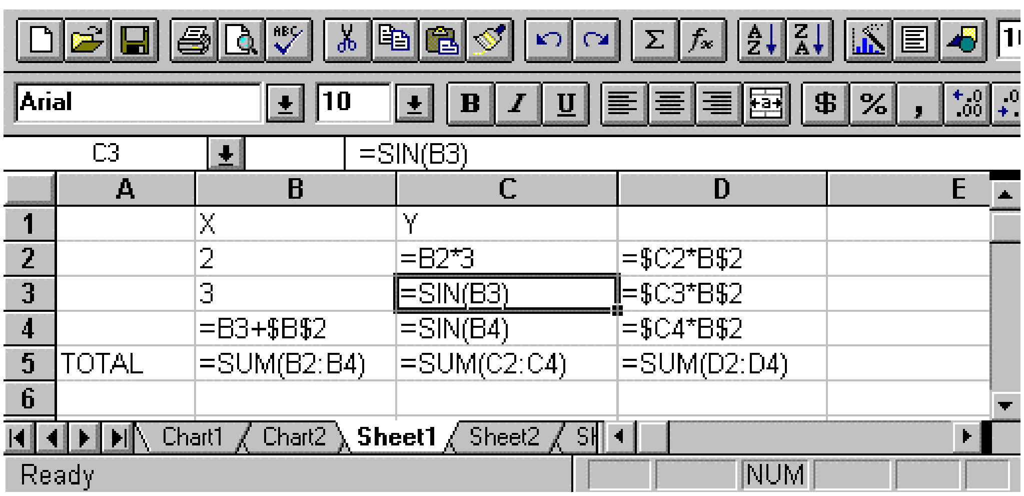
A cell may either contain a constant or an equation (sometimes also referred to as "formula"). The difference between the two is whether the object is preceded by an equal sign: anything preceded by an equal sign is considered an equation, anything else a constant.

Constants

Typical constants in a spreadsheet are text labels such as cell A5, "TOTAL." Any number entered that is not preceded by an equal sign is also considered a constant.

Equations (or Formulas)

Though the sample spreadsheet above contains numerous equations they can not be seen because equations are normally not displayed. Usually, the computer executes the equation and displays only the resulting value. Figure D.2 is a picture of the same spread sheet as shown in figure 1 but revealing the "real" contents of the cells:



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2. Figure 2

As you can see now, the majority of the cells actually contain equations. For example, all cells in column D contain equations while cells B1, B2 and B3 contain constants.

If you want to see the content of a particular cell just move the cursor to that cell and the "real" content of that cell is displayed in the window above the column headers. For example, as you can see from figure 2, cell C3 contains the equation: =SIN(B3). This information is normally displayed in the window directly above the column header (see figure D.1).

More Cell Addresses

The majority of the cells in a typical spreadsheet usually contain equations. Though such equations do not have to relate to values stored in other cells (for example: =2*7 is perfectly OK) most equations do. When an equation refers to a value in another cell, it can do so using an absolute, a relative or a mixed reference for the address. As long as you do not move or copy a cell referring to another cell, the way absolute and relative addresses function are identical.

Absolute Reference or Address

To make a reference absolute, it is preceded by a dollar sign. The address can be absolute with respect to the column (for example: =$C4), the row (for example, =C$4) or both, colon and row (for example =$C$4).

Relative Reference or Address

Any address not preceded by a dollar sign is relative. As previously mentioned, addresses may either be entirely relative, (for example: =C4) or they can contain both absolute and relative references.

Address Range

For certain mathematical functions, an entire range of cells can be specified. This is indicated by inserting a colon between the starting cell and ending cell, for example A5:C9 indicates all cells in the range from and including A5 to C9. As you can see, though most of the times a range only spans part of a column or row, it can span multiple columns and rows. Another example is shown in cell B5, figure2, which adds the contents of cells, B2, B3 and B4.

How to Enter the Address in an Equation

Two methods exist: First, you click and move to the appropriate cell and then type the equation with the necessary address. This is difficult because it's easy to mistype the address. A better method is, to move to the cell and enter the equation till you have to enter the address. At this point move the cursor to the actual cell that you are referring to and hit return; the computer will automatically insert the address into the equation. If you are referring to a range of cells you can select those directly by moving the cursor to the first cell, then while holding down the shift key, move to the last cell in the range.

Copying

As previously mentioned, the difference between absolute and relative addresses becomes apparent when copying the content of a cell to a different location.

Copying a Relative Address

Consider the following example: we want to multiply by 2 all constants in column A and store the results in column B. As you can see from the spreadsheet in table 1, we have already entered the corresponding equation in cell B5.

	A	B	C
5	5	=2*A5	
6	6		
7	7		

3. Table 1 (Note: these and the following tables display what is stored in the cells, not what the computer would display on the screen!)

Instead of entering subsequent equations in cells B6 and B7 we decide to copy the content of cell B5 into B6 i.e. we use Excel's copy/paste feature. When we copy the content of cell B5 into cell B6 the following happens:

	A	B	C
5	5	=2*A5	
6	6	=2*A6	
7	7		

4. Table 2

Though we expected cell B6 to be =2*A5, the computer automatically updated the referring address because it was a relative reference.

When copying cells with relative addresses to a different location one should read them like: this reference is in regard to the cell directly to the left; multiply its content by 2. In other words, when copying relative addresses the computer will do some thinking for us.

Copying an Absolute Address

Now we repeat the example shown in table 1 using an absolute address:

	A	B	C
5	5	=2*$A$5	
6	6		
7	7		

5. Table 3

When the content of cell B5 is copied into B6, the address remains unchanged.

	A	B	C
5	5	=2*$A$5	
6	6	=2*$A$5	
7	7		

6. Table 4

Warning About Copying Data to Different Places in a Spreadsheet!

Though these examples appear rather trivial they lead to a very important point. What is shown in the above tables is not what you would normally see on the computer screen because the computer displays only the results from your calculations and so you don't know if the results were obtained using constants, or formulas with either absolute or relative addresses. Therefore, if you copy a data to another place in the spreadsheet or to another spreadsheet using the copy/paste feature, the copied data may not be the same as the original. This is almost always the case if the data your copying has relative addresses.

You have two options to avoid this problem. Either make the addresses in the original data absolute; this can be annoying at times because you have to go back and change your equations. The second approach is to use the **copy/paste special** feature: **values**. If you use this feature, the program copies the equations, evaluates them and finally only stores the resulting values as constants in the new location.

Making Multiple Copies: Filling in

Filling in: Fill Down, Fill Right

So far our examples for copying cells have been more for illustration purposes and have not yet revealed the power of spreadsheets. Most of the times, we do not just want to copy one cell but a whole range of cells. For example, going back to table 1, if we could copy the first cell multiple times in one keystroke then we could multiply all values in the first column. The process of copying a cell multiple times is called filling and depending if it is done in a row or column, it's called **fill right** or **fill down**. The fill-process involves three steps:

1) enter the equation or constant in the first cell that you want to be duplicated

2) select an adjacent range of cells into which the equation or constant will be copied. (To highlight or select, move the cursor to the first cell, hold down the shift key while moving the cursor to the last cell in the range.)

3) select the "fill down or fill right" feature under the EDIT / FILL menu or as short-cut, press CTRL-R or CTRL-D, respectively.

Filling "in" Equations Containing Relative Addresses

Let's repeat the example in table 1. After having entered the equation in the first cell, (B5), we select or highlight cells B5:B7. After having selected fill down, the spread sheet looks like:

	A	**B**	**C**
5	5	=2*A5	
6	6	=2*A6	
7	7	=2*A7	

7. Table 5

Note how all the relative addresses are automatically updated

Filling "in" Equations Containing Absolute Addresses

If we use a fill-down on the example in table 3, all cells would be filled with the equation: = 2*$A$5.

Filling "in" Equations With Containing Relative and Absolute Addresses

For a more advanced example consider what happens if we decide to fill down a “mixed” referenced address. For example we want to multiply the values in column A by a value at the top of column B.

	A	B	C
5		2	
6	5	=$B$5*A6	
7	6		

8. Table 6

The equation in cell B6 of table 6 could be filled down and would produce the desired effect. What about if we want to multiply column A with various constants specified in row 5? Certainly we could enter in cell C6: =$C$5*A6 and a similar equation in column D. This is cumbersome. A better way is shown in cell B6, Table 7:

	A	B	C
5		2	
6	5	=B$5*$A6	
7	6		

9. Table 7

Now we can both fill the value in cell B6 down and right and with very little effort set up the following table:

	A	B	C
5		2	
6	5	=B$5*$A6	=C$5*$A6
7	6	=B$5*$A7	=C$5*$A7

10. Table 8

Advanced Topics

Equations

Excel has an extensive library of advanced mathematical and statistical functions. The ones most often used are:

=if(condition, A, B) if condition is true, A will be executed, else B

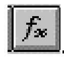
=sum(RANGE) totals all values for cells specified in RANGE

=max(RANGE) finds the largest value for the cells specified in RANGE

=min(RANGE) finds the smallest value for the cells specified in RANGE

=count(RANGE) returns a number indicating how many cells in RANGE contain values

=average(RANGE) calculates the average for the cells specified in RANGE

These are just some of the many functions that are provided. For a listing of the functions and their use click on the “function wizard” button, .

Graphs

To Create Graphs

It’s very easy to make graphs: select or highlight the data and then use the charting options to plot it. If you should change your data, Excel automatically updates your graphs also.

There is a “chart wizard” button  but it can be cumbersome. Instead, use the direct step by step approach to create charts:

1) Select the columns or ranges (hold down the CTRL key if selecting non adjacent columns) containing the data and any header that you want to plot. Do not select any data that you want to use for errors bars, i.e. your σ_i.

2) Select: **Insert, Chart**.

3) A menu will guide you through different charting options; select the ones you want. If you want to plot *x* vs. *y*, which is the case most of the time, then select **XY (Scatter)**. **Warning**: if you select **Line graph** instead, the x data will not be plotted but appears instead as evenly spaced labels along the x-axis! Be careful! Go to the next panel by clicking on NEXT.

4) The next window displays the data range selected for plotting and confirms that the data series are stored in columns rather than rows. Usually, you do not need make any changes in this window.

5) Go to the next panel, which will give you the option to enter chart titles and axis labels. Proceed to the next panel.

6) In this panel, you have to choose if you want the new graph to be inserted as a new sheet or as an object in the current spreadsheet. Always select *As New Sheet*. Do not *select As Object in…* because this generally leads to tiny charts that are difficult to see, annoying to resize, place and print. Select "FINISH."

7) You should see a new tab added to your worksheets, by default called CHART1, containing the new graph. To view your spreadsheet again click on the tab on the very bottom of the chart, most likely it will say *Sheet1*; to go back to the chart click on *Chart1*. You can change these labels by double clicking on them.

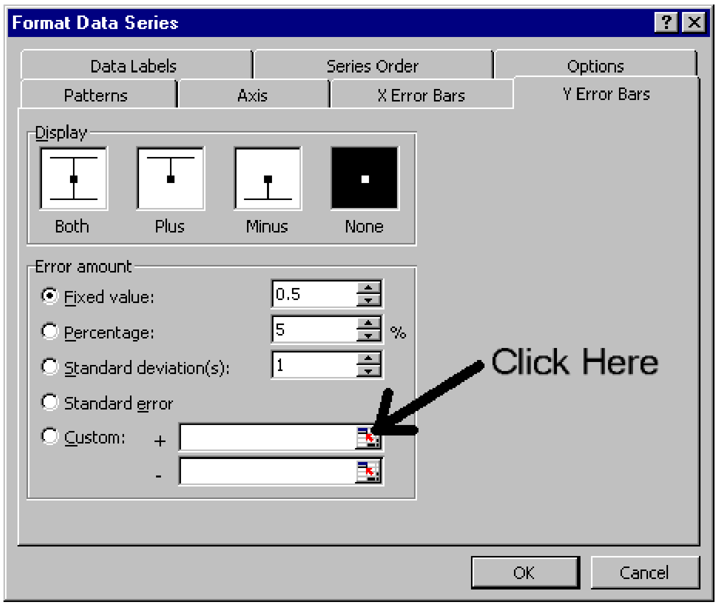
Adding Error Bars to Your Graph

To add error bars to your graphs, in your spreadsheet (ideally adjacent to the real data) add a column with the σ_I for each data point:

1) With the **right** mouse button click on any data point in your graph; once selected the color of the data points will change and a menu appears.

2) On that menu, select: **Format Data Series**.

3) Click on the *Y-Error Bars* tab. A window like the one shown below is displayed.



4) Click on the colored button that the arrow in the above picture points to. A new window called *Format Data Series Custom* + will open. See below.

5) Select the worksheet containing the data with the σ_I: Select it, by clicking on its tab, at the bottom of the chart.

6) On that sheet, highlight the σ_I. Note that the address range is automatically filled in *the Format Data Series-Custom+* window as shown above.

7) In the *Format Data Series-Custom+* window, click on the colored button to the right, directly below the "x" button. It will return you to Y-Error Bars window.

8) Click on the colored button in the *Y-Error Bars* window directly below the one you just clicked on, i.e. the one to the right in the *Custom - window*. A new window *called Format Data Series Custom* - will open. Repeat steps 4 through 7.

9) When done, click Enter in the *Y-Error Bars* window. Your graph should now display the appropriate error bars.

Note: make sure you match the y_i and the σ_i. If the addresses are not matched, strange stuff can appear on the graph.

Adding a Trendline to Your Data

Though Excel has some line fitting capabilities, you should use these with great care! They are only useful in that they give you a quick, first glance at the data. For more serious fitting, use your own program! Anyway, here it is:

1) Just like in step one for selecting the error bars, click on one of the data points in the chart.

2) Select in the *Add Trendline* menu and select the type you want, i.e. usually linear.

3) Click on the *Options* tab and select *Display Equation on Chart*.

4) Since the font size for the displayed equation is very small, click on the equation, click on *Font* and then Increase the *Size* to 20.

Basic Tips:

Do Not Start a Spreadsheet at Cell A1!

Always leave a few columns to the right and a few rows on top free. As you will work on your spreadsheet, they will come in very handy because you may want to add something. In case you do not need them, delete them when you are done.

Always List the Results in the Top Rows!

You will have to break with the habit of listing your results at the bottom of a column. Unless the columns are very short, always display your results on top. If you list them on the bottom, you or the user has to go searching for them; if they are on top, they are visible and easy to find.