WELCOME TO THE PHYSICS LABORATORY

Physics is our human attempt to explain the workings of the world. The success of that attempt is evident in the technology of our society. You have already developed your own physical theories to understand the world around you. Some of these ideas are consistent with accepted theories of physics while others are not. This laboratory manual is designed, in part, to help you recognize where your ideas agree with those accepted by physics and where they do not. It is also designed to help you become a better physics problem solver.

You are presented with contemporary physical theories in lecture and in your textbook. In the laboratory you can apply the theories to real-world problems by comparing your application of those theories with reality. You will clarify your ideas by: answering questions and solving problems *before* you come to the lab room, performing experiments and having discussions with classmates *in the lab room*, and occasionally by writing lab reports *after you leave*. Each laboratory has a set of problems that ask you to make decisions about the real world. As you work through the problems in this laboratory manual, remember: **the goal is <u>not</u> to make lots of measurements**. The goal is for you to examine your ideas about the real world.

The three components of the course - lecture, discussion section, and laboratory section - serve different purposes. The laboratory is where physics ideas, often expressed in mathematics, meet the real world. Because different lab sections meet on different days of the week, you may deal with concepts in the lab before meeting them in lecture. In that case, the lab will serve as an introduction to the lecture. In other cases the lecture will be a good introduction to the lab.

The amount you learn in lab will depend on the time you spend in preparation before coming to lab.

Before coming to lab each week you must read the appropriate sections of your text, read the assigned problems to develop a fairly clear idea of what will be happening, and complete the prediction and method questions for the assigned problems.

Often, your lab group will be asked to present its predictions and data to other groups so that everyone can participate in understanding how specific measurements illustrate general concepts of physics. You should always be prepared to explain your ideas or actions to others in the class. To show your instructor that you have made the appropriate connections between your measurements and the basic physical concepts, you will be asked to write a laboratory report. Guidelines for preparing lab reports can be found in the lab manual appendices and in this introduction. An example of a good lab report is also available in the appendix. Please do not hesitate to discuss any difficulties with your fellow students or the lab instructor.

Relax. Explore. Make mistakes. Ask lots of questions, and have fun.

WHAT TO DO TO BE SUCCESSFUL IN THIS LAB:

Safety comes first in any laboratory. If in doubt about any procedure, or if it seems unsafe to you, STOP. Ask your lab instructor for help. YOU MUST READ ALL WARNINGS FOUND IN THE LAB MATERIALS.

A. **What to bring to each laboratory session:**

1. Bring a graph-ruled lab journal, to all lab sessions. Your journal is your "extended memory" and should contain everything you do in the lab and all of your thoughts as you are going along. Your lab journal is a legal document; you should **never** tear pages from it. Your lab journal **must** be bound (as *University of Minnesota 2077-S*) and must **not** allow pages to be easily removed (as spiral bound notebooks).

2. Bring a "scientific" calculator.

3. Bring this lab manual.

B. **Prepare for each laboratory session:**

Each laboratory consists of a series of related problems that can be solved using the same basic concepts and principles. Often all lab groups will work on the same problem, at times groups will work on different problems and share results.

1. Before beginning a new lab, carefully read the Introduction, Objectives and Preparation sections. Read sections of the text specified in the *Preparation* section.

2. Each lab contains several different experimental problems. Before you come to a lab, complete the assigned *Prediction* and *Method Questions*. The Method Questions help you build a prediction for the given problem. It is usually helpful to answer the Method Questions before making the prediction. **These individual predictions will be checked (graded) by your lab instructor *immediately* at the beginning of each lab session.**
This preparation is crucial if you are going to get anything out of your laboratory work. There are at least two other reasons for preparing:

   a) There is nothing duller or more exasperating than plugging mindlessly into a procedure you do not understand.

   b) The laboratory work is a **group** activity where every individual contributes to the thinking process and activities of the group. Other members of your group will be unhappy if they must consistently carry the burden of someone who isn't doing his/her share.

C. **Laboratory Reports**

About once every two weeks you will be assigned to write up one of the experimental problems. Your report must present a clear and accurate account of what you and your group members did, the results you obtained, and what the results mean. A report must not be copied or fabricated. (That would be scientific fraud.) Copied or fabricated lab reports will be treated in the same manner as cheating on a test, and will result in **a failing grade for the course and possible expulsion from the University**. Your lab report should describe your predictions, your experiences, your observations, your measurements, and your conclusions. A description of the lab report format is discussed at the end of this introduction. **Consult the course syllabus for info on grading.**

D. **Attendance**
Attendance is required at all labs **without exception**. If something disastrous keeps you from your scheduled lab, contact your lab instructor **immediately**. The instructor will arrange for you to attend another lab section that same week. **There are no make-up labs in this course.**

E. **Grades**

Satisfactory completion of the lab is required as part of your course grade. *Consult the course syllabus for further info.*

F. **The laboratory class forms a local scientific community. There are certain basic rules for conducting business in this laboratory.**

   1. *In all discussions and group work, full respect for all people is required.* All disagreements about work must stand or fall on reasoned arguments about physics principles, the data, or acceptable procedures, never on the basis of power, loudness, or intimidation.

   2. *It is OK to make a <u>reasoned</u> mistake. It is in fact, one of the most efficient ways to learn.*

This is an academic laboratory in which to learn things, to test your ideas and predictions by collecting data, and to determine which conclusions from the data are acceptable and reasonable to other people and which are not.

What do we mean by a "reasoned mistake"? We mean that after careful consideration and after a substantial amount of thinking has gone into your ideas you simply give your best prediction or explanation as you see it. Of course, there is always the possibility that your idea does not accord with the accepted ideas. Then someone says, "No, that's not the way I see it and here's why." Eventually persuasive evidence will be offered for one viewpoint or the other.

"Speaking out" your explanations, in writing or vocally, is one of the best ways to learn.

3. *It is perfectly okay to share information and ideas with colleagues. Many kinds of help are okay. Since members of this class have highly diverse backgrounds, you are encouraged to help each other and learn from each other.*

   However, it is never okay to copy the work of others.

   Helping others is encouraged because it is one of the best ways for you to learn, but copying is inappropriate and unacceptable. Write out your own calculations and answer questions in your own words. It is okay to make a reasoned mistake; it is wrong to copy.

   No credit will be given for copied work. It is also subject to University rules about plagiarism and cheating, and may result in dismissal from the course and the University. See the University course catalog for further information.

4. *Hundreds of other students use this laboratory each week. Another class probably follows directly after you are done. Respect for the environment and the equipment in the lab is an important part of making this experience a pleasant one.*

   The lab tables and floors should be clean of any paper or "garbage." Please clean up your area before you leave the lab. The equipment must be either returned to the lab instructor or left neatly at your station, depending on the circumstances.

   A note about Laboratory equipment:
   At times equipment in the lab may break or may be found to be broken. If this happens you should inform your TA.

Remember, safety comes first in any laboratory. If equipment appears to be broken in such a way as to cause a danger, do not use the equipment and inform your TA immediately!

In summary, the key to making any community work is **RESPECT**.

Respect yourself and your ideas by behaving in a professional manner at all times.
Respect your colleagues (fellow students) and their ideas.
Respect your lab instructor and his/her effort to provide you with an environment in which you can learn.
Respect the laboratory equipment so that others coming after you in the laboratory will have an appropriate environment in which to learn.

1301 LAB 1 PROBLEM 1a: INTRODUCTION TO MEASUREMENT AND UNCERTAINTY

Welcome! This lab exercise and the one that follows are meant to introduce you to measurement procedures, uncertainties in measurement, and the computer software you will be using throughout this course. It will be worth your time to read through this entire lab and the suggested reading material.

Since this physics laboratory design may be new to you, this first problem contains extra info and an explanation of the various parts of the instructions. The extra info and explanation of the instructions are italicized.

*These lab instructions may be unlike any you have seen before. You will not find worksheets or step-by-step instructions. Instead, each lab consists of a problem that you solve before coming to the laboratory by making an organized set of decisions (a problem-solving strategy) based on your initial knowledge. The **Prediction** and **Warm Up** questions are designed to help you examine your thoughts about physics. These labs are your opportunity to compare your ideas about what "should" happen with what really happens. The labs will have little value in helping you learn physics unless you take time to predict what will happen before you do something.*

*While in lab, take your time and try to answer all the questions in this lab manual. In particular, answering each of the **Exploration** questions can save you time and frustration later by helping you understand the behavior and limitations of your equipment before you make measurements. Make sure to complete the lab problem, including all **Analysis** and **Conclusions**, before moving on to the next problem.*

The first paragraphs of each lab problem describe a real-world situation. Before coming to lab, you will solve a physics problem to predict something about that situation. The measurements and analysis you perform in lab will allow you to test your prediction against the behavior of the real world.

Like all future lab problems you will see in this class, an attempt is made in this problem to build a context for the experimentation you will undertake.

Let's pretend that you perform quality control for BuggyMagic, the leading manufacturer of constant velocity toy buggies. Recently the company started manufacturing buggies at a second factory. Coincidently, the customer service department started fielding complaints around the same time that the new factory came on line. The customer service department couldn't elaborate on many specifics, only that many customers felt the buggies offered from the second factory were "inconsistent" and had "less magic" than the original buggies. The customer service manager at your company decided that you needed to check out the buggies ASAP.

You fly to the new factory with all sorts of sophisticated measurement and analysis equipment and a brand new buggy from the original factory, but sadly, when you arrive, your luggage has been lost by the airline. All you have is a smartphone (stopwatch) and a small flag affixed to a rather short wood dowel handed out by a local welcoming committee. You'll need to do your best at assessing the buggies with what you have.

EQUIPMENT

The "Equipment" section contains a brief description of the apparatus you can use to test your prediction. Working through the exploration section will familiarize you with the apparatus you will be using.

You have a stopwatch, wood dowel of fixed length and a toy buggy.

WARM UP

These are your first lab "Warm Ups", which are to be done before the lab meets, written in your lab notebook, and turned in to your TA. You may want to refer back to the appendices during the lab.

*Typically, Warm Up questions are a series of questions intended to help you solve the problem stated in the opening paragraph. They may help you make the prediction, help you plan how to analyze data, or help you think through the consequences of a prediction that is an educated guess. **Warm Up questions should be answered and written in your lab journal before you come to lab.***

1) Read the appendix **Significant Figures**. Do the exercises at the end and write the results in your lab notebook under a section called "Warm Ups".

2) Read the appendix **Accuracy, Precision and Uncertainty** and write the answers to the exercises in your lab notebook.

3) Read the appendix **Review of Graphs**.

4) Work out a plan as specified in the Prediction.

You should also start reading the **Video and Analysis** appendix. You will be using the software and video camera extensively throughout the semester and for part B of this lab.

PREDICTION

One purpose of lab is for you to test your conceptions about the physical world. Before you collect and analyze data, you will make a "Prediction" about what you expect to happen in your experiment. You should spend a few minutes at the start of each lab to discuss and compare predictions with your group members. It is not necessary that your predictions are completely correct, but you should understand the rationale behind your prediction.

With the limited tools on hand, what sorts of measurements can you make? Devise a plan to measure the length and velocity of the buggies measured in dowels and dowels/sec in order to verify the 'sameness' of the buggies and later convert to metric units.

> ### EXPLORATION

Each lab will have an "Exploration" section before the "Measurement" section. This is where you can run informal trials to develop your procedure and see how the equipment responds to the activity. The data from these exploratory trials do not need to be included in your final data set.

Can you determine the length of the dowel without using a meterstick or ruler? Can you determine if something is 0.5 dowel, or 0.75 dowel long? How accurate can you be with your measurement? If you measure the buggy with the dowel, what qualities of your dowel and buggy can help you estimate the uncertainty in your measurement?

How accurate is your stopwatch? How accurate is your reaction time? If you measure the time it takes for the buggy to travel 1 dowel length, how accurate can you be?

> ### MEASUREMENT

After you have explored your equipment, you are ready to begin the "Measurement" process. You should record a careful description of your procedure, with enough relevant detail that your results could be replicated by another person. Remember that along with your data points, you should also consider the uncertainties associated with your measurements.

1) Length

Use the dowel to measure the length of the buggy. How long is it? Have each person in the group measure the buggy using the dowel in successive turn. This should give 3-4 measurements per group for the length. Individually record measurements and then combine them after everyone is done. Record your procedure and associated measurements in your lab notebook.

What is your estimated uncertainty in your measurements?

Using the instructions in the appendix **Accuracy, Precision and Uncertainty**, calculate the mean and average deviation of the combined data set for the length of the buggy. Compare your estimated uncertainty to your average deviation. Do they agree within significant figures?

Note on Assumptions: *When physicists are trying to solve a problem, they often make assumptions about the situation. Depending on how accurate the results need to be (i.e. how small the uncertainty), making estimates saves a lot of time if it turns out to be 'good enough' for the task. You will see phrases such as 'friction is negligible', 'ignoring air resistance', or 'assuming that earth is a sphere' in your textbook or in class. The assumptions made must always be stated since it gives the audience important information about the precision of the results.*

2) Time

Each member of your group should use a stopwatch simultaneously to measure the time it takes the buggy to travel 1 dowel length. Do the times agree? Did you start the buggy, then set it down? Or, did you start the buggy already sitting on the table? Does it matter? Which point on the buggy are you using for your measurement? This kind of question might seem trivial, but it is an example of the amount to detail you should be recording in your notebook.

Record your procedure and associated measurements in your lab notebook. Calculate the mean and average deviation of the times for the buggy to travel one dowel length.

Note on rejecting data: *One must be very careful about rejecting data. In general, you should keep all of your data even if it does not seem to match with what you are expecting. For this class, the only reason you might 'throw away' data is if you can say EXACTLY what was wrong with it. For example, if you just did a run with the cart and someone forgot to say "Go!" at the right time, then you know that time measurement is wrong. You may not, however, ignore the data points that just seem too big or too small. Hopefully you see by now that ALL MEASUREMENTS HAVE UNCERTAINTY. This is nothing to apologize for as it is expected for any measurement.*

3) Constant Velocity

Now, have each member measure and record the time it takes the buggy to travel two, then three, and finally four dowel lengths.

Did the measurements become more or less consistent as each person did more trials? Did you "formalize" the procedure after the first couple trials (e.g., agree upon the start procedure, decide what viewing angle to measure from)? Could you make the average deviation smaller with this equipment or are you close to the limit of the accuracy that can be expected? You should redo any trials as needed.

Take at least 4 time measurements **for each** of your 4 distances. You will want to format the data in a 4x4 table. Find the average time and the average deviation of times for each distance.

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ANALYSIS

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The quantities that can be derived from your measurements are typically more interesting than the direct measurements themselves. In the "Analysis" section, you will process your data to see these results. It is useful to complete your data processing after you take a set of data, so that you can see if something is going wrong and fix it before continuing. If you see that you need to modify your measurement plan and redo your measurements, make sure to record the changes to your plan in your

notebook.

Use an entire page of your lab notebook to make a graph with time along the <u>vertical</u> axis and (the more accurate) distance in dowels along the <u>horizontal</u> axis. (This does not make your graph look like those in the **Review of Graphs** appendix; usually we put time along the horizontal axis.) Plot your average time for each distance with the 'error bars' on the graph. The error bars are the range of the average deviation of the measurement.

Example: If your time is 3.40 +/- 0.4 seconds, then you should put a dot at 3.40, a vertical line through the dot that extends from 3.00 to 3.80, and 'cross' the line at the top and bottom.

Now draw your best fit line through the four data points, as directed in the **Review of Graphs** appendix. You are now able to find the average velocity from the best fit line.

To get the uncertainty of the measured velocity, use the equation for propagation of errors for multiplication/division as shown in the appendix material. You can propagate the error for each position/time pair and then average the values to find the average uncertainty.

$$R = \frac{X \cdot Y}{Z} \quad \rightarrow \quad \delta R = |R| \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2 + \left(\frac{\delta Z}{Z}\right)^2}$$

What is the average velocity in dowels/sec plus or minus the average uncertainty? Does this value make sense?

<div style="text-align:center; border:1px solid black; padding:4px;">**CONCLUSION**</div>

After you have analyzed your data, you can come to a "Conclusion" about your experimental problem. State your results in the most general terms supported by your data and analysis. You should also compare your results to your prediction.

Which of the three measurements (length, time, or velocity) gives the most uncertainty of measurement? Would you consider this uncertainty significant, moderate, or insignificant? Why?

If you compare your data with a different lab group's data, would you consider the buggies to be manufactured to be the same? Why, or why not?

Using a meterstick, can you convert your earlier measurements? How accurately can you measure the dowel? Does additional error get added (propagated) into your earlier measurements if you convert the units? If so, is it substantial?

Give a value for the velocity of the buggy, with uncertainty, in meters/sec.

1301 LAB 1 PROBLEM 1b: MEASUREMENT AND UNCERTAINTY USING VIDEO ANALYSIS

Welcome! This lab exercise is meant to introduce you to measurement procedures and uncertainties in measurement while using the computer software you will be using throughout the course.

Back in your office at BuggyMagic headquarters, you realize that you could have used your smartphone and taken a simple video of the buggy's motion and likely achieved far better results. You would like to see if video is an effective analysis technique, one that might be deployed as an automated quality control to ensure that you always produce quality buggies.

You know that your smartphone is equipped with a decent camera. You Google the specifications of your phone's camera and quickly realize that video analysis will be a more complex system with the potential to introduce error in different ways compared to hand measurements.

You decide to focus on one of the major sources of systematic errors that can be modeled in a simple way, before comparing random measurement uncertainty between the two methods. Luckily, you now have a ruler and can measure the length of the dowel.

EQUIPMENT

You have a video camera and computer with MotionLab analysis software, a wood dowel and meterstick, and a toy buggy.

The video cameras used in lab can be configured to record 30-120 frames per second with a resolution of 1280x720. The resolution implies that the camera has 1280 pixels that can be selected in one axis, 720 in the other. The camera clock is accurate. Each frame is recorded precisely $1/30^{th}$, $1/60^{th}$, or $1/120^{th}$ of a second after the next. The exposure time of each frame is a fraction of that time; the lower the exposure setting, the more discreet (and darker) the image will be. If you record motion that is fast without adjusting the exposure time, the motion will be blurred.

You ALWAYS need to have a calibration object in every video. Any object that has a known length works as a calibration object. When you analyze your video, you select the ends of the calibration object using your mouse and enter its length. This procedure calibrates the software to correspond pixel measurements to locations and lengths **in the plane of motion**.

The primary way error is introduced into a measurement while using videos is through a poor calibration procedure. If you are interested in measurements three feet away from the camera and you calibrate it using something five feet away from the camera, your results will be different than expected by an unknown factor. Similarly, if you are interested in the motion of a cart, it is important that it moves roughly the same distance in front of the camera the whole time. Your video device should be set up as parallel as possible to the plane of motion.


WARM UP

Using simple geometry, figure out how important putting the calibration object in the plane of motion is.

1.   Draw a right triangle with vertices A, B, C where the tip, A, represents the point-like camera. Imagine that the line passing through BC is the plane of motion, which means the buggy will move on this line during the experiment.
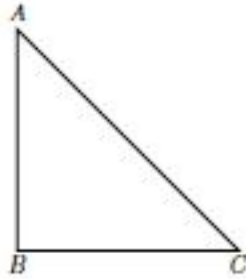
2.   Imagine putting a calibration object (dowel) with length, $l_c = |DE|$, where DE is parallel to BC and the points D and E are on the edges AB and AC, respectively.
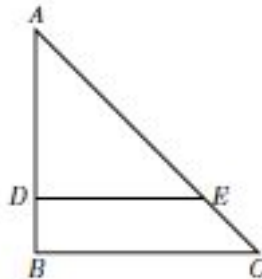
3.   Note that because we did not put the dowel in the plane of motion, the camera mistakenly thinks that the length, $l_o = |BC|$, equals the length of the calibration object l_c. The percentage error in calibration is then the difference of the two divided by the actual distance we want to measure. If we do not know the lengths, l_c and l_o, what other parameters do we need to be able to calculate the percentage error? What happens to this percentage error if you put your point-like camera farther from the dowel?

PREDICTION

In a video where the dowel is not positioned in the plane of motion, calculate the following percentage error

$$Error \% = \frac{l_o - l_c}{l_o}$$

using the parameters you found in the Warm Up section.

On the same graph, sketch a position versus time (x vs. t) plot you expect for the constant velocity motion of the buggy measured with the dowel placed in front of the buggy and with

the dowel placed in the plane of motion. Does this way of miscalibration make it look like the buggy is moving faster or slower than it really is?

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EXPLORATION

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Open the ProCam application on the video device.

Position the buggy and dowel in front of the camera. Experiment by placing the dowel in different orientations and at different distances in front of and behind the buggy. Move the camera different distances from the camera side of the buggy as well. Does it matter how far the camera is away from the setup, or how big the field of view of the camera is? Is it better to have the camera near the setup for a close-up view, or further back?

Practice recording videos of the motion across the lab table. Does the buggy stay approximately the same distance from the camera the entire time?

Make sure everyone in your group gets the chance to operate the camera.

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MEASUREMENT

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Record two videos of the buggy's motion across the table. In one, place the dowel in the same plane of motion as the buggy. Record a second video with the dowel 10cm in front of the buggy, while keeping the camera and the buggy's plane of motion the same as before. Record the distance between the camera and the dowel as well.

When you are satisfied with your videos, connect the video device to the computer and import the videos of the buggy's motion to the computer. Open MotionLab to begin your video analysis. You should take turns analyzing the videos.

Although the directions to analyze a video are given in the instructions box in the upper left corner within MotionLab, the following is a short summary that will be useful to do the exploration for this and any other lab video (You should also read the **Video and Analysis** appendix).

1. Once MotionLab is started you will be prompted to open a movie file.

2. With the video loaded, a calibration screen automatically opens. **Advance the video with the "Fwd >" button in the Video Controls to the frame where the first data point will be taken. This step is very important because it sets up the origin of your time axis (t=0).**

3. To tell the analysis program the real size of the video images, select the calibration object in the plane of motion that you can measure. Drag the red cursor, located in the center of the video display, to one end of the calibration object. You can use the zoom slider in the Video Controls box to zoom in on the calibration object for more accuracy in locating the ends. Make sure to use the same part of the cursor for each point selected; either the central circle or the tip of one of the cross-hairs will work the same if used consistently. Click the "Accept >" button

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when the red cursor is in place. Move the red cursor to the other end and select "Accept >". Enter the length of the object in the "Length" box and specify the "Units" then select "Accept >". You do not need to rotate the reference frame for this lab. Select the "Quit Calibration" button to complete the calibration sequence.

4. Based on the data you took by hand earlier; enter your prediction equations for how you expect the position to behave. For the x-position graph, use the function that matches the kinematic equation relating position, velocity, and time (*Remember! z is time!). Fill in the function with your previous measurement values. Make sure the units all agree! Once your x-position prediction is ready, select "Accept >" and repeat the procedure for the y-position. (Do you expect the cart to move in the y-direction?)

5. Once you have made predictions for the x- and y-positions, a data acquisition screen will automatically open. Select a specific point on the cart. Drag the red cursor over this point and click the "Add Point" button and you will see the data on the appropriate graph on your computer screen. The video will automatically advance one frame. Again, drag the red cursor over the same point selected on the object and accept the data point. Experiment with advancing the video several frames and taking a data point. Should that change your results? Decide how many data points are necessary for reliable results.

6. Once you have added enough points, click the "Quit Data Acq" button and fit your data. Sometimes you will not see your data because the scale of the graph is not in the right place. If you click the buttons in the center of the screen called "Autorange x", "Autorange y", etc. the graph will automatically <u>scale to the data points</u>. This may not include the prediction equation in the newly scaled window. You may need to further re-scale the axes by highlighting the highest or lowest value on the graph and typing in values to expand the ranges. Decide which equation and constants are the best approximations for <u>your data</u> and accept your "x-fit" and "y-fit".

7. The program will ask you to enter your prediction for velocity in the x- and y-directions. Choose the function that matches the kinematic equation relating velocity and time. Fill in your prediction values (NOT the best fit value from the position graph). Accept your v_x and v_y predictions, and you will see the data on the last two graphs.

8. Fit your data for these velocities in the same way that you did for position. Accept your fit and click the "Print Results" button to view a PDF document of your graphs that can be e-mailed to you and your group members. You must save the file on the computer in order to send it.

9. Export your data to an Excel file by clicking the "Save" button. Save it in the LabData folder, using a unique name you will remember.

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ANALYSIS

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When you have finished with your video analysis, rewrite the fit equations for each case. Match the *dummy letters* with the appropriate *kinematic quantities*. If you have constant values, assign them the correct units.

Compare the average velocities determined from each of your videos. Did having the calibration object in front of the plane of motion increase or decrease your calculated velocity? How much effect did the incorrect calibrations have on the results? How do the results fit with your prediction?

If you can measure within a pixel or two, what does this say about the random measurement uncertainty in your distance? If the time interval between each video frame is also only a fraction of a second, how would these affect the uncertainty in the velocity value you found from MotionLab? Qualitatively, how would the random measurement uncertainty for your velocity in Problem 1b compare with your random measurement uncertainty from Problem 1a?

Compare the average velocity of the buggy from your earlier hand measurements with that determined using computer analysis for the correctly calibrated video. Do the computer measurements fall within the expected uncertainty that you determined from Problem 1a?

Why is there one fewer data point in a *velocity vs. time* graph than in the corresponding *position vs. time* graph?

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CONCLUSION

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What is the average velocity in meters/sec, as determined from the video analysis? Does this value make sense? Do your graphs match what you expected for constant velocity motion? Qualitatively, how does the uncertainty determined from the hand measurement of Problem 1a compare to the uncertainty determined from the video analysis in 1b?

Which data points have the smallest deviation from the fit line? Which have the largest? Do measurements near the edges of the video give the same velocity as you find for the center of the image (within the uncertainties of your measurements in Problem 1a)? If not, you may be seeing the effects of camera lens distortion in your data. Does this affect what you will do for future measurements?

If you can rule out data points near the edges due to lens distortion, will that notably change the average velocity? In that case, how does the velocity of the buggy then compare with your results from Problem 1a?

How good was the simple model you used to determine the effect of the wrong calibration, compared to your results from the experiment? When you are working with video analysis, what is the dominant source of error, and how will you account for this?

1301 LAB 1 PROBLEM 2: CONSTANT VELOCITY

You have an internship managing a network of closed-circuit "Freeway cameras" for MnDOT Metro Traffic Engineering. Your boss wants to use images from those cameras to determine velocities of cars, particularly during unusual circumstances such as traffic accidents. Your boss knows that you have taken physics and asks you to prepare a presentation. During the presentation, you must demonstrate possibilities for determining a car's average velocity from graphs of its position vs. time, instantaneous velocity vs. time, and instantaneous acceleration vs. time. You decide to model the situation with a small digital camera and a toy car that moves at a constant velocity.

Read: Mazur Chapter 2.

EQUIPMENT

You have a motorized toy car, which moves with a constant velocity on an aluminum track. You also have a stopwatch, a meter stick, a video camera and a computer with video analysis applications written in LabVIEW™ to help you analyze the motion.

WARM UP

To find schemes for determining a car's velocity, you need to think about representing its motion. The following questions should help.

1. How would you expect an *instantaneous velocity vs. time graph* to look for an object with constant velocity? Make a rough sketch and explain your reasoning. Assign appropriate labels and units to your axes. Write an equation that describes this graph. What is the meaning of each quantity in your equation? In terms of the quantities in your equation, what is the velocity?

2. How would you expect an *instantaneous acceleration vs. time graph* to look for an object moving with a constant velocity? Make a rough sketch and explain your reasoning. Remember axis labels and units. Write down an equation that describes this graph. In this case, what can you say about the velocity?

3. How would you expect a *position vs. time graph* to look for an object moving with constant velocity? Make a rough sketch and explain your reasoning. What is the relationship between this graph and the instantaneous velocity versus time graph? Write down an equation that describes this graph. What is the meaning of each quantity in your equation? In terms of the quantities in your equation, what is the velocity?

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PREDICTION

</div>

Sketch graphs of position vs. time, instantaneous velocity vs. time, and instantaneous acceleration vs. time for the toy car. How could you determine the speed of the car from each graph?

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EXPLORATION

</div>

Place one of the metal tracks on your lab bench and place the toy car on the track. Turn on the car and observe its motion. Qualitatively determine if it actually moves with a constant velocity. Use the meter stick and stopwatch to determine the speed of the car. Estimate the uncertainty in your speed measurement.

Move the camera closer to the car. How does this affect the video image? Try moving it farther away. Raise the height of the camera tripod. How does this affect the image? Decide where you want to place the camera to get the most useful image.

Practice taking videos of the toy car. Write down the best situation for taking a video in your journal for future reference. When you have a good movie, transfer it to the computer.

Open MotionLab to analyze your movie.

Although the directions to analyze a video are given during the procedure in a box with the title "INSTRUCTIONS," the following is a short summary of them that will be useful to do the exploration for this and any other lab.

1. Open the video that you are interested in by clicking the "AVI" button.

2. Advance the video with the "Fwd >" button to the frame where the first data point will be taken, then select "Accept" from the main controls. This step is very important because it sets up the origin of your time axis (t=0).

3. To tell the analysis program the real size of the video images, select some object in the plane of motion that you can measure. Drag the red cursor, located in the center of the video display, to one end of the calibration object. Click "Accept" button when the red cursor is in place. Move the red cursor to the other end and select "Accept". Enter the length of the object in the "Length" box and specify the "Units". Select the "Accept" button again, then select the "Quit Calibration" button to exit the calibration routine.

4. Enter your prediction equations of how you expect the position to behave. Notice that the symbols used by the equations in the program are *dummy letters*, which means that you have to identify those with the quantities involved in your prediction. In order to do the best guess you will need to take into account the scale and the values from your practice trials using the stopwatch and the meterstick. Once your x-position prediction is ready, select "Accept" in the main controls. Repeat the previous procedure for the y-position.

5. Once both your x and y position predictions are entered, the data collection routine will begin. Select a specific point on the object whose motion you are analyzing. Drag the red cursor over this point and click the "Add Point" button from the data acquisition controls and you will see the data on the appropriate graph on your computer screen, after this the video will advance one frame. Again, drag the green cursor over the selected spot on the object and select "Add Point." Keep doing this until you have enough data, then select "Quit Data Acq".

6. Decide which equation and constants are the best approximations for your data, and then select "Accept" from the main controls.

7. At this level the program will ask you to enter your predictions for velocity in x- and y-directions. Choose the appropriate equations and give your best approximations for the constants. Once you have accepted your v_x - and v_y –predictions, you will see the data on the last two graphs.

8. Fit your data for these velocities in the same way that you did for position. Accept your fit and click the "Print" button to get a hard copy of your graphs.

Now you are ready to answer some questions that will be helpful for planning your measurements.

What would happen if you calibrate with an object that is not on the plane of the motion? What would happen if you use different points on your car to get your data points?

MEASUREMENT

1. Record the time the car takes to travel a known distance. Estimate the uncertainty in time and distance measurements.

2. Take a good video of the car's motion. Analyze the video with MotionLab to predict and fit functions for *position vs. time* and *velocity vs. time*.

If possible, every member of your group should analyze a video. Record your procedures, measurements, prediction equations, and fit equations in a neat and organized manner so that you can understand them a month from now. Some future lab problems will require results from earlier ones.

ANALYSIS

Calculate the average speed of the car from your stopwatch and meter stick measurements. Determine if the speed is constant within your measurement uncertainties.

As you analyze data from a video, be sure to *write down* each of the prediction and fit equations for position and velocity.

When you have finished making a fit equation for each graph, rewrite the equations in a table but now matching the *dummy letters* with the appropriate *kinetic quantities*. If you have constant values, assign them the correct units.

<div align="center">

CONCLUSIONS

</div>

Compare the car's speed measured with video analysis to the measurement using a stopwatch. Did your measurements and graphs agree with your answers to the Warm-up Questions? If not, why? Do your graphs match what you expected for constant velocity motion? What are the limitations on the accuracy of your measurements and analysis?

1301 LAB 1 PROBLEM 3: MOTION DOWN AN INCLINE

You have a job working with a team studying accidents for the state safety board. To investigate one accident, your team needs to determine the acceleration of a car rolling down a hill without any brakes. Everyone agrees that the car's velocity increases as it rolls down the hill but your team's supervisor believes that the car's acceleration also increases uniformly as it rolls down the hill. To test your supervisor's idea, you determine the acceleration of a cart as it moves down an inclined track in the laboratory.

Read: Mazur Chapter 3.

EQUIPMENT

You have a stopwatch, meterstick, endstop, wood block, video camera and a computer with video analysis software. You will also have a cart to roll down an inclined track.

WARM UP

The following questions should help you to explore three different scenarios involving the physics given in the problem.

1. How would you expect an *instantaneous acceleration vs. time graph* to look for a cart moving with a constant acceleration? With a uniformly increasing acceleration? With a uniformly decreasing acceleration? Make a rough sketch of the graph *for each possibility* and explain your reasoning. To make the comparison easier, it is useful to draw these graphs next to each other. Remember to assign labels and units to your axes. Write down an equation for each graph. Explain what the symbols in each of the equations mean. What quantities in these equations can you determine from your graph?

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct an instantaneous velocity versus time graph just below each of your acceleration versus time graphs from question 1, with the same scale for each time axis. Write down an equation for each graph. Explain what the symbols in each of the equations mean. What quantities in these equations can you determine from your graph?

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct a position versus time graph just below each of your velocity versus time graphs from question 2, with the same scale for each time axis. Write down an equation for each graph. Explain what the symbols in each of the equations mean. What quantities in these equations can you determine from your graph?

PREDICTION

Consider the questions printed in italics, below, to make a rough sketch of how you expect the *acceleration vs. time* graph to look for a cart under the conditions given in the problem. Explain your reasoning.

*Do you think the cart's acceleration **changes** as it moves down the track? If so, how does the acceleration change (increase or decrease)? Or, do you think the acceleration is constant (does not change) as the cart moves down the track?*

EXPLORATION

If necessary, try leveling the table by adjusting the levelers in the base of each table leg. You can test that the table is level by observing the motion of the cart on a level track.

You will use a wood block and the aluminum track to create an incline. This set up will give you an angle with respect to the table. How are you going to measure this angle? *Hint: Think trigonometry!*

Start with a small angle and with the cart at rest near the top of the track. Observe the cart as it moves down the inclined track. Try a range of angles. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!** If the angle is too large, you may not get enough video frames, and thus enough position and time measurements, to measure the acceleration accurately. If the angle is too small the acceleration may be too small to measure accurately with the precision of your measuring instruments. Select the best angle for this measurement.

Where is the best place to release the cart so enough of its motion captured on video?

When placing the camera, consider which part of the motion you wish to capture. Try different camera positions until you get the best possible video. Make sure you have an object in your video to calibrate with. *Hint: Your video may be easier to analyze if the motion on the video screen is purely horizontal. Why? It could be useful to rotate the camera!*

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking.

Write down your measurement plan.

MEASUREMENT

Follow the measurement plan you wrote down.

Record all of your measurements; you may be able to re-use some of them in other lab problems. Be sure to record your measurements with the appropriate number of significant figures and with your estimated uncertainty. Otherwise, the data is nearly meaningless.

When you have finished making measurements, you should have printouts of position and velocity graphs and good records (including uncertainty) of: your determination of the incline angle, the time it takes the cart to roll a known distance down the incline starting from rest, the length of the cart, and prediction and fit equations for position and velocity.

Make sure that everyone gets the chance to operate the computer.

ANALYSIS

Calculate the cart's average acceleration from the distance and time measurements you made with a meter stick and stopwatch.

Look at your graphs and rewrite all of the equations in a table but now matching the *dummy letters* with the appropriate kinetic quantities. If you have constant values, assign them the correct units, and explain their meaning.

From the velocity vs. time graph, determine if the acceleration is constant, increasing, or decreasing as the cart goes down the ramp. Use the function representing the velocity vs. time graph to calculate the acceleration of the cart as a function of time. Make a graph of that function. Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

Compare the accelerations for the cart you found with your video analysis to your acceleration measurement using a stopwatch.

CONCLUSION

How do the graphs of your measurements compare to your predictions?

Was your boss right about how a cart accelerates down a hill? If yes, state your result in the most general terms supported by your analysis. If not, describe how you would convince your boss of your conclusions. What are the limitations on the accuracy of your measurements and analysis?

1301 LAB 1 PROBLEM 4: MOTION UP AND DOWN AN INCLINE

A proposed ride at the Valley Fair amusement park launches a roller coaster car up an inclined track. Near the top of the track, the car reverses direction and rolls backwards into the station. As a member of the safety committee, you have been asked to describe the acceleration of the car throughout the ride.   The launching mechanism has been well tested. You are only concerned with the roller coaster's trip up and back down. To test your expectations, you decide to build a laboratory model of the ride.

Read: Mazur Chapter 3.

EQUIPMENT

You have a stopwatch, meterstick, track endstop, wood block, video camera and a computer with video analysis software. You will also have a cart to roll up an inclined track.

WARM UP

The following questions should help you examine the situation.

1. Sketch a graph of the *instantaneous acceleration vs. time graph* you expect for the cart as it rolls up and then back down the track **after** an initial push. Sketch a second *instantaneous acceleration vs. time graph* for a cart moving up and then down the track with the direction of a constant acceleration always down along the track **after** an initial push. On each graph, label the instant where the cart reverses its motion near the top of the track. Explain your reasoning for each graph. Write down the equation(s) that best represents each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs?

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct an *instantaneous velocity vs. time graph* just below each of acceleration vs. time graph from question 1, with the same scale for each time axis. (The connection between the derivative of a function and the slope of its graph will be useful.) On each graph, label the instant where the cart reverses its motion near the top of the track. Write an equation for each graph. If there are constants in your equations, what kinematics quantities do they represent?   How would you determine these constants from your graphs?   Can any of the constants be determined from the constants in the equation representing the acceleration vs. time graphs?

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct an *instantaneous position vs. time graph* just below each of your velocity vs. time graphs from question 2, with the same scale for each time axis. (The

connection between the derivative of a function and the slope of its graph will be useful.) On each graph, label the instant where the cart reverses its motion near the top of the track. Write down an equation for each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of the constants be determined from the constants in the equations representing velocity vs. time graphs?

4. Which graph do you think best represents how position of the cart will change with time? Adjust your prediction if necessary and explain your reasoning.

PREDICTION

Make a rough sketch of how you expect the acceleration vs. time graph to look for a cart with the conditions discussed in the problem. The graph should be for the entire motion of going up the track, reaching its highest point, and then coming down the track.

*Do you think the acceleration of the cart moving up an inclined track will be **greater than**, **less than**, or **the same as** the acceleration of the cart moving down the track? What is the acceleration of the cart at its highest point? Explain your reasoning.*

EXPLORATION

If necessary, try leveling the table by adjusting the levelers in the base of each table leg. You can test that the table is level by observing the motion of the cart on a level track.
What is the best way to change the angle of the inclined track in a reproducible way? How are you going to measure this angle with respect to the table? (Think about trigonometry.)

Start the cart up the track with a gentle push. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP ON ITS WAY DOWN!** Observe the cart as it moves up the inclined track. At the instant the cart reverses direction, what is its velocity? Its acceleration? Observe the cart as it moves down the inclined track. Do your observations agree with your prediction? If not, discuss it with your group.

Where is the best place to put the camera? Which part of the motion do you wish to capture?

Try different angles. If the angle is too large, the cart may not go up very far and will give you too few video frames for the measurement. If the angle is too small it will be difficult to measure the acceleration. Take a practice video and play it back to make sure you have captured the motion you want *Hint: To analyze motion in only one dimension (like in the previous problem) rather than two dimensions, it could be useful to rotate the camera!*

What is the total distance through which the cart rolls? Using your stopwatch, how much time does it take? These measurements will help you set up the graphs when using the computer, and can provide for a check on your video analysis of the cart's motion.

Write down your measurement plan.

MEASUREMENT

Follow your measurement plan to make a video of the cart moving up and then down the track at your chosen angle. Record the time duration of the cart's trip, and the distance traveled. Make sure you get enough points for each part of the motion to determine the behavior of the acceleration. *Don't forget to measure and record the angle (with estimated uncertainty).*

Work through the complete set of calibration, prediction equations, and fit equations for a single (good) video before making another.

Make sure everyone in your group gets the chance to operate the computer.

ANALYSIS

From the time given by the stopwatch and the distance traveled by the cart, calculate its average acceleration. Estimate the uncertainty.

Look at your graphs and rewrite all of the equations in a table but now matching the *dummy letters* with the appropriate kinetic quantities. If you have constant values, assign them the correct units, and explain their meaning.

Can you tell from your graph where the cart reaches its highest point?

From the *velocity vs. time graph* determine if the acceleration changes as the cart goes up and then down the ramp. Use the *function* representing the velocity vs. time graph to calculate the acceleration of the cart as a function of time. Make a graph of that function. Can you tell from this *instantaneous acceleration vs. time graph* where the cart reaches its highest point? Is the average acceleration of the cart equal to its instantaneous acceleration in this case?

Compare the acceleration function you just graphed with the average acceleration you calculated from the time on the stopwatch and the distance the cart traveled.

CONCLUSION

How do your position vs. time, velocity vs. time graphs compare with your answers to the warm up questions and the prediction? What are the limitations on the accuracy of your measurements and analysis?

Did the cart have the same acceleration throughout its motion? Did the acceleration change direction? Was the acceleration zero at the top of its motion? Describe the acceleration of the cart through its entire motion **after** the initial push. Justify your answer with kinematics arguments and experimental results. If there are any differences between your predictions and your experimental results, describe them and explain why they occurred.

1301 LAB 1 PROBLEM 5: MOTION DOWN AN INCLINE WITH AN INITIAL VELOCITY

Because of your physics background, you have a summer job with a company that is designing a new bobsled for the U.S. team to use in the next Winter Olympics. You know that the success of the team depends crucially on the initial push of the team members – how fast they can push the bobsled before they jump into the sled. You need to know in more detail how that initial velocity affects the motion of the bobsled. In particular, your boss wants you to determine if the initial velocity of the sled affects its acceleration down the ramp. To solve this problem, you decide to model the situation using a cart moving down an inclined track.

Read: Mazur Chapter 3.

EQUIPMENT

You have a stopwatch, meterstick, endstop, wood block, video camera and a computer with video analysis software. You will also have a cart to roll down an inclined track.
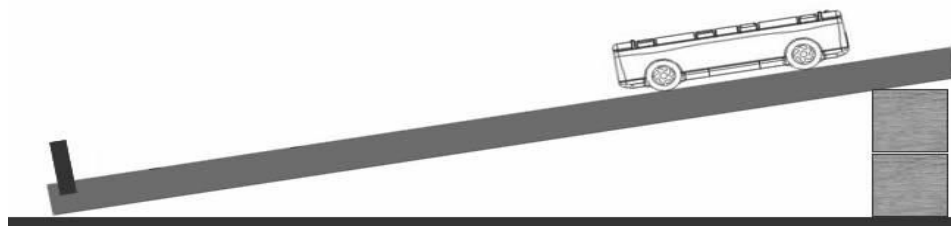
WARM UP

The following questions should help you (a) understand the situation and (b) interpret your measurements.

1. Sketch a graph of *instantaneous acceleration vs. time graph* when the cart rolls down the track **after** an initial push (your graph should begin **after** the initial push.) Compare this to an *instantaneous acceleration vs. time graph* for a cart released from rest. (To make the comparison easier, draw the graphs next to each other.) Explain your reasoning for each graph. Write down the equation(s) that best represents each of the graphs. If there are constants in your equations, what kinematics quantities do they represent? How would you determine the constants from your graphs?

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct an *instantaneous velocity vs. time graph*, **after** an initial push, just below each of your *acceleration vs. time graphs* from question 1. Use the same scale for your time axes. (The connection between the derivative of a function and the slope of its graph will be useful.) Write down the equation that best represents each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine the constants from your graphs? Can any of the constants be determined from the equations representing the *acceleration vs. time graphs*?

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct a *position vs. time graph*, **after** an initial push, just below each

velocity vs. time graph from question 2. Use the same scale for your time axes. (The connection between the derivative of a function and the slope of its graph will be useful.) Write down the equation that best represents each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of these constants be determined from the equations representing the *velocity vs. time graphs*?

PREDICTION

Do you think the cart launched down the inclined track will have a larger acceleration, smaller acceleration, or the same acceleration as the cart released from rest?

EXPLORATION

If necessary, try leveling the table by adjusting the levelers in the base of each table leg. You can test that the table is level by observing the motion of the cart on a level track.

Slant the track at an angle. (Hint: Is there an angle that would allow you to reuse some of your measurements and calculations from other lab problems?)

Determine the best way to gently launch the cart down the track in a consistent way without breaking the equipment. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!**

Where is the best place to put the camera? Is it important to have most of the motion in the center of the picture? Which part of the motion do you wish to capture? Try taking some videos before making any measurements.

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking.

Write down your measurement plan. Make sure everyone in your group gets the chance to operate the camera and the computer.

MEASUREMENT

Using the plan you devised in the exploration section, make a video of the cart moving down the track at your chosen angle. Make sure you get enough points for each part of the motion to determine the behavior of the acceleration. *Don't forget to measure and record the angle (with estimated uncertainty).*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case?

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and total time to determine the maximum and minimum value for each axis before taking data.

ANALYSIS

Choose a function to represent the *position vs. time* graph. How can you estimate the values of the constants of the function from the graph? You may waste a lot of time if you just try to guess the constants. What kinematics quantities do these constants represent?

Choose a function to represent the velocity versus time graph. How can you calculate the values of the constants of this function from the function representing the position versus time graph? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematics quantities do these constants represent?

From the velocity versus time graph, determine the acceleration as the cart goes down the ramp **after** the initial push. Use the function representing the velocity versus time graph to calculate the acceleration of the cart as a function of time. Make a graph of that function.

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer.*

CONCLUSIONS

Look at the graphs you produced through video analysis. How do they compare to your answers to the warm-up questions and your predictions? Explain any differences. What are the limitations on the accuracy of your measurements and analysis?

What will you tell your boss? Does the **acceleration** of the bobsled down the track depend on the initial velocity the team can give it? Does the **velocity** of the bobsled down the track depend on the initial velocity the team can give it? State your result in the most general terms supported by your analysis.

1301 LAB 1 PROBLEM 6: MASS AND MOTION DOWN AN INCLINE

Your neighbors' child has asked for your help in constructing a soapbox derby car. In the soapbox derby, two cars are released from rest at the top of a ramp. The one that reaches the bottom first wins. The child wants to make the car as heavy as possible to give it the largest acceleration. Is this plan reasonable?

Read: Mazur Chapter 3.

EQUIPMENT

You have a stopwatch, meterstick, track endstop, wood block, camera and a computer with video analysis software. You will also have a cart to roll down an inclined track and additional cart masses to add to the cart.
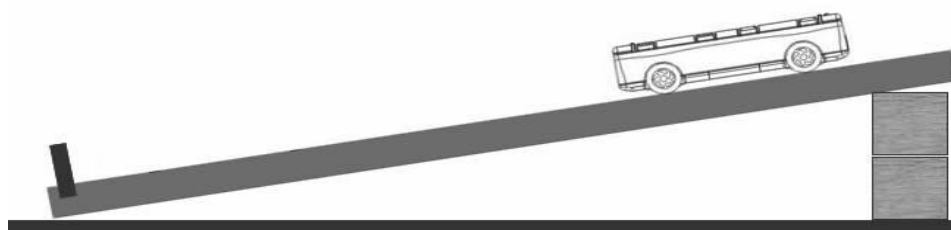
WARM UP

The following questions should help you (a) understand the situation and (b) interpret your measurements.

1. Make a sketch of the *acceleration vs. time graph* for a cart released from rest on an inclined track. On the same axes sketch an *acceleration vs. time graph* for a cart on the same incline, but with a much larger mass. Explain your reasoning. Write down the equations that best represent each of these accelerations. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs?

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct an *instantaneous velocity vs. time* graph for each case. (The connection between the derivative of a function and the slope of its graph will be useful.) Write down the equation that best represents each of these velocities. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of these constants be determined from the equations representing the accelerations?

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct a *position vs. time* graph for each case. The connection between the derivative of a function and the slope of its graph will be useful. Write down the equation that best represents each of these positions. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of these constants be determined from the equations representing the velocities?

Do you think that increasing the mass of the cart increases, decreases, or has no effect on the cart's acceleration?

EXPLORATION

If necessary, try leveling the table by adjusting the levelers in the base of each table leg. You can test that the table is level by observing the motion of the cart on a level track.

Slant the track at an angle. (Hint: Is there an angle that would allow you to reuse some of your measurements and calculations from other lab problems?)

Observe the motion of several carts of different mass when released from rest at the top of the track. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!** From your estimate of the size of the effect, determine the range of mass that will give the best results in this problem. Determine the first two masses you should use for the measurement.

How do you determine how many different masses do you need to use to get a conclusive answer? How will you determine the uncertainty in your measurements? How many times should you repeat these measurements? Explain.

What is the total distance through which the cart rolls? How much time does it take? These measurements will help you set up the graphs for your computer data taking.

Write down your measurement plan.

Make sure everyone in your group gets the chance to operate the camera and the computer.

MEASUREMENT

Using the plan you devised in the exploration section, make a video of the cart moving down the track at your chosen angle. Make sure you get enough points for each part of the motion to determine the behavior of the acceleration. *Don't forget to measure and record the angle (with estimated uncertainty).*

Choose an object in your picture for calibration. Choose your coordinate system. Is a rotated coordinate system the easiest to use in this case?

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and the total time to determine the maximum and minimum value for each axis before taking data.

Make several videos with carts of different mass to check your qualitative prediction. If you analyze your data from the first two masses you use *before* you make the next video, you can determine which mass to use next. As usual you should minimize the number of measurements you need.

<div align="center">

ANALYSIS

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Choose a function to represent the *position vs. time* graph. How can you estimate the values of the constants of the function from the graph? You may waste a lot of time if you just try to guess the constants. What kinematics quantities do these constants represent?

Choose a function to represent the *velocity vs. time* graph. How can you calculate the values of the constants of this function from the function representing the *position vs. time* graph? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematics quantities do these constants represent?

From the *velocity vs. time* graph determine the acceleration as the cart goes down the ramp. Use the function representing the velocity-versus-time graph to calculate the acceleration of the cart as a function of time.

Make a graph of the cart's acceleration down the ramp as a function of the cart's mass. Do you have enough data to convince others of your conclusion about how the acceleration of the cart depends on its mass?

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer*.

<div align="center">

CONCLUSION

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Did your measurements of the cart's motion agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

What will you tell the neighbors' child? Does the **acceleration** of the car down its track depend on its total mass? Does the **velocity** of the car down its track depend on its mass? State your result in the most general terms supported by your analysis.

1301 LAB 1 PROBLEM 7: MASS AND THE ACCELERATION OF A FALLING BALL

You have a job with the National Park Service. Your task is to investigate the effectiveness of spherical canisters filled with fire-retarding chemicals to help fight forest fires. The canisters would be dropped by low-flying planes or helicopters. They are specifically designed to split open when they hit the ground, showering the nearby flames with the chemicals. The canisters could contain different chemicals, so they will have different masses. In order to drop the canisters accurately, you need to know if the motion of a canister depends on its mass. You decide to model the situation by measuring the free-fall acceleration of balls with similar sizes but different masses.

Read: Mazur Chapter 3.

EQUIPMENT

You have a collection of balls each with approximately the same diameter. You also have a stopwatch, meterstick, camera and a computer with video analysis software.

WARM UP

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1. Sketch a graph of *acceleration as a function of time* for a constant acceleration. Below it, make graphs for *velocity* and *position* as functions of time. Write down the equations that best represent each graph. If there are constants in each equation, what kinematics quantities do they represent? How would you determine these constants from your graphs?

2. Make two more sketches of the *acceleration vs. time graph*: one for a heavy falling ball and another for a falling ball with one quarter of the heavy one's mass. Explain your reasoning. Write the equation that best represents each of acceleration. If there are constants in your equations, what kinematics quantities do they represent? How would you determine the constants from your graphs? How do they differ from each other, and from your constant acceleration graph?

3. Use the relationships between acceleration and velocity and velocity and position of the ball to construct an *instantaneous velocity vs. time graph* and a *position vs. time graph* for each case from the previous question. The connection between the derivative of a function and the slope of its graph will be useful. Write down the equations that best represent each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of these constants be determined from the constants in the equations representing the acceleration and velocity?

4. Compare your graphs to those for constant acceleration. What are the differences, if any, that you might observe in your data? The similarities?

5. Write down an outline of how you will determine the acceleration of the object from video data.

<div style="border:1px solid;">PREDICTION</div>

Make a sketch of how you expect the *average acceleration vs. mass graph* to look for falling objects such as the balls in the problem.

*Do you think that the free-fall acceleration **increases**, **decreases**, or **stays the same** as the mass of the object increases? Make your best guess and explain your reasoning.*

<div style="border:1px solid;">EXPLORATION</div>

Review your lab journal from earlier problems. Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice dropping one of the balls until you can get the ball's motion to fill the screen. Determine how much time it takes for the ball to fall and estimate the number of video points you will get in that time. Are there enough points to make the measurement? Adjust the camera position to give you enough data points.

Although the ball is an obvious choice to use to calibrate the video, you might have better results calibrating on a larger object. For calibration purposes, you can hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object close to the camera and then further away. What do you notice about the size of the reference object in the video image? The best place to put the reference object to determine the distance scale is at the position of the falling ball.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see some blurring of the image. You can adjust the camera settings to give you a discrete image.

Write down your measurement plan.

<div style="border:1px solid;">MEASUREMENT</div>

Measure the mass of a ball and make a video of its fall according to the plan you devised in the exploration section.

Record the position of the ball in enough frames of the video so that you have the sufficient data to accomplish your analysis. Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance

the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

Complete your data analysis as you go along (before making the next video), so you can determine how many different videos you need to make. Don't waste time in collecting data you don't need or, even worse, incorrect data. Collect enough data to convince yourself and others of your conclusion.

Repeat this procedure for different balls.

ANALYSIS

Choose a function to represent the *position vs. time graph.* How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Choose a function to represent the *velocity vs. time graph.* How can you calculate the values of the constants of this function from the function representing the *position vs. time graph*? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent?

From the *velocity vs. time graph(s)* determine the acceleration of the ball. Use the function representing the *velocity vs. time graph* to calculate the acceleration of the ball as a function of time. Is the average acceleration different for the beginning of the video (when the object is moving slowly) and the end of the video (when the object is moving fast)?

Determine the average acceleration of the object in free fall for each value of its mass and graph this result. Do you have enough data to convince others of your conclusions about your predictions?

CONCLUSION

Did the data support your predicted relationship between acceleration and mass? (Make sure you read the **Review of Graphs** appendix to determine if your data really supports this relationship.) If not, what assumptions did you make that were incorrect? Explain your reasoning.

What are the limitations on the accuracy of your measurements and analysis?

Do your results hold regardless of the masses of balls? Would the acceleration of a falling Styrofoam ball be the same as the acceleration of a falling baseball? Explain your rationale. Make sure you have some data to back up your claim. Will the acceleration of a falling canister depend on its mass? State your results in the most general terms supported by your analysis.

1301 LAB 1 PROBLEM 8: ACCELERATION OF A BALL WITH AN INITIAL VELOCITY

You have designed an apparatus to measure air quality in your city. To quickly force air through the apparatus, you will launch it straight downward from the top of a tall building. A very large acceleration may destroy sensitive components in the device; the launch system's design ensures that the apparatus is protected during its launch. You wonder what the acceleration of the apparatus will be once it exits the launcher. Does the object's acceleration after it has left the launcher depend on its velocity when it leaves the launcher? You decide to model the situation by throwing balls straight down.

Read: Mazur Chapter 3.

EQUIPMENT

You have a ball, stopwatch, meterstick, camera and a computer with video analysis software. The launcher is your hand.

WARM UP

The following questions will help you examine three possible scenarios. They should help you to understand your prediction and analyze your data.

1. How would you expect an acceleration *vs. time graph* to look for a ball moving downward with a constant acceleration? With a uniformly increasing acceleration? With a uniformly decreasing acceleration? Sketch the graph for each scenario and explain your reasoning. To make the comparison easier, draw these graphs next to each other. Write down the equation that best represents each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graph?

2. Write down the relationships between the acceleration and the velocity and the velocity and the position of the ball. Use these relationships to construct the graphs for *velocity vs. time* and *position vs. time* just below each acceleration graph from question 1. Use the same scale for each time axis. Write down the equation that best represents each graph. If there are constants in your equations, what kinematics quantities do they represent? How would you determine these constants from your graphs? Can any of the constants be determined from the equations representing the acceleration and velocity graphs?

3. Does your prediction agree with one of the scenarios you just explored? Explain why or why not.

4. Write down an outline of how you will determine the acceleration of the object from the video data.

PREDICTION

Sketch a graph of a ball's acceleration as a function of time **after** it is launched in the manner described above. How will your graph change if the object's initial velocity increases or decreases?

*Do you think that the acceleration **increases**, **decreases**, or **stays the same** as the initial velocity of the object changes? Make your best guess and explain your reasoning.*

EXPLORATION

Review your lab journal from earlier problems. Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice throwing the ball straight downward until you can get the ball's motion to fill most of the video screen **after** it leaves your hand. Determine how much time it takes for the ball to fall and estimate the number of video points you will get in that time. Is it sufficient to make the measurement? Adjust the camera position to get enough data points.

Although you could calibrate on the ball, you might have better results calibrating on a larger object. For calibration purposes, you can hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object close to the camera and then further away. What do you notice about the size of the reference object in the video image? The best place to put the reference object to determine the distance scale is at the position of the falling ball.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see some blurring of the image. You can adjust the camera settings to give you a discrete image.

Write down your measurement plan.

MEASUREMENT

Make a video of the ball being tossed downwards. Repeat this procedure for different initial velocities.

Record the position of the ball in enough frames of the video so that you have sufficient data to accomplish your analysis. Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

Graph your data as you go along (before making the next video), so you can determine how many different videos you need to make and how you should change the ball's initial velocity for each video. Don't waste time collecting data you don't need or, even worse, incorrect data. Collect enough data to convince yourself and others of your conclusion.

Repeat this procedure for different launch velocities.

ANALYSIS

Choose a function to represent the *position vs. time graph*. How can you estimate the values of the constants of the function from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Choose a function to represent the *velocity vs. time graph*. How can you calculate the values of the constants of this function from the function representing the *position vs. time graph*? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Determine the launch velocity of the ball from this graph. Is this value reasonable?

From the *velocity vs. time graph*(s) determine the acceleration of the ball. Use the function representing the *velocity vs. time graph* to calculate the acceleration of the ball as a function of time. Is the average acceleration different for the beginning of the video (just after launch) and the end of the video?

Determine the acceleration of the ball just after launch and at the end of the video. How do they compare with the gravitational acceleration? Do you have enough data to convince others of your conclusions about your predictions?

Repeat the analysis for another launch velocity and compare the results.

CONCLUSION

Did the data support your predicted relationship between acceleration and initial velocity? (Make sure you read the **Review of Graphs** appendix to determine if your data really supports this relationship.) If not, what assumptions did you make that were incorrect? Explain your reasoning.

What are the limitations on the accuracy of your measurements and analysis?

Will the survival of your apparatus depend on its launch velocity? State your results in the most general terms supported by your analysis.

1301 LAB 1 PROBLEM 9: MOTION ON A LEVEL SURFACE WITH AN ELASTIC CORD

You are helping a friend design a new ride for the State Fair. In this ride, a cart is pulled from rest along a long straight track by a stretched elastic cord (like a bungee cord). Before building it, your friend wants you to determine if this ride will be safe. Since sudden changes in velocity can lead to whiplash, you decide to find out how the acceleration of the cart changes with time. In particular, you want to know if the greatest acceleration occurs when the sled is moving the fastest or at some other time. To test your prediction, you decide to model the situation in the laboratory with a cart pulled by an elastic cord along a level surface.

Read: Mazur Chapter 3.

EQUIPMENT

You have a stopwatch, meterstick, camera and a computer with video analysis software. You also have a cart to roll on a level track and elastic cord.

WARM UP

The following questions should help you (a) understand the situation and (b) interpret your results.

1. Make a qualitative sketch of how you expect an *acceleration vs. time graph* to look for a cart pulled by an elastic cord. Explain your reasoning. For a comparison, make an *acceleration vs. time graph* for a cart moving with constant acceleration. Point out the differences between the two graphs.

2. Write down the relationship between the acceleration and the velocity of the cart. Use that relationship to construct a qualitative *velocity vs. time graph* for each case. (The connection between the derivative of a function and the slope of its graph will be useful.) Point out the differences between the two *velocity vs. time* graphs.

3. Write down the relationship between the velocity and the position of the cart. Use that relationship to construct a qualitative *position vs. time* graph for each case. (The connection between the derivative of a function and the slope of its graph will be useful.) Point out the differences between the two graphs.

PREDICTION

Make a qualitative sketch of how you expect the *acceleration vs. time* graph to look for a cart pulled by an elastic cord. Just below that graph make a qualitative graph of the *velocity vs. time*

on the same time scale. Identify on each graph where the velocity is largest and where the acceleration is largest.

<div style="text-align: center;">

EXPLORATION

</div>

Test that the track is level by observing the motion of the cart. If necessary, try leveling the track by adjusting the levelers in the base of each table leg.

Attach an elastic cord to the cart and track. Gently move the cart along the track to stretch out the elastic. **Be careful not to stretch the elastic too tightly.** Start with a small stretch and release the cart. **BE SURE TO CATCH THE CART BEFORE IT HITS THE END STOP!** Slowly increase the starting stretch until the cart's motion is long enough to get enough data points on the video, but does not cause the cart to come off the track or snap the elastic.

Practice releasing the cart smoothly and capturing videos.

Write down your measurement plan.

Make sure everyone in your group gets the chance to operate the camera and the computer.

<div style="text-align: center;">

MEASUREMENT

</div>

Using the plan you devised in the exploration section, make a video of the cart's motion. Make sure you get enough points to determine the behavior of the acceleration.

Choose an object in your picture for calibration. Choose your coordinate system.

Why is it important to click on the same point on the car's image to record its position? Estimate your accuracy in doing so.

Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the cart travels and total time to determine the maximum and minimum value for each axis before taking data.

<div style="text-align: center;">

ANALYSIS

</div>

Can you fit your position-versus time data with an equation based on constant acceleration? Do any other functions fit your data better?

From the *position vs. time* graph or your fit equation for it, predict an equation for the *velocity vs. time* graph of the cart.

From the *velocity vs. time graph*, sketch an *acceleration vs. time graph* of the cart. Can you determine an equation for this *acceleration vs. time* graph from the fit equation for the *velocity vs. time* graph?

Do you have enough data to convince others of your conclusion?

As you analyze your video, *make sure everyone in your group gets the chance to operate the computer.*

CONCLUSION

How does your acceleration-versus-time graph compare with your predicted graph? Are the position-versus-time and the velocity-versus-time graphs consistent with this behavior of acceleration? What is the difference between the motion of the cart in this problem and its motion along an inclined track? What are the similarities? What are the limitations on the accuracy of your measurements and analysis?

What will you tell your friend? Is the acceleration of the cart greatest when the velocity is the greatest? How will a cart pulled by an elastic cord accelerate along a level surface? State your result in the most general terms supported by your analysis.

1301 LAB 2 PROBLEM 1: COLLISIONS WHEN OBJECTS STICK TOGETHER

You work with the Minnesota Traffic Safety Board helping to write a report about different kinds of traffic accidents. Your boss wants you to concentrate on the scenario where a moving vehicle hits a stationary vehicle and they stick together.

For the report, you are asked to determine the velocity of these vehicles once they have collided in terms of their masses and the initial velocity of the moving vehicle. One of your team members believes that if the combined mass of the vehicles is constant, the final velocity doesn't depend on which car is more massive. You decide to determine this by measuring the final velocity of three different cart collisions: one in which the moving cart is more massive, one in which the stationary cart is more massive, and one in which the moving and stationary carts are equally massive.

You know that in a traffic collision, some of the initial energy of motion is "dissipated" in the deforming (damaging) of the vehicles. Given a constant combined mass of the vehicles, your boss is also interested in investigating whether the damage done depends on the distribution of mass between cars.

Read: Mazur Chapter 5.

EQUIPMENT

You have a track, set of carts, cart masses, meterstick, stopwatch, two endstops and video analysis equipment.

For this problem, cart A is given an initial velocity towards a stationary cart B. Velcro at the end of each cart allow the carts to stick together after the collision.

WARM UP

The following questions will help you with your prediction and with the analysis of your data.

1. Draw two pictures, one showing the situation before the collision and the other one after the collision. Is it reasonable to neglect friction? Draw velocity vectors on your sketch. Define your system. If the carts stick together, what must be true about their final velocities?

2. Write down the momentum of the system before and after the collision.

3. Write down the energy of the system before and after the collision.

4. Which conservation principle (energy or momentum) should you use to predict the final velocity of the stuck-together carts? Do you need both? Why? Write your equation for final velocity in terms of the cart masses and initial velocity of cart A.

3. Write an equation for the efficiency of the collision in terms of the final and initial kinetic energy of the carts, and then in terms of the cart masses and their initial and final velocities. Combining this efficiency equation with the final velocity found in question 4, what does variables does your efficiency depend on?

<div align="center">

PREDICTION

</div>

Consider the three cases described in the problem, with the same total mass of the carts for each case (mA + mB = constant).

Calculate the final velocity of the combined carts for each case.

Rank the collisions from most efficient to least efficient. Use your calculations to determine which collision will cause the most damage.

<div align="center">

EXPLORATION

</div>

Practice rolling the cart so the carts will stick together after colliding. Carefully observe the carts to determine whether either cart leaves the grooves in the track. Minimize this effect so that your results are reliable.

Try giving the moving cart various initial velocities over the range that will give reliable results. Note qualitatively the outcomes. Choose initial velocities that will give you useful videos.

Try varying the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Is the same range of initial velocities useful with different masses? If the moving cart should have approximately the same kinetic energy for each collision, how should its speed depend on its mass? What masses will you use in your final measurement?

<div align="center">

MEASUREMENT

</div>

Record the masses of the two carts. Make a video of the collision. Examine your video and decide if you have enough frames to determine the velocities you need. Do you notice any peculiarities that might suggest the data is unreliable?

Analyze your data as you go along (before making the next video), so you can determine how many different videos you need to make, and what the carts' masses should be for each video. Collect enough data to convince yourself and others of your conclusion about how the final velocity and energy efficiency of this type of collision depends on the relative masses of the carts.

ANALYSIS

Determine the velocity of the carts before and after the collision using video analysis. Treating the initial velocity as a known value, use the equations from your prediction to calculate final velocity, and compare this to your measured final velocity.

For each video, calculate the kinetic energy of the carts before and after the collision. Calculate the energy efficiency of each collision, once with the kinetic energy and once with just the cart masses using the equation found in the warm-up questions. Do these methods agree? Into what other forms of energy do you think the cart's initial kinetic energy is most likely to transform?

Graph how the energy efficiency varies with mass of the initially moving cart (keeping the total mass of both carts constant). What function describes this graph?

CONCLUSION

How do your measured and predicted values of the final velocity compare? Compare both magnitude and direction. What are the limitations on the accuracy of your measurements and analysis?

When a moving shuttle collides with a stationary shuttle and they dock (stick together), how does the final velocity depend on the initial velocity of the moving shuttle and the masses of the shuttles? State your results in the most general terms supported by the data.

What conditions must be met for a system's *total momentum* to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment. What conditions must be met for a system's *total energy* to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment.

Which case ($m_A = m_B$, $m_A > m_B$, or $m_A < m_B$) is the energy efficiency the largest? The smallest? Does this make sense? (Imagine extreme cases, such as a flea running into a truck and a truck running into a flea. In which case must the incoming "vehicle" be moving faster to satisfy your boss's assumption about initial kinetic energy? Which collision might cause more damage to the flea? To the truck?)

Can you approximate the results of this type of collision by assuming that the energy dissipated is small?

Suppose two equal mass cars traveling with equal speeds in opposite directions collide head on and stick together. What fraction of the energy is dissipated? Try it.

1301 LAB 2 PROBLEM 2: COLLISIONS WHEN OBJECTS BOUNCE APART

You are working for NASA with the group designing a docking mechanism that would allow two space shuttles to connect with each other. The mechanism is designed for one shuttle to move carefully into position and dock with a stationary shuttle. Since the shuttles may be carrying different payloads and have consumed different amounts of fuel, their masses may be different when they dock: the shuttles could be equally massive, the moving shuttle could be more massive, or the stationary shuttle could have a larger mass.

Your supervisor wants you to consider the case which could result from the pilot missing the docking mechanism or the mechanism failing to function. In this case the shuttles gently collide and bounce off each other. Your supervisor asks you to calculate the final velocity of both shuttles as a function of (a) the initial velocity of the initially moving shuttle, (b) the masses of both shuttles, and (c) the fraction of the moving shuttle's initial kinetic energy that is *not dissipated* during the collision (the "energy efficiency"). You may assume that the total mass of the two shuttles is constant. You decide to check your calculations in the laboratory using the most efficient bumper you have, a magnetic bumper.

Read: Mazur Chapter 4, especially Section 4.8.

EQUIPMENT

You have a track, set of carts, cart masses, meterstick, stopwatch, two endstops and video analysis equipment.

Cart A is given an initial velocity towards a stationary cart B. Magnets at the end of each cart are used as bumpers to ensure that the carts bounce apart after the collision.

WARM UP

The following questions are designed to help you with your prediction and the analysis of your data.

1. Draw two pictures that show the situation before the collision and after the collision. Draw velocity vectors on your sketch. If the carts bounce apart, do they have the same final velocity? Define your system.

2. Write down the momentum of the system before and after the collision. Is the system's momentum conserved during the collision? Why or why not?

3. If momentum is conserved, write the momentum conservation equation for this situation; identify all of the terms in the equation.

4. Write down the energy of the system before and after the collision.

5. Assuming kinetic energy is conserved, write down the energy conservation equation for this situation and identify all the terms in the equation. Is this an accurate assumption for this type of collision? Why or why not?

6. Solve the equations you wrote in previous steps to find the final velocity of each cart in terms of the cart masses, the energy efficiency of the collision, and the initial speed of the moving cart.

PREDICTION

Restate the problem such that you understand and identify its goal then get the equations necessary to test your lab model.

EXPLORATION

Practice setting the cart into motion so that the carts don't touch when they collide. Carefully observe the carts to determine whether or not either cart leaves the grooves in the track. Minimize this effect so that your results are reliable.

Try giving the moving cart various initial velocities over the range that will give reliable results. Note qualitatively the outcomes. Keep in mind that you want to choose an initial velocity that gives you a good video.

Try varying the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Be sure the carts still move freely over the track. What masses will you use in your final measurement?

MEASUREMENT

Record the masses of the two carts. Make a video of their collision. Examine your video and decide if you have enough frames to determine the velocities you need. Do you notice any peculiarities that might suggest the data is unreliable?

Analyze your data as you go along (before making the next video), so you can determine how many different videos you need to make, and what the carts' masses should be for each video. Collect enough data to convince yourself and others of your conclusion about how the final velocities of both carts in this type of collision depend on the velocity of the initially moving cart, the masses of the carts.

ANALYSIS

Determine the velocities of the carts (with uncertainty) before and after each collision from your video. Calculate the momentum and kinetic energy of the carts before and after the collision.

Now use your Prediction equation to calculate the final velocity (with uncertainty) of each cart, in terms of the cart masses and the initial velocity of the moving cart.

CONCLUSION

Did your measurement agree with your prediction? Why or why not? Was the collision perfectly elastic in the three different cases? What are the limitations on the accuracy of your measurements and analysis?

What conditions must be met for a system's *total momentum* to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment. What conditions must be met for a system's *total energy* to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment.

1301 LAB 2 PROBLEM 3: ACCELERATION OF AN OBJECT DOWN AN INCLINE

You are an engineer working for an automobile manufacturer. Your team is investigating possible designs for brakes. Once a prototype of a brake has been built, its performance must be understood in a wide variety of situations to ensure that your company's cars will always stop in an acceptable time and distance. You have been assigned the task of investigating brake performance when the car is accelerating at various speeds, for example, when rolling down a hill. You need to devise a way of producing controlled acceleration to perform your tests. With the rolling-down-a-hill scenario in mind, you decide to use straight tracks inclined at various angles; now, you need to know how the acceleration varies as a function of the angle.

Read: Mazur Sections 5.1–5.4, 5.7, 7.2, and 7.9.

EQUIPMENT

You will have a track, wooden blocks, cart, endstop, camera and computer with video analysis software.

WARM UP

1. When the car is at the top of the hill, what form(s) does its energy take? Write down an expression for the total energy of a car at the top of a hill.

2. Now imagine that the car is rolling down the same hill and is just reaching the bottom. What form(s) does its energy take? Write down an expression for the total energy of the car.

3. What can you say about the values of the car's energy at the top and bottom of the hill? Write an equation relating your answers for questions 1 and 2.

4. Assume that the car's acceleration is constant. Write an equation relating the magnitude of the acceleration and the distance through which the car accelerates.

5. Use your answers to questions 3 and 4 to find the magnitude of the car's acceleration.

PREDICTION

Write down an expression for the magnitude of the acceleration of a car accelerating under the influence of gravity down a hill at a given angle of steepness.

EXPLORATION

Try simulating hills of varying steepness and cars rolling down them. Under what circumstances will friction be the most significant? The least significant? Is friction helpful or harmful in your investigation? What will you do about this friction? How will you measure the angle of inclination? How will you measure the acceleration of the car?

Write down your measurement plan in your lab book.

MEASUREMENT

Execute your measurement plan. Don't forget to measure any necessary quantities (lengths, angles, etc.) that won't be measured by MotionLab, and don't forget to record the uncertainties in all of your measurements (both in and out of MotionLab)!

ANALYSIS

Use your measurements made outside of MotionLab to calculate what the acceleration of the car should be, given your prediction. Compare this to the acceleration that you measured with MotionLab.

CONCLUSION

How does the acceleration of an object moving down an inclined plane under the influence of gravity vary with the angle of inclination?
What are the limitations of your investigation? Are there any confounding factors that might change this result in extreme cases or in a real car? How important are they?

1301 LAB 2 PROBLEM 4: ENERGY AND VELOCITY

You are an engineer working for a top-secret government lab hidden deep underground. The lab has outgrown its small, underground reactor and needs to decrease its power budget because using power from the surface would be suspicious. Moving materials to the lab from the surface on the electric elevator is very power-intensive, so you suggest a system that would simply allow them to fall down a shaft in a suspended basket, pulling a weight on a track to control the descent. You need to know how fast the basket will be moving when it reaches the bottom.

Read: Mazur Sections 5.1–5.4, 5.7, 7.2, and 7.9.

EQUIPMENT

You have a meterstick, stopwatch, mass set, cart masses, a pulley & table clamp, string and the video analysis equipment.
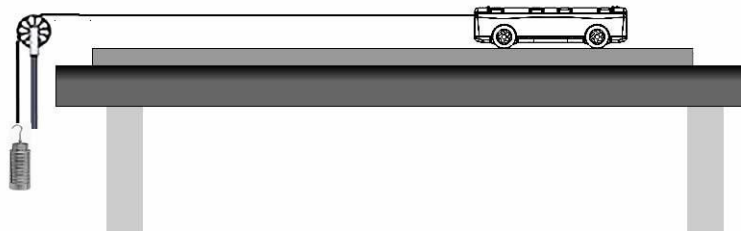
Released from rest, a cart is pulled along a level track by a hanging mass as shown. You can vary the hanging mass and the cart's mass which are connected by a light string. The mass falls from a height shorter than the track's length.

WARM UP

1. What types of energy are present just before the cart begins its motion? Write down expressions for the magnitudes of each of these types of energy.

2. What is the relationship between the velocity of the cart and the velocity of object A? What happens to the cart when object A hits the floor?

3. What types of energy will be present just as the falling mass hits the ground? What types of energy will be present then? Write down expressions for the magnitudes of each of these types of energy.

4. Write down an equation relating the energy from question 1 to the energy from question 3.

5. Solve your equation from question 3 to find the final velocity of the cart.

PREDICTION

Restate the problem in terms of quantities you know or can measure. Beginning with basic physics principles, show how you get an equation that gives the velocity you need. Make sure that you state any approximations or assumptions that you are making.

EXPLORATION

Adjust the length of the string such that object A hits the floor well before the cart runs out of track. You will be analyzing a video of the cart both **before** and **after** object A has hit the floor. Adjust the string length to give you a video that is long enough to allow you to analyze enough frames of motion.

Choose a mass for the cart and find a range of masses for object A that allows the cart to achieve a reliably measurable velocity before object A hits the floor. Make sure you include masses of object A that range from at least 1/2 that of the cart to masses that are a small fraction of the cart. Practice catching the cart **before** it hits the clamp on the end of the track.

Make sure that the assumptions for your prediction apply to the situation in which you are making the measurement. For example, if you are neglecting friction, make sure that the cart's wheels turn freely. Also check that the pulley wheel turns freely.

Write down your measurement plan.

MEASUREMENT

Carry out your measurement plan.

Complete the entire analysis of one case before making videos and measurements of the next case. **A different person should operate the computer for each case.**

Make sure you measure and record the mass of the cart and object A. Record the height through which object A falls and the time this takes to occur.

Take a video that will allow you to analyze the data during both time intervals. Make measurements for at least two different heights of release.

ANALYSIS

Determine the velocity of the cart just after the hanging object hits the floor. See if this velocity agrees with your prediction. Examine the dependence of these velocities on the masses and the height of release.

What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

How does the velocity of the cart depend on the masses and the distance traveled just before the hanging object strikes the floor? Were you able to predict the maximum velocity? If not, why not? Were there any forms of energy change that were ignored in your predictions?

1301 LAB 2 PROBLEM 5: ENERGY AND EFFICIENCY I

You are working at a company that designs pinball machines and have been asked to devise a test to determine the efficiency of some new magnetic bumpers. You know that when a normal pinball rebounds off traditional bumpers, some of the initial energy of motion is "dissipated" in the deformation of the ball and bumper, thus slowing the ball down. The lead engineer on the project assigns you to determine if the new magnetic bumpers are more efficient. The engineer tells you that the efficiency of a collision is the ratio of the final kinetic energy to the initial kinetic energy of the system.

To limit the motion to one dimension, you decide to model the situation using a cart with a magnet colliding with a magnetic bumper. You will use a level track, and use a video data acquisition system to measure the cart's velocity before and after the collision. You begin to gather your camera and data acquisition system when your colleague suggests a method with simpler equipment. Your colleague claims it would be possible to release the cart from rest on an inclined track and make measurements with just a meter stick. You are not sure you believe it, so you decide to measure the energy efficiency both ways, and determine the extent to which you get consistent results. *For this problem, you will use the level track. For* **Energy and Efficiency II**, *you will work with the inclined track.*

Read: Mazur Chapter 5.

EQUIPMENT

You have a meterstick, stopwatch, track, endstop, cart and video analysis equipment.

WARM UP

It is useful to have an organized problem-solving strategy. The following questions will help with your prediction and the analysis of your data.

1. Make a drawing of the cart on the level track before and after the impact with the bumper. Define your system. Label the velocity and kinetic energy of all objects in your system before and after the impact.

2. Write an expression for the efficiency of the bumper in terms of the final and initial kinetic energy of the cart.

3. Write an expression for the energy dissipated during the impact with the bumper in terms of the kinetic energy before the impact and the kinetic energy after the impact.

PREDICTION

Calculate the energy efficiency of the bumper discussed in the problem in terms of the least number of quantities that you can easily measure in the situation of a level track. Calculate the energy dissipated during the impact with the bumper in terms of those measurable quantities.

EXPLORATION

Test that the track is level by observing the motion of the cart. If necessary, try leveling the track by adjusting the levelers in the base of each table leg.

Review your exploration notes for measuring a velocity using video analysis. Practice pushing the cart with different velocities, slowly enough that the cart will never contact the bumper (end stop) during the impact when you make a measurement. Find a range of velocities for your measurement. Set up the camera and tripod to give you a useful video of the collision immediately before and after the cart collides with the bumper.

MEASUREMENT

Take the measurements necessary to determine the kinetic energy before and after the impact with the bumper. What is the most efficient way to measure the velocities with the video equipment? Take data for several different initial velocities.

ANALYSIS

Calculate the efficiency of the bumper for the level track. Does your result depend on the velocity of the cart before it hits the bumper?

CONCLUSION

What is the efficiency of the magnetic bumpers? How much energy is dissipated in an impact? State your results in the most general terms supported by your analysis.

If available, compare your value of the efficiency (with uncertainty) with the value obtained by the different procedure given in the problem **Energy and Efficiency II**. Are the values consistent? Which way to measure the efficiency of the magnetic bumper do you think is better? Why?

1301 LAB 2 PROBLEM 6: ENERGY AND EFFICIENCY II

You are working at a company that designs pinball machines and have been asked to devise a test to determine the efficiency of some new magnetic bumpers. You know that when a normal pinball rebounds off traditional bumpers, some of the initial energy of motion is "dissipated" in the deformation of the ball and bumper, thus slowing the ball down. The lead engineer on the project assigns you to determine if the new magnetic bumpers are more efficient. The engineer tells you that the efficiency of a collision is the ratio of the final kinetic energy to the initial kinetic energy of the system.

To limit the motion to one dimension, you decide to model the situation using a cart with a magnet colliding with a magnetic bumper. You plan to use a level track, and use a video data acquisition system to measure the cart's velocity before and after the collision. You begin to gather your camera and data acquisition system when your colleague suggests a method with simpler equipment. Your colleague claims it would be possible to release the cart from rest on an inclined track and make measurements with just a meter stick. You are not sure you believe it, so you decide to measure the energy efficiency both ways, and determine the extent to which you get consistent results. *For this problem, you will use the inclined track.*

Read: Mazur Chapter 5.

EQUIPMENT

You have a meterstick, stopwatch, cart masses, a wooden block to create the incline, and the video analysis equipment.
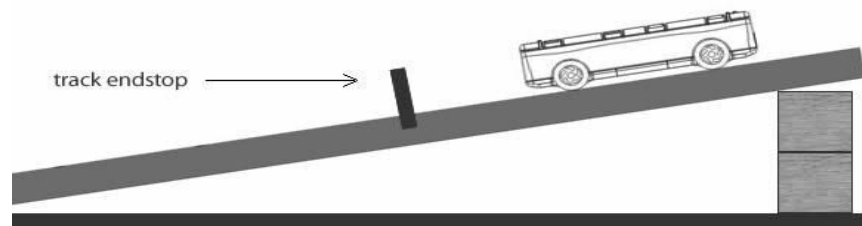
WARM UP

The following questions will help you to make your prediction and analyze your data.

1. Make a drawing of the cart on the ***inclined track*** at its initial position (before you release the cart) and just before the cart hits the bumper. Define the system and label the initial height of the cart above the bumper. Write the kinetic and potential energy of the cart at these two points.
2. Use the principle of the conservation of energy to relate the total energy of the cart at its initial position to the total energy just before it hits the bumper.
3. Now make another drawing of the cart on the inclined track just after the collision with the bumper *and* at its maximum rebound height. Label the rebound height of the cart above the bumper. Write the kinetic and potential energy of the cart at these two points.
4. Use the principle of the conservation of energy to relate the total energy of the cart just after it hits the bumper to the total energy when the cart reaches its rebound height.
5. Write an expression for the efficiency of the bumper in terms of the kinetic energy of the cart just before the impact and the kinetic energy of the cart just after the impact. Rewrite

this expression in terms of the cart's initial height above the bumper and the cart's maximum rebound height above the bumper.

6. Write an expression for the energy dissipated during the impact with the bumper in terms of the kinetic energy of the cart just before the impact and the kinetic energy of the cart after the impact. Re-write this expression in terms of the cart's initial height above the bumper and the cart's maximum rebound height above the bumper.

PREDICTION

Calculate the energy efficiency of the bumper in terms of the least number of quantities that you can easily measure in the situation of an inclined track.

EXPLORATION

Find a useful range of heights and inclined angles that will not cause damage to the carts or bumpers. Make sure that the cart will never contact bumper (end stop) during the impact. Decide how you are going to consistently measure the *height* of the cart.

MEASUREMENT

Take the measurements necessary to determine the kinetic energy of the cart before and after the impact with the bumper. Take data for several different initial heights.

ANALYSIS

Calculate the efficiency of the bumper for the inclined track. Does your result depend on the velocity of the cart before it hits the bumper?

CONCLUSION

What is the efficiency of the magnetic bumpers? How much energy is dissipated in an impact? State your results in the most general terms supported by your analysis.

If available, compare your value of the efficiency (with uncertainty) with the value obtained by the different procedure given in the problem **Energy and Efficiency I**. Are the values consistent? Which way to measure the efficiency of the magnetic bumper do you think is better? Why?

1301 LAB 2 PROBLEM 7: MECHANICAL ENERGY OF A SPRING

You are designing the suspension for a new high-performance sports car. You need to understand the behavior of the springs that you will be using in the car. In particular, you want to understand how the suspension will dissipate the energy of the car bouncing up and down. Before you can understand that dissipation, however, you need to understand the more fundamental idea of the energy stored in the springs that you will be using. You decide to model the situation by suspending a mass from a spring in your laboratory.

Read: Mazur Sections 5.2–5.4, 5.7, 7.2, 7.3, 7.9, 9.7

EQUIPMENT

You have springs, a table clamp, a rod, a meter stick, a mass set, and the video analysis equipment.

WARM-UP

1. Make a sketch of the system before a mass is hung from the end of the spring. Draw a coordinate system, placing the origin at the end of the hanging spring when the spring is unstretched. Approximate the spring as massless.

2. Draw the spring system when a mass is hung from the spring and resting at its equilibrium position. Draw a force diagram for the object hanging at rest from the end of the spring and solve for the spring constant in terms of the object's mass, the gravitational acceleration constant, and the object's equilibrium position.

3. Consider the situation where an object, attached to the end of the spring, is initially held at the position at which the spring is unstretched and then released so that it begins oscillating. Make a sketch of this system when the object is at some arbitrary position. Determine the different kinds of energy found in the system while the object is oscillating.

4. What is the total energy of the system just before the object is released? How is this related to the other forms of energy in the system?

5. Assuming that total energy is conserved, although the values of the different forms of energy change during the oscillation, determine how the spring potential energy depends on the kinetic energy and the gravitational potential energy of the object and, in turn, how

the kinetic and gravitational potential energies of the object depend on its position and speed.

6. Finally, write down the theoretical form for the spring potential energy. How could we plot the spring potential energy (as determined from the answer to problem 5) as a function of position to easily show that this theoretical form holds? Will a plot of spring potential energy versus position be linear? How could we adjust position or spring potential energy to make this plot linear? What would be the slope of this plot? (The section "Using Linear Relationships to Make Graphs Clear" in the appendix "A Review of Graphs" will help you answer this question.)

PREDICTION

Knowing the value of spring potential energy, determine how it can be plotted versus some function of position to yield a linear plot, and determine the slope of this plot.

EXPLORATION

Secure one end of the spring safely to the metal rod and select a mass that gives a regular oscillation without excessive wobbling at the hanging end of the spring. **The largest choice for the mass of the object should not result in the object pulling the spring past its elastic limit (about 40cm). Beyond that point you will damage the spring.** However, the smallest choice for the mass should be much greater than the mass of the spring to fulfill the massless spring assumption. Practice releasing the mass from the unstretched position of the spring so that its vertical motion is smooth.

Practice making a video to record the motion of the spring-object system. What quantities do you need to measure in order to calculate the kinetic and gravitational potential energies of the system?

MEASUREMENT

Record the mass of the object. Make a video of the motion of the hanging object. Make sure your video includes at least two full cycles of oscillation so you have sufficient data to analyze. Make sure to fit the data for both position and velocity as these equations will be necessary for your analysis. Also save the data in a text file that can be imported into Excel.

ANALYSIS

Copy your position vs. time and velocity vs. time data into Excel. Using position and velocity, make a table of the gravitational potential and kinetic energies at all times. Make another column that provides the spring potential energy by subtracting the gravitational potential

energy and the kinetic energy from the total constant energy. Plot the spring potential energy versus some function of the position to (hopefully) obtain a linear plot. What is the slope of this plot? What constant can be extracted from this slope? How else can you find this constant? Repeat this procedure for objects with different masses hanging from the spring if you have time.

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CONCLUSION

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How do your results compare to your prediction? If your results don't match your prediction, e.g., if your plot of spring potential energy versus a function of position is not linear, what might be the reason for this deviation?

Since you are ultimately interested in dissipation of energy by the car's suspension anyway, does your data seem to show any energy dissipation? How does dissipation affect your initial prediction?

How would a change in mass change the total energy of the system? Does changing the mass change the value of the spring constant or the form of the spring's potential energy? What would happen to the form of the spring's potential energy if you had defined your origin for the object at another location (not the position of the unstretched spring)?

1301 LAB 2 PROBLEM 8: MECHANICAL ENERGY OF A SPRING II

Suppose that you and your neighbor carpool to work each morning. One day before work, you try to start your car, but nothing happens. You realize that you left the headlights on all night, and the car's battery is now dead. Neither you nor your neighbor has jumper cables, so you will need to get to a car mechanic. However, your car has a manual transmission, so it still might be possible to start your car. In order for this to happen, you need to push your car until it reaches a certain speed, and then you will be able to start the ignition. Your neighbor has a crazy idea to achieve this: she has been building a large, hand-crank driven mechanical spring in her garage, and she wants to use it on your car! Her plan is the following: you sit in the driver's seat and put the car into the drive gear, while she cranks the spring and then releases it to try to help the car achieve the minimum required speed.

Read: Mazur Sections 5.2–5.4, 5.7, 7.2, 7.3, 7.9, 9.7

EQUIPMENT

You have a meter stick, a stopwatch, cart masses, a cart, a cart launcher, and video analysis equipment. Since the cart launcher has a spring mechanism, it will act like a spring. The cart launcher mounts to the track in the track's side T-slot.
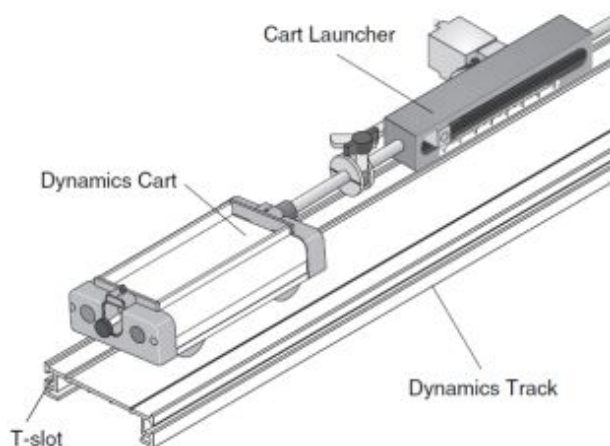
WARM-UP

1. Sketch a graph of the potential energy of the spring versus its displacement from the equilibrium position. What is the shape of this graph (linear, parabolic, etc.)?

2. On the same graph as above, sketch the kinetic energy of the spring versus its displacement from the equilibrium position. What is the shape of this graph?

3. Assume that frictional effects are negligible. On the same graph as above, sketch the total energy of the spring versus its displacement from the equilibrium position. What is the shape of this graph? How is total energy related to potential energy and kinetic energy? Write down an equation for this relationship.

4. Suppose that your neighbor has another spring with double the spring constant. How would the graph change? Sketch a similar graph based on questions 1-3 with double the spring constant.

5. Suppose that your other neighbors want to join the fun. If they get in the car too, the mass will increase. How would the graph change? Sketch a similar graph based on questions 1-3 with double the mass.

6. Compare the three graphs made in 1-3, 4, and 5, respectively. Do they each have the same shape, or different shapes? How do their maximum values compare? How do their minimum values compare?

7. Rewrite the equation from question 3 in terms of the spring's velocity, the spring's mass, the spring constant, and the spring's displacement from its equilibrium position. Now solve for the velocity of the spring.

PREDICTION

For a given mass, you will need to determine the displacement of the spring needed to reach the minimum required velocity for the car to start. How will the velocity depend on the displacement of the spring?

In order to vary the velocity of the spring, which variables can you easily change? Which ones will require equipment that is not listed in the equipment section?

EXPLORATION

Become familiar with the cart launcher and its behavior. Without the cart, compress, lock, and release the spring mechanism of the cart launcher for a number of different displacements. The cart launcher works well for spring displacements between 1.0 and 4.0 cm. Find an upper limit for spring displacement at which you and your partners are comfortable setting up the cart launcher.

Choose a range of displacement values for the cart launcher and test these values using the cart. For each displacement value, place the cart at the end of the launcher and make sure that the cart does not wobble, fall of the track, or move too fast or too slow after the spring mechanism is released. Make sure that someone in your group is able to catch the cart at the other end of the track.

Write down your measurement plan.

MEASUREMENT

Carry out your measurement plan.

Complete the entire measurement and analysis of one displacement value before moving on to the next value. **A different person should operate the computer for each displacement value.**

Notice that the velocity of the cart changes as the spring mechanism releases. The cart will reach a constant velocity very soon after it is completely released from the cart launcher. Make sure that you measure the velocity of the cart **after** this point in the video, i.e. when it has reached a constant value.

ANALYSIS

For each displacement value, determine the velocity of the cart just after the cart is released from the cart launcher. Make a graph of the cart's velocity versus the displacement of the spring. See if this agrees with your predicted equation.

What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

How do your results compare to your predictions? Does your data indicate that the cart's velocity depends on the displacement of the spring linearly? Some of your data may not show a linear relationship. What could cause this deviation from your prediction?

In order to vary the potential energy of the spring (and thus the final kinetic energy of the cart), which variable did you change? Would your results be different if you had chosen another one (e.g., cart mass versus spring displacement)?

1301 LAB 2 PROBLEM 9: ENERGY AND FRICTION

You work for an auto company, which has experienced work stoppages when novice forklift drivers suddenly stop, causing crates of auto parts to slide off the forklift and spill on the floor. Your team is investigating the conditions under which such accidents will occur, in order to improve driver training. What factors are important? Your task is to calculate the distance a crate slides after the forklift has come to a sudden stop, as a function of the forklift's initial speed. You assume that the crate is not tied down, and that the surface supporting the crate is horizontal. To test your prediction, you will model the situation with a cart on the track.

Read: Mazur Chapter 5.

EQUIPMENT

You have a cart, 500g (flat topped) cart masses, track, endstop, wood/cloth friction block, mass set and the video analysis equipment to determine the velocity of the cart before the collision.
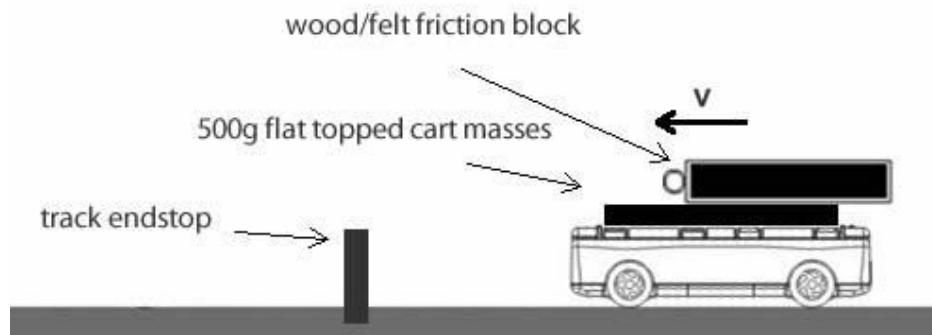
You need to use a pair of flat topped 500g cart masses and the wood/cloth block should be placed sideways on the surface to provide enough sliding distance.

WARM UP

The following questions will help with the prediction, and analysis of your data.

1. Draw three pictures: one showing the situation just before the collision of the cart with the end-stop, one immediately after the collision when the cart is stopped but the block has not yet begun to slow down, and the third when the wood block has come to rest. Draw velocity vectors on your sketches, and label any important distances. What is the relationship between the cart's velocity and the wood block's velocity in each picture? Define your system. Write down the energy of the system for each picture.

2. Write down the energy conservation equation for this situation, between the second and third pictures. Is any energy transferred into or out of the system?

3. Draw a force diagram for the wood/cloth block as it slides across the cart. Identify the forces that do work on the block (i.e., result in the transfer of energy in or out of the system). Write an equation relating the energy transferred by these forces to the distance the block slides.

4. Complete your prediction, and graph sliding distance vs. initial forklift speed.

PREDICTIONS

Calculate the distance the block slides in the situation described in the problem as a function of the cart's speed before the collision. Illustrate your prediction graphically.

EXPLORATION

Practice setting the cart with masses into motion so the cart sticks to the end stop. What adjustments are necessary to make this happen consistently? Place the wood/cloth block on the cart. Try giving the cart various initial velocities. Choose a range of initial velocities that give you good video data. Make sure that the wood/cloth block does not begin to slide on the cart before the collision. Try several masses for the cart and the block. Note qualitatively the outcomes when the cart sticks to the end stop.

MEASUREMENT

Make the measurements that you need to check the prediction. Because you are dealing with friction, it is especially important that you repeat each measurement several times under the same conditions to see if it is reproducible.

ANALYSIS

Make a graph of the distance the block travels as a function of the cart's initial speed. Does this result depend on the mass of the block or the mass of the cart? If the graph is not linear, graph the *distance vs. some power of the speed* to produce a linear graph (see the appendix **Review of Graphs**). (Use your prediction to guess which power of speed to use.) What is the meaning of the slope of that line?

CONCLUSION

Do your results agree with your predictions? What are the limitations on the accuracy of your measurements and analysis? As a check, determine the coefficient of kinetic friction between the block and the cart from your results. Is it reasonable?

Does the distance that the crate slides depend on the mass of the forklift, or the mass of the crate? If the sliding distance varies linearly with some power of the forklift's initial speed, what is that power? What would you tell forklift drivers about the effect of doubling their speed? In a sentence or two, relate this result to conservation of energy.

1301 LAB 3 PROBLEM 1: FORCE AND MOTION

You are a volunteer in the city's children's summer program. In one activity the children build and race model cars along a level surface. To give each car a fair start, another volunteer builds a special launcher with a string attached to the car at one end. The string passes over a pulley and from its other end hangs a block. The car starts from rest when the block is allowed to fall. After the block hits the ground, the string no longer exerts a force on the car and it continues along the track. You decide to calculate how the launch velocity of the car depends on the mass of the car, the mass of the block, and the distance the block falls. You hope to use the calculation to impress other volunteers by predicting the winner of each race.

Read: Mazur Chapter 8.

EQUIPMENT

You have a meterstick, stopwatch, mass set, cart masses, a pulley & table clamp, string and the video analysis equipment.
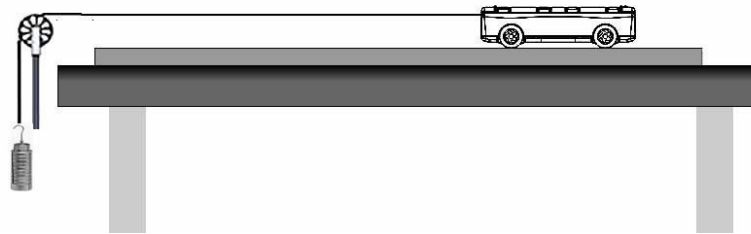
Released from rest, a cart is pulled along a level track by a hanging mass as shown. You can vary the hanging mass and the mass of the cart which are connected by a light string. The mass falls through a height shorter than the track's length.

WARM UP

To figure out your prediction, it is useful to have an organized problem-solving strategy such as the one outlined in the following questions. You might also find the Problem Solving techniques in the Competent Problem Solver useful.

1. Make three sketches of the problem situation, one for each of three instants: when the cart starts from rest, just <u>before</u> object A hits the floor, and just <u>after</u> object A hits the floor. Draw vectors to show the directions and relative magnitudes of the two objects' velocities and accelerations at each instant. Draw vectors to show all of the forces on object A and the cart at each instant. Assign appropriate symbols to all of the quantities describing the motion and the forces. If two quantities have the same magnitude, use the same symbol but write down your justification for doing so. (For example, the cart and object A have the same magnitude of velocity when the cart is pulled by the string. Explain why.) Decide on your coordinate system and draw it.

2. The "known" quantities in this problem are the mass of object A, the mass of the cart, and the height above the floor where object A is released. Assign a symbol to each known quantity. Identify all the unknown quantities. What is the relationship between what you

really want to know (the velocity of the cart after object A hits the floor) and what you can calculate (the velocity of the cart just before object A hits the floor)?

3. Identify and write the physics principles you will use to solve the problem. (Hint: forces determine the objects' accelerations so Newton's 2nd Law may be useful. You need to relate the magnitudes of forces on different objects to one another, so Newton's 3rd Law is probably also useful. Will you need any kinematics principles?) Write down any assumptions you have made which are necessary to solve the problem and justified by the physical situation. (For example, why will it be reasonable to ignore frictional forces in this situation?)

4. Draw one free-body diagram for object A, and a separate one for the cart after they start accelerating. Check to see if any of these forces are related by Newton's 3rd Law (Third Law Pairs). Draw the acceleration vector for the object next to its free-body diagram. Next, draw two separate coordinate systems; place vectors to represent each force acting on the cart on one coordinate system, and those acting on Object A on the second one (force diagrams). (The origin (tail) of each vector should be the origin of the coordinate system.) For each force diagram, write down Newton's 2nd law along each axis of the coordinate system. Make sure all of your signs are correct in the Newton's 2nd law equations. (For example, if the acceleration of the cart is in the + direction, is the acceleration of object A + or -? Your answer will depend on how you define your coordinate system.)

5. You are interested in the final velocity of the cart, but Newton's 2nd Law only gives you its acceleration; write down any kinematics equations which are appropriate to this situation. Is the acceleration of each object constant, or does it vary while object A falls?

6. Write down an equation, from those you have collected in steps 4 and 5 above, which relates what you want to know (the velocity of the cart just before object A hits the ground) to a quantity you either know or can find out (the acceleration of the cart and the time from the start until just before object A hits the floor). Now you have two new unknowns (acceleration and time). Choose one of these unknowns and write down a new equation (again from those collected in steps 4 and 5) which relates it to another quantity you either know or can find out (distance object A falls). If you have generated no additional unknowns, go back to determine the other original unknown (acceleration). Write down a new equation that relates the acceleration of the cart to other quantities you either know or can find (forces on the cart). Continue this process until you generate no new unknowns. At that time you should have as many equations as unknowns.

7. Solve your mathematics to give the prediction.

   Make a graph of the cart's velocity <u>after object A has hit the floor</u> as a function of the mass of object A, keeping constant the cart mass and the height through which object A falls.

   Make a graph of the cart's velocity <u>after object A has hit the floor</u> as a function of the mass of the cart, keeping constant the mass of object A and the height through which object A falls.

   Make a graph of the cart's velocity <u>after object A has hit the floor</u> as a function of the distance object A falls, keeping constant the cart mass and the mass of object A.

8. Does the shape of each graph make sense to you? Explain your reasoning.

Calculate the cart's velocity **after object A has hit the floor.** Express it as an equation, in terms of quantities mentioned in the problem, and draw graphs to show how the velocity changes with each variable.

EXPLORATION

Adjust the length of the string such that object A hits the floor well before the cart runs out of track. You will be analyzing a video of the cart *after* object A has hit the floor. Adjust the string length to give you a video that is long enough to allow you to analyze several frames of motion.

Choose a mass for the cart and find a useful range of masses for object A that allows the cart to achieve a reliably measurable velocity before object A hits the floor. Practice catching the cart <u>before</u> it hits the end stop on the track. Make sure that the assumptions for your prediction are good for the situation in which you are making the measurement. Use your prediction to determine if your choice of masses will allow you to measure the effect that you are looking for. If not, choose different masses.

Choose a mass for object A and find a useful range of masses for the cart.

Now choose a mass for object A and one for the cart and find a useful range of falling distances for object A.

Write down your measurement plan. (Hint: What do you need to measure with video analysis? Do you need video of the cart? Do you need video of object A?)

MEASUREMENT

Carry out the measurement plan you determined in the Exploration section.

Complete the entire analysis of one case before making videos and measurements of the next case.

Make sure you measure and record the masses of the cart and object A (with uncertainties). Record the height through which object A falls and the time it takes to fall (measured with the stopwatch).

ANALYSIS

Determine the cart's velocity just after object A hits the floor from your video.

From the time and distance object A fell in each trial, calculate the cart's velocity just after object A hits the floor. Compare this value to the velocity you measured from the video. Are

they consistent with each other? What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

How does the velocity from your prediction equation compare with the two *measured* velocities (measured with video analysis, and also with stopwatch / meter stick measurements) compare in each case? Did your measurements agree with your initial prediction? If not, why?

Does the launch velocity of the car depend on its mass? The mass of the block? The distance the block falls?

If the same mass block falls through the same distance, but you change the mass of the cart, does the force the string exerts on the cart change? Is the force of the string on object A *always* equal to the weight of object A? Is it *ever* equal to the weight of object A? Explain your reasoning.

1301 LAB 3 PROBLEM 2: FORCES IN EQUILIBRIUM

You have a summer job with a research group studying the ecology of a rainforest in South America. To avoid walking on the delicate rainforest floor, the team members walk along a rope walkway that the local inhabitants have strung from tree to tree through the forest canopy. Your supervisor is concerned about the maximum amount of equipment each team member should carry to safely walk from tree to tree. If the walkway sags too much, the team member could be in danger, not to mention possible damage to the rainforest floor. You are assigned to set the load standards.

Each end of the rope supporting the walkway goes over a branch and then is attached to a large weight hanging down. You need to determine how the sag of the walkway is related to the mass of a team member plus equipment when they are at the center of the walkway between two trees. To check your calculation, you decide to model the situation using the equipment shown below.

Read: Mazur Chapter 8.

EQUIPMENT

You have a meterstick, two pulleys, two table clamps, string and three mass sets.
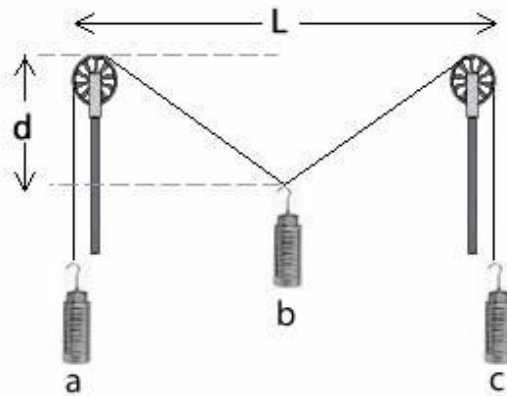
The system consists of a central object, B, suspended halfway between two pulleys by a string. The whole system is in equilibrium. The counterweight objects A and C, which have the same mass, allow you to determine the force exerted on the central object by the string.

WARM UP

It is useful to have an organized problem-solving strategy such as the one outlined in the following questions. You can refer to the problem **Force and Motion** if needed (where a more detailed set of Warm up questions is provided) to solve this problem.

1. Draw a sketch of the setup. Draw vectors that represent the forces on objects A, B, C, and point P. Use trigonometry to show how the vertical displacement of object B is related to the horizontal distance between the two pulleys and the angle that the string between the two pulleys sags below the horizontal.

2. The "known" (measurable) quantities in this problem are L, m and M; the unknown quantity is the vertical displacement of object B.

3. Write down the acceleration for each object. Draw separate force diagrams for objects A, B, C and for point P (if you need help, see your text). Use Newton's third law to identify pairs of forces with equal magnitude. What assumptions are you making?

Which angles between your force vectors and your horizontal coordinate axis are the same as the angle between the strings and the horizontal?

4. For each force diagram, write Newton's second law along each coordinate axis.

5. Solve your equations to predict how the vertical displacement of object B depends on its mass (M), the mass (m) of objects A and C, and the horizontal distance between the two pulleys (L). Use this resulting equation to make a graph of how the vertical displacement changes as a function of the mass of object B.

6. From your resulting equation, analyze what is the limit of mass (M) of object B corresponding to the fixed mass (m) of object A and C. What will happen if M>2m?

<div align="center">

PREDICTION

</div>

Write an equation for the vertical displacement of the central object B in terms of the horizontal distance between the two pulleys (L), the mass (M) of object B, and the mass (m) of objects A and C.

<div align="center">

EXPLORATION

</div>

Start with just the string suspended between the pulleys (no central object), so that the string looks horizontal. Attach a central object and observe how the string sags. Decide on the origin from which you will measure the vertical position of the object.

Try changing the mass of objects A and C (keep them equal for the measurements but you will want to explore the case where they are not equal).

Do the pulleys behave in a frictionless way for the entire range of weights you will use? How can you determine if the assumption of frictionless pulleys is a good one? Add mass to the central object to decide what increments of mass will give a good range of values for the measurement. Decide how measurements you will need to make.

<div align="center">

MEASUREMENT

</div>

Measure the vertical position of the central object as you increase its mass. Make a table and record your measurements with uncertainties.

ANALYSIS

Graph the *measured* vertical displacement of the central object as a function of its mass. On the same graph, plot the *predicted* vertical displacement.

Where do the two curves match? Are there places where the two curves start to diverge from one another? What does this tell you about the system?

What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

What will you report to your supervisor? How does the vertical displacement of an object suspended on a string between two pulleys depend on the mass of that object? Did your measurements of the vertical displacement of object B agree with your predictions? If not, why? State your result in the most general terms supported by your analysis.

What information would you need to apply your calculation to the walkway through the rainforest?

Estimate reasonable values for the information you need, and solve the problem for the walkway over the rainforest.

1301 LAB 3 PROBLEM 3: FRICTIONAL FORCE

You have joined a team trying to win a solar powered car race and have been asked to investigate the effect of friction on the strategy of the race. In any race, sometimes the car coasts and sometimes it speeds up. One of your team has suggested that the frictional force is larger when a force causes an object to speed up than when it coasts and slows down "naturally" because of friction. Do you agree? You suggest making a laboratory model to measure the frictional force when it is speeding up and when it is coasting. You can't measure force directly; to make the model useful you must calculate how *measurable* quantities will be affected by the friction force. Your model consists of a cart pulled along a level track by a light string. The string passes over a pulley and is tied to some weights hanging down. After the weights hit the ground, the cart continues to coast along the track. A pad between the cart and the track provides a variable friction force.

Read: Mazur Chapter 10, especially Section 10.4.

EQUIPMENT

You have a cart, track, meterstick, mass set, stopwatch, pulley & table clamp, cart masses and video equipment. You can change the hanging mass and the cart. A small bolt with a Velcro pad is the friction accessory. It screws into the bottom of the cart.
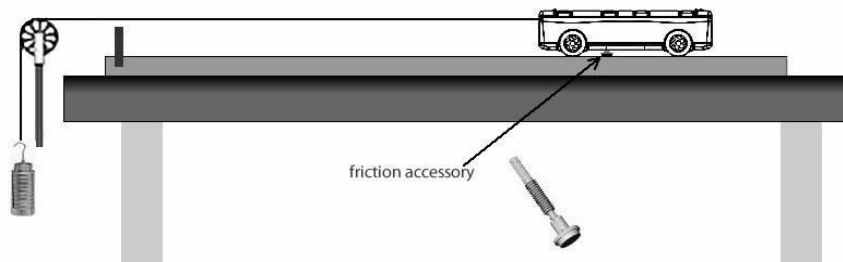
friction accessory

WARM UP

It is useful to have an organized problem-solving strategy such as the one outlined in the following questions. You can refer to the problem **Force and Motion** if needed (where a more detailed set of Warm up questions is provided) to solve this problem.

1. Make a drawing of the problem situation while the cart's speed is increasing, and another one while the cart's speed is decreasing. Draw vectors for each drawing to represent all quantities that describe the *motions* of the block and the cart and the *forces* acting on them. Assign appropriate symbols to each quantity. If two quantities have the same magnitude, use the same symbol. Choose a coordinate system and draw it.

2. List the "known" (controlled by you)_ and "unknown" (to be measured or calculated) quantities in this problem.

3. Write down what principles of Physics you will use to solve the problem. Will you need any of the principles of kinematics? Write down any assumptions you have made that are necessary to solve the problem and are justified by the physical situation.

4. Start with the time interval in which the string exerts a force on the cart (before object A hits the floor). Draw separate free-body and force diagrams for object A and for the cart after they start accelerating. Check to see if any force pairs are related by Newton's 3rd Law. For each force diagram (one for the car and one for object A), write down Newton's 2nd law along each axis of the coordinate system. Be sure all signs are correct.

5. Write down an equation, from those you have collected in step 4 above, that relates what you want to know (the frictional force on the cart) to a quantity you either know or can find out (the acceleration of the cart). Is the force the string exerts on the cart equal to, greater than, or less than the gravitational pull on object A? Explain. Solve your equations for the frictional force on the cart in terms of the masses of the cart, the mass of object A, and the acceleration of the cart.

6. Now deal with the time interval in which the string does not exert a force on the cart (after object A hits the floor). Draw a free-body and force diagram for the cart. Write down Newton's 2nd law along each axis of the coordinate system. Be sure your signs are correct. Solve your equation for the frictional force on the cart in terms of the masses of the cart, the mass of object A, and the acceleration of the cart. You can now determine the frictional force on the cart for each case by measuring the acceleration of the cart.

PREDICTION

Express the frictional force on the cart in terms of quantities that you can measure in the experiment. Make an educated guess about the relationship between the frictional forces in the two situations.

EXPLORATION

Adjust the length of the string such that object A hits the floor well before the cart runs out of track. You will be analyzing a video of the cart both *before and after* object A has hit the floor. Consider how to distinguish these two cases in the same video.

Choose a mass for the cart and find a mass for object A that allows you to reliably measure the cart's acceleration both *before* and *after* object A hits the floor. Because you are comparing the case of the string pulling on the cart with the case of the string not pulling on the cart, make sure the force of the string on the cart is as large as possible. Practice catching the cart <u>before</u> it hits the end stop on the track. Use your prediction to determine if your choice of masses will allow you to measure the effect you are looking for. If not, choose different masses.

Write down your measurement plan. (Do you need video of the cart? Do you need video of object A?)

MEASUREMENT

Carry out the measurement plan you determined in the Exploration section.

Measure and record the mass of the cart and object A (with uncertainties). Record the height through which object A (the mass hanger) falls and the time it takes to fall. Make enough measurements to convince yourself and others of your conclusion.

ANALYSIS

Using the height and time of object A's fall for each trial, calculate the cart's acceleration *before* object A hits the floor. Use the video to determine the cart's acceleration *before* and *after* object A. Is the "before" acceleration from the video consistent with the one you calculate based on time and height of fall?

Use acceleration and determine the friction force before and after object A hits the floor. What are the limitations on the accuracy of your measurements and analysis?

CONCLUSION

Was the frictional force the same whether or not the string exerted a force on it? Does this agree with your initial prediction? If not, why?

1301 LAB 3 PROBLEM 4: NORMAL AND KINETIC FRICTIONAL FORCE I

You work for a consulting firm with contracts to test the mechanical properties of different materials. A customer wants you to determine the coefficient of kinetic friction for wood on aluminum. You decide to measure the coefficient of kinetic friction by graphing the frictional force as a function of the normal force when a wood block slides down an aluminum track. The coefficient of kinetic friction is the slope of that graph. Because there is measurement uncertainty no matter how you do the measurement, you decide to vary the normal force in two different ways. You divide your group into two teams. The other team will vary the normal force by changing the angle of incline of the track (**Normal and Kinetic Frictional Force II**). *Your team will vary the normal force by changing the mass of the block.*

Read: Mazur Sections 10.1–4.

EQUIPMENT

You have wooden blocks and either an aluminum plane, or track, to make an incline. You also have a friction block with felt and wood sides, masses to tape to the block, a meterstick, stopwatch and video analysis equipment.
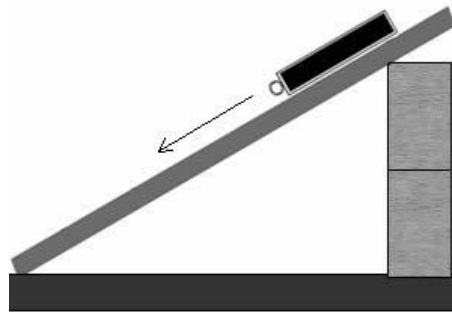
WARM UP

To figure out your prediction you must determine how to calculate the normal force and the kinetic frictional force from quantities you can measure in this problem. It is useful to have an organized problem-solving strategy such as the one outlined in the following questions.

1. What do you expect for the shape of a graph of kinetic friction force vs. normal force? What do you expect for the slope?

2. Make a drawing of the problem situation similar to the one in the Equipment section. Draw vectors to represent all quantities that describe the motion of the block and the forces on it. What measurements can you make with a meter stick to determine the angle of incline? Choose a coordinate system. What is the reason for using the coordinate system you picked?

3. What measurements can you make to enable you to calculate the kinetic frictional force on the block? What measurements can you make to enable you to calculate the normal force on the block? Do you expect the kinetic frictional force the track exerts on the wooden block to **increase**, **decrease**, or **stay the same** as the normal force on the wooden block increases? Explain your reasoning.

4. Draw a free-body diagram of the wooden block as it slides down the aluminum track. Draw the acceleration vector for the block near the free-body diagram. Transfer the force vectors to your coordinate system. What angles between your force vectors and your coordinate axes are the same as the angle between the aluminum track and the table? Determine all of the angles between the force vectors and the coordinate axes.

5. Write down Newton's 2nd Law for the sliding block along each coordinate axis.

6. Using the equations from step 5, determine an equation for the kinetic frictional force in terms of quantities you can measure. Next determine an equation for the normal force in terms of quantities you can measure. In your experiment, the measurable quantities include the mass of the block, the angle of incline and the acceleration of the cart.

PREDICTION

To make sense of your experimental results, you need to determine the relationship between the coefficient of kinetic friction and the quantities that you can measure in experiment. You can look up the accepted value of the coefficient of friction from the Table of Coefficients of Friction near the end of this laboratory. Explain your reasoning.

EXPLORATION

Find an angle at which the wooden block accelerates smoothly down the aluminum track. Try this when the wooden block has different masses on top of it. Select an angle and series of masses that will make your measurements most reliable.

MEASUREMENT

At the chosen angle, take a video of the wooden block's motion. Keep the track fixed when block is sliding down. *Make sure you measure and record that angle. You will need it later.*

Repeat this procedure for different block masses to change the normal force. Make sure the block moves smoothly down the incline for each new mass. Make sure every time you use the same surface of the block to contact the track.

Collect enough data to convince yourself and others of your conclusion about how the kinetic frictional force on the wooden block depends on the normal force on the wooden block.

ANALYSIS

For each block mass and video, calculate the magnitude of the kinetic frictional force from the acceleration. Also determine the normal force on the block.

Graph the magnitude of the kinetic frictional force against the magnitude of the normal force, for a constant angle of incline. Use the graph to find the coefficient of kinetic friction.

CONCLUSION

What is the coefficient of kinetic friction for wood on aluminum? How does this compare to the value on the table? Does the shape of the measured graph match the shape of the predicted graph? Over what range of values does the measured graph best match the predicted graph?

What are the limitations on the accuracy of your measurements and analysis?

If available, compare your value of the coefficient of kinetic friction (with uncertainty) with the value obtained by the different procedure given in the next problem. Are the values consistent? Which way of varying the normal force to measure the coefficient of friction do you think is better? Why?

1301 LAB 3 PROBLEM 5: NORMAL AND KINETIC FRICTIONAL FORCE II

You work for a consulting firm with contracts to test the mechanical properties of different materials. A customer wants your group to determine the coefficient of kinetic friction for wood on aluminum. You decide to measure the coefficient of kinetic friction by graphing the frictional force as a function of the normal force when a wood block slides down an aluminum track. The coefficient of kinetic friction is the slope of that graph. Because there is experimental measurement uncertainty no matter how you do the measurement, you decide to vary the normal force in two different ways. You divide your group into two teams. The other team will vary the normal force by changing the mass of the block (**Normal and Kinetic Frictional Force I**). *Your team will vary the normal force by changing the angle of incline of the aluminum ramp.*

Read: Mazur, Sections 10.1–4.

EQUIPMENT

You have wooden blocks and either an aluminum plane, or track, to make an incline. You also have a friction block with felt and wood sides, additional blocks to vary the incline, a meterstick, stopwatch and video analysis equipment.
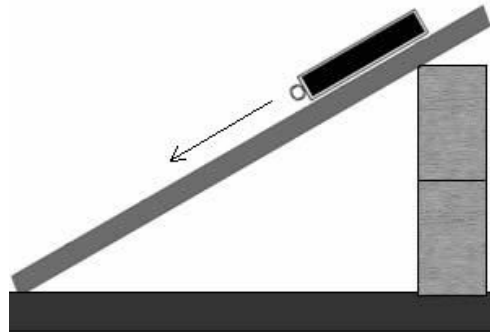
WARM UP

To figure out your prediction you must determine how to calculate the normal force and the kinetic frictional force from the quantities you can measure in this problem. It is useful to have an organized problem-solving strategy such as the one outlined in the following questions.

1. What do you expect for the shape of a graph of kinetic friction force vs. normal force? What do you expect for the slope?

2. Make a drawing of the problem situation similar to the one in the Equipment section. Draw vectors to represent all quantities that describe the motion of the block and the forces on it. What measurements can you make with a meter stick to determine the angle of incline? Choose a coordinate system. What is the reason for using the coordinate system you picked?

3. What measurements can you make to enable you to calculate the kinetic frictional force on the block? What measurements can you make to enable you to calculate the normal force on the block? Do you expect the normal force the track exerts on the wooden block to **increase**, **decrease**, or **stay the same** as the angle of the track increases? How do you expect

the kinetic frictional force the track exerts on the wooden block to change if the normal force changes? Explain your reasoning.

4. Draw a free-body diagram of the wooden block as it slides down the aluminum track. Draw the acceleration vector for the block near the free-body diagram. Transfer the force vectors to your coordinate system. What angles between your force vectors and your coordinate axes are the same as the angle between the aluminum track and the table? Determine all of the angles between the force vectors and the coordinate axes.

5. Write down Newton's 2nd Law for the sliding block along each coordinate axis.

6. Using the equations from step 5, determine an equation for the kinetic frictional force in terms of quantities you can measure. Next determine an equation for the normal force in terms of quantities you can measure. In our experiment, the measurable quantities include the mass of the block, the angle of incline and the acceleration of the cart.

PREDICTIONS

To make sense of your experimental results, you need to determine the relationship between the coefficient of kinetic friction and the quantities that you can measure in experiment. You can look up the accepted value of the coefficient of friction from the Table of Coefficients of Friction near the end of this laboratory. Explain your reasoning.

EXPLORATION

Find a mass for which the wooden block accelerates smoothly down the aluminum track. Try this several different angles of the aluminum track.

Try different block masses. Select a mass that gives you the greatest range of track angles for reliable measurements.

MEASUREMENT

With the chosen block mass fixed, take a video of its motion. *Make sure you measure and record each angle.*

Repeat this procedure for different track angles. Make sure the block moves smoothly down the incline for each angle. Use the same surface of the block with each trial.

Collect enough data to convince yourself and others of your conclusion about how the kinetic frictional force on the wooden block depends on the normal force on the wooden block.

ANALYSIS

For each angle and video, calculate the magnitude of the kinetic frictional force from the acceleration. Also determine the normal force on the block.

Graph the magnitude of the kinetic frictional force against the magnitude of the normal force for a constant block mass. Use the graph to find the coefficient of kinetic friction.

<div style="border:1px solid black; text-align:center;">CONCLUSION</div>

What is the coefficient of kinetic friction for wood on aluminum? How does this compare to the value on the table? Does the shape of the measured graph match the shape of the predicted graph? Over what range of values does the measured graph best match the predicted graph? What are the limitations on the accuracy of your measurements and analysis?

If available, compare your value of the coefficient of kinetic friction (with uncertainty) with the value obtained by the procedure of the preceding problem. Are the values consistent? Which way of varying the normal force to measure the coefficient of friction do you think is better? Why?

1301 LAB 3 PROBLEM 6: MEASURING SPRING CONSTANTS (HOOKE'S LAW)

You are selecting springs for a large antique clock; to determine the forces they will exert in the clock, you need to know their spring constants. The book you have recommends a static approach: hang objects of different weights on the spring and measure the displacements. You have to figure out how to calculate spring constants from your measurements.

Read: Mazur Chapter 8, especially Sections 8.6 and 8.9.

EQUIPMENT

You have springs, a table clamp, rod, meterstick, stopwatch, mass set and video analysis equipment. You should hang the spring from a rod that extends from a table clamp.

WARM UP

To figure out your predictions, it is useful to apply a problem-solving strategy such as the one outlined below:

You hang objects of several different masses on a spring and measure the vertical displacement of each object.

1. Make two sketches of the situation, one before you attach a mass to a spring, and one after a mass is suspended from the spring and is at rest. Draw a coordinate system and label the position where the spring is unstretched, the stretched position, the mass of the object, and the spring constant. Assume the springs are massless.

2. Draw a force diagram for the object hanging *at rest* from the end of the spring. Label the forces. Newton's second law gives the equation of motion for the hanging object. Solve this equation for the spring constant.

3. Use your equation to sketch the displacement (from the unstretched position) versus weight graph for the object hanging at rest from the spring. How is the slope of this graph related to the spring constant?

PREDICTION

Restate the problem. What relationships must you calculate to prepare for your experiment?

EXPLORATION

Select a series of masses that give a usable range of displacements. The smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. **The largest mass should not pull the spring past its elastic limit (about 40 cm). Beyond that**

point you will damage the spring. Decide on a procedure that allows you to measure the displacement of the spring-object system in a consistent manner. Decide how many measurements you will need to make a reliable determination of the spring constant.

MEASUREMENT

Record the masses of different hanging objects and the corresponding displacements.

Analyze your data as you go along so you can decide how many measurements you need to make to determine the spring constant accurately and reliably.

ANALYSIS

Make a graph of displacement versus weight for the object-spring system. From the slope of this graph, calculate the value of the spring constant, including the uncertainty

CONCLUSION

How do the values of the spring constants compare to the stiffness of the spring? Did the springs behave in a linear fashion over the range of the experiment?

1301 LAB 3 PROBLEM 7: FORCE, IMPULSE, AND MOMENTUM

Your 15-year-old is about to get her driving permit and you are concerned about the bumpers on your cars because they are expensive to fix, even after low-speed impacts. You decide to engineer a "5-mph" bumper that will encounter a slow collision with a fixed object using a spring attachment, which will avoid damaging the car's actual bumper. You need to know what kind of spring to purchase for this experiment, so you decide to model the situation using a cart with a spring attached to one end and a fixed end stop. Springs with high spring constant values are very expensive, so you want to find the smallest spring constant you can use.

Read Mazur Chapter 4 and Sections 8.9 and 8.10.

EQUIPMENT

For this lab you have a cart with a spring attached to it, track with end stop, a wood block and a computer with analysis software.
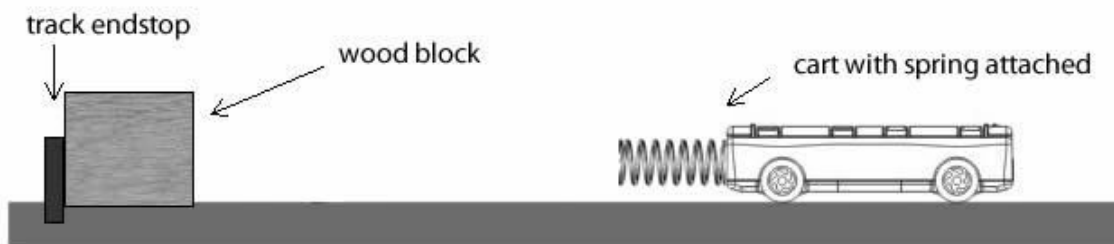
WARM-UP

Note: This lab does not have the same approach as other labs. In this lab, you are assuming that two values are equal, and you are measuring them to see if they really are, instead of measuring an "experimental" value to see if it is equal to a "theoretical" value.

The first question deals with the changing momentum of the system.

1) Draw two pictures of the cart: one before the cart hits the end stop and another one after it has bounced off and is no longer in contact with the spring. Label all kinematic quantities and constants in the system. Use the conservation of momentum to write relationship between the motion before and after the collision. What variables in this relationship are measurable with the equipment you have access to?

The remaining questions deal with the force and impulse of the collision.

2) Draw at least four pictures of the cart during the collision with the spring and the end stop, including two pictures when the spring is being compressed and two as it expands. Include in each picture the amount of compression in the spring and the direction of the force from the spring on the cart.

3) Write down the relationship of how the compression of the spring and the force exerted by the spring on the cart are related in each case. *Be sure to name each force something*

unique ($\vec{F}_1$, $\vec{F}_2$, etc.). Which quantities in this relationship are measurable with your equipment?

4) Using your four pictures, assume that the time between pictures is <u>equal</u> and that the force in the picture is <u>constant</u> until the next picture. Graph the force of the spring versus time for the duration of the collision.

5) Find the total impulse by adding together all of the individual areas under the curve in the force-versus-time graph.

6) What are the assumptions made for this model?

PREDICTION

How do you expect the impulse to compare to the changing momentum? Do you expect the duration of the collision to affect the validity of this comparison?

EXPLORATION

Be very careful with the springs attached to the ends of the carts! They cannot be reattached if they break off. Do not pull on them or bend them side to side.

Try varying the mass of the cart to see how that affects the length of the collision time. Does varying the mass increase or decrease the collision time? Does varying the incoming speed of the cart affect the collision time? Which one has a greater effect? Decide if you would like to minimize or maximize the collision time. Given the assumptions of the problem, which do you think would give more accurate results? *Hint: it is best to maximize the number of data points with the spring in contact with the endstop.*

Think about what quantities you need to obtain from the video and what resolution you will need in the video. Be sure that you will be able to see all the interactions necessary in the video.

Once you have found an acceptable speed and mass of the cart, record a video.

Write down your measurement plan for finding the impulse of the cart as it relates to 1) the changing momentum of the cart and 2) the force over time from the spring. Be sure to include your procedure for finding the spring constant.

If (and only if!) you are unable to complete the procedure for finding the spring constant for your cart during the time allotted, you should assume a value of 355 N/m.

MEASUREMENT

Carry out your measurement plan. Make sure that your video is clear enough to get both the initial and final velocities of the cart and the compression of the spring in each frame.

Think about the quantities that you need to measure and the most efficient way to make these measurements. You will be able to skip many of the "typical" analysis steps ("Prediction x vs t," etc.) in the MotionLab program since you are only using it to acquire data, not to predict behavior.

Discuss how to use the analysis software to find the impulse as it relates to the change in momentum of the cart.

Discuss how to use the procedure from Warm-Up Questions 4 & 5 and the video of the collision to find the impulse as it relates to the force over time.

ANALYSIS

How do the two different methods of finding impulse compare? Which method gives a larger value? Is this what you were expecting? Were the assumptions made for this model reasonable or unreasonable for the situation? Do you see a difference between your collision time measurements and another group's collision time measurement?

CONCLUSION

Did this model provide a sufficient answer to the kind of spring you should purchase? Which impulse calculation would you be doing for this scenario: the force over time or the change in momentum? Which do you think is a better estimate of the actual impulse?

1301 LAB 3 TABLE: COEFFICIENTS OF FRICTION

Surfaces	μs **	μk **
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Steel on lead	0.9	0.9
Copper on cast iron	1.1	0.3
Copper on glass	0.7	0.5
Wood on wood	0.25 - 0.5	0.2
Glass on glass	0.94	0.4
Metal on metal (lubricated)	0.15	0.07
Teflon on Teflon	0.04	0.04
Rubber on concrete	1.0	0.8
Ice on ice	0.1	0.03
Wood on Aluminum		0.25-0.3

** All values are approximate.

1301 LAB 4 PROBLEM 1: PROJECTILE MOTION AND VELOCITY

A toy company has hired you to produce an instructional videotape for would-be jugglers. To plan the videotape, you decide to separately determine how the horizontal and vertical component of a ball's velocity change as it flies through the air. To catch the ball, a juggler must be able to predict its position, so you decide to calculate functions to represent the horizontal and vertical positions of a ball after it is tossed. To check your analysis, you decide to analyze a video of a ball thrown in a manner appropriate to juggling.

Read: Mazur Sections 10.1–3 and 10.6–**7**.

EQUIPMENT

You have a ball, stopwatch, meterstick, camera and a computer.

WARM UP

The following questions will help you determine the details of your prediction and analyze your data.

1. Make a large (about one-half page) sketch of the trajectory of the ball on a coordinate system. Label the horizontal and vertical axes of your coordinate system.

2. On your sketch, draw acceleration vectors for the ball (show directions and relative magnitudes) at five different positions: two when the ball is going up, two when it is going down, and one at its maximum height. Explain your reasoning. Decompose each acceleration vector into its vertical and horizontal components.

3. On your sketch, draw velocity vectors for the ball at the same positions as your acceleration vectors (use a different color). Decompose each velocity vector into vertical and horizontal components. Check that the change of the velocity vector is consistent with the acceleration vector. Explain your reasoning.

4. *On your sketch*, how does the *horizontal* acceleration change with time? How does it compare to the gravitational acceleration? Write an equation giving the ball's horizontal acceleration as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

5. *On your sketch*, how does the ball's horizontal velocity change with time? Is this consistent with your statements about the ball's acceleration from the previous question? Write an equation for the ball's horizontal velocity as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

6. *Based on the equation of the ball's horizontal velocity*, write an equation for the ball's horizontal position as a function of time. Graph this equation. If there are constants in your equation,

what kinematic quantities do they represent? How would you determine these constants from your graph?

7. *On your sketch*, how does the ball's vertical acceleration change with time? How does it compare to the gravitational acceleration? Write an equation giving the ball's vertical acceleration as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

8. *On your sketch*, how does the ball's vertical velocity change with time? Is this consistent with your statements about the ball's acceleration questioning the previous question? Write an equation for the ball's vertical velocity as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

9. *Based on the equation describing the ball's vertical velocity*, write an equation for the ball's vertical position as a function of time. Graph this equation. If there are constants in your equation, what kinematic quantities do they represent? How would you determine these constants from your graph?

PREDICTION

1. Write down equations to describe the horizontal and vertical velocity components of the ball as a function of time. Sketch a graph to represent each equation.

    *Do you think the **horizontal** component of the object's velocity **changes** during its flight? If so, how does it change? Or do you think it is **constant** (does not change)? Make your best guess and explain your reasoning. What about the **vertical** component of its velocity?*

2. Write down the equations that describe the horizontal and vertical position of the ball as a function of time. Sketch a graph to represent each equation.

EXPLORATION

Review your lab journal from earlier problems.

Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice throwing the ball until you can get the ball's motion **after** it leaves your hand to reliably fill the video screen. Determine how much time it takes for the ball to travel and estimate the number of video points you will get in that time. Do you have enough points to make the measurement? Adjust the camera position to get enough data points.

Although you could calibrate on the ball, you might have better results calibrating on a larger object. For calibration purposes, you can hold an object of known length *in the plane of motion* of the ball, near the center of the ball's trajectory. Where you place your reference object does make a difference in your results. Check your video image when you put the reference object

close to the camera and then further away. What do you notice about the size of the reference object in the video image? The best place to put the reference object to determine the distance scale is at the position of the falling ball.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see some blurring of the image. You can adjust the camera settings to give a discrete image.

Write down your measurement plan.

MEASUREMENT

Make a video of the ball being tossed. Make sure you have enough useful frames for your analysis.

Take the position of the ball in enough frames of the video so that you have the sufficient data to accomplish your analysis. Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

ANALYSIS

Choose a function to represent the *horizontal position vs. time graph* and another for the *vertical position vs. time graph*. How can you estimate the values of the constants of the functions from the graph? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent?

Choose a function to represent the *velocity vs. time graph* for each component of the velocity. How can you calculate the values of the constants of these functions from the functions representing the *position vs. time graphs*? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Determine the launch velocity of the ball from this graph. Is this value reasonable? Determine the velocity of the ball at its highest point. Is this value reasonable?

From the *velocity vs. time graphs* determine the acceleration of the ball independently for each component of the motion. Use the functions representing the *velocity vs. time graph* for each component to calculate each component of the ball's acceleration as a function of time. Is the acceleration constant from just after launch to just before the ball is caught? What is its direction? Determine the magnitude of the ball's acceleration at its highest point. Is this value reasonable?

<div align="center">

CONCLUSION

</div>

Did your measurements agree with your initial predictions? Why or why not? Did your measurements agree with those taken by other groups? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

How do the horizontal components of a juggled ball's velocity and position depend on time? How do the vertical components of a juggled ball's velocity and position depend on time? State your results in the most general terms supported by your analysis. At what position does the ball have the minimum velocity? Maximum velocity?

1301 LAB 4 PROBLEM 2: BOUNCING

You work for NASA designing a low-cost landing system for a Mars mission. The payload will be surrounded by padding and dropped onto the surface. When it reaches the surface, it will bounce. The height and the distance of the bounces will get smaller with each bounce so that it finally comes to rest on the surface. Your boss asks you to determine how the ratio of the horizontal distance covered by two successive bounces depends on the ratio of the heights of the two bounces and the ratio of the horizontal components of the initial velocity of the two bounces. After making the calculation you decide to check it in your laboratory on Earth.

Read: Mazur Sections 10.1–3 and 10.6-7.

EQUIPMENT

You have a ball, stopwatch, meterstick, and a computer with a video camera.

WARM UP

The following questions will help you make the prediction.

1. Draw a sketch of the situation, including velocity and acceleration vectors at all relevant times. Decide on a coordinate system. Define the positive and negative directions. During what time interval does the ball have motion that is easiest to calculate? Is the acceleration of the ball during that time interval constant or is it changing? Why? Are the time durations of two successive bounces equal? Why or why not? Label the horizontal distances and the maximum heights for each of the first two bounces. What reasonable assumptions will you probably need to make to solve this problem? How will you check these assumptions with your data?

2. Write down the basic kinematics equations that apply to the time intervals you selected, under the assumptions you have made. Clearly distinguish the equations describing horizontal motion from those describing vertical motion.

3. Write an equation for the horizontal distance the ball travels in the air during the first bounce, in terms of the initial horizontal velocity of the ball, its horizontal acceleration, and the time it stays in the air before reaching the ground again.

4. The equation you just wrote contains the time of flight, which must be re-written in terms of other quantities.   Determine it from the vertical motion of the ball. First, select an equation that gives the ball's vertical position during a bounce as a function of its initial vertical velocity, its vertical acceleration, and the time elapsed since it last touched the ground.

5. The equation in the previous step involves two unknowns, which can both be related to the time of flight. How is the ball's vertical position when it touches the ground at the **end** of its first bounce related to its vertical position when it touched the ground at the **beginning** of its first bounce? Use this relationship and the equation from step 4 to write **one** equation

involving the time of flight. How is the time of flight related to the time it takes for the ball to reach its maximum height for the bounce? Use this relationship and the equation from step 4 to write **another** equation involving the time of flight. Solve these two equations to get an equation expressing the time of flight as a function of the height of the bounce and the vertical acceleration.

6. Combine the previous steps to get an equation for the horizontal distance of a bounce in terms of the ball's horizontal velocity, the height of the bounce, and the ball's vertical acceleration.

7. Repeat the above process for the next bounce; take the ratio of horizontal distances to get your prediction equation.

PREDICTION

Calculate the ratio asked for by your boss. (Assume that you know the ratio of the heights of the two bounces and the ratio of the horizontal components of the initial velocity for the two bounces.)

Be sure to state your assumptions so your boss can decide if they are reasonable for the Mars mission.

EXPLORATION

Review your lab journal from any previous problem requiring analyzing a video of a falling ball.

Position the camera and adjust it for optimal performance. *Make sure everyone in your group gets the chance to operate the camera and the computer.*

Practice bouncing the ball without spin until you can get at least two full bounces to fill the video screen. Three is better so you can check your results. It will take practice and skill to get a good set of bounces. Everyone in the group should try to determine who is best at throwing the ball.

Determine how much time it takes for the ball to have the number of bounces you will record and estimate the number of video frames you will get in that time. Is that enough to make the measurement? Adjust the camera position to get enough data points.

Although you could calibrate on the ball, you might have better results calibrating on a larger object. Place an object of known length in the plane of motion of the ball, near the center of the ball's trajectory, for calibration purposes. Where you place your reference object does make a difference to your results. Determine the best place to put the reference object for calibration.

Step through the video and determine which part of the ball is easiest to consistently determine. When the ball moves rapidly you may see some blurring of the image. You can adjust the camera settings to give a discrete image.

Write down your measurement plan.

<div style="border:1px solid black; text-align:center;">**MEASUREMENT**</div>

Make a video of the ball being tossed. Make sure you have enough frames to complete a useful analysis.

Take the position of the ball in enough frames of the video so that you have the sufficient data to accomplish your analysis. Make sure you set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the ball travels and total time to determine the maximum and minimum value for each axis before taking data.

<div style="border:1px solid black; text-align:center;">**ANALYSIS**</div>

Analyze the video to get the horizontal distance of two successive bounces, the height of the two bounces, and the horizontal components of the ball's velocity for each bounce. You may wish to calibrate the video independently for each bounce so you can begin your time as close as possible to when the ball leaves the ground. (Alternatively, you may wish to avoid repeating some work with the "Save Session" and "Open Session" commands.) The point where the bounce occurs will usually not correspond to a video frame taken by the camera so some estimation will be necessary to determine this position. (Can you use the "Save Data Table" command to help with this estimation?)

Choose a function to represent the *horizontal position vs. time graph* and another for the *vertical position graph* for the first bounce. How can you estimate the values of the constants of the functions? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? How can you tell where the bounce occurred from each graph? Determine the height and horizontal distance for the first bounce.

Choose a function to represent the *velocity vs. time graph* for each component of the velocity for the first bounce. How can you calculate the values of the constants of these functions from the functions representing the *position vs. time graphs*? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? How can you tell where the bounce occurred from each graph? Determine the initial horizontal velocity of the ball for the first bounce. What is the horizontal and vertical acceleration of the ball between bounces? Does this agree with your expectations?

Repeat this analysis for the second bounce, and the third bounce if possible.

What kinematics quantities are approximately the same for each bounce? How does that simplify your prediction equation?

CONCLUSION

How do your graphs compare to your predictions and warm up questions? What are the limitations on the accuracy of your measurements and analysis?

Will the ratio you calculated be the same on Mars as on Earth? Why?

What additional kinematic quantity, whose value you know, can be determined with the data you have taken to give you some indication of the precision of your measurement? How close is this quantity to its known value?

1301 LAB 4 PROBLEM 3: ACCELERATION AND CIRCULAR MOTION

You have been appointed to a citizen committee investigating the safety of a proposed new ride called "The Spinner" at the Mall of America. The ride consists of seats mounted on each end of a steel beam. For most of the ride, the beam rotates about its center in a horizontal circle at a constant speed. Several committee members insist that a person moving in a circle at constant speed is not accelerating, so there is no need to be concerned about the ride's safety. You disagree and sketch a diagram showing that each component of the velocity of a person on the ride changes as a function of time even though the speed is constant. Then you calculate the magnitude of a person's acceleration. The committee is still skeptical, so you build a model to show that your calculations are correct.

Read: Mazur Section 11.1.

EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown below. You also have a stopwatch, meterstick and the video analysis equipment.
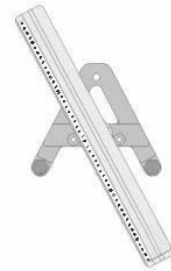
WARM UP

The following questions will help with your prediction and data analysis.

1. Draw the trajectory of an object moving in a horizontal circle with a constant speed. Choose a convenient origin and coordinate axes. Draw the vector that represents the position of the object at some time when it is not along an axis.

2. Write an equation for one component of the position vector as a function of the radius of the circle and the angle the vector makes with one axis of your coordinate system. Calculate how that angle depends on time and the constant angular speed of the object moving in a circle (Hint: see equation 3-19, integrate both sides by time). You now have an equation that gives a component of the position as a function of time. Repeat for the component perpendicular to the first component. Make a graph of each equation. If there are constants in the equations, what do they represent? How would you determine the constants from your graph?

3. From your equations for the components of the position of the object and the definition of velocity, use calculus to write an equation for each component of the object's velocity. Graph each equation. If there are constants in your equations, what do they represent? How would you determine these constants? Compare these graphs to those for the components of the object's position.

4. From your equations for the components of the object's velocity, calculate its speed. Does the speed change with time or is it constant?

5. From your equations for the components of the object's velocity and the definition of acceleration, use calculus to write down the equation for each component of the object's acceleration. Graph each equation. If there are constants in your equations, what do they represent? How would you determine these constants from your graphs? Compare these graphs to those for the components of the object's position.

6. From your equations for the components of the acceleration of the object, calculate the magnitude of the object's acceleration. Is it a function of time or is it constant?

PREDICTION

Calculate the time dependence of the velocity components of an object moving like the ride's seats. Use this to calculate the object's acceleration.

EXPLORATION

Practice spinning the beam at different speeds. How many rotations does the beam make before it slows down appreciably? Use the stopwatch to determine which spin gives the closest approximation to constant speed. At that speed, how many video frames will you get for one rotation? Will this be enough to determine the characteristics of the motion?

Check to see if the spinning beam is level.

Move the apparatus to the floor and adjust the camera tripod so that the camera is directly above the middle of the spinning beam. Practice taking some videos. How will you make sure that you always click on the same position on the beam?

Decide how to calibrate your video.

MEASUREMENT

Take the position of a fixed point on the beam in enough frames of the video so that you have sufficient data to accomplish your analysis -- at least two complete rotations. Set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the object travels and total time to determine the maximum and minimum value for each axis before taking data.

ANALYSIS

Analyze your video by digitizing a single point on the beam for at least two complete revolutions.

Choose a function to represent the graph of *horizontal position vs. time* and another for the graph of *vertical position vs. time*. How can you estimate the values of the constants in the

functions? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell from the graph when a complete rotation occurred?

Choose a function to represent the *velocity vs. time graph* for each component of the velocity. How can you calculate the values of the constants of these functions from the functions representing the position vs. time graphs? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell when a complete rotation occurred from each graph?

Use the equations for the velocity components to calculate the speed of the object. Is the speed constant? How does it compare with your measurements using a stopwatch and meter stick?

Use the equations for the velocity components to calculate the equations that represent the components of the acceleration of the object. Use these components to calculate the magnitude of the total acceleration of the object as a function of time. Is the magnitude of the acceleration a constant? What is the relationship between the acceleration and the speed?

CONCLUSION

How do your graphs compare to your predictions and warm up questions? What are the limitations on the accuracy of your measurements and analysis?

Is it true that the velocity of the object changes with time while the speed remains constant?

Is the instantaneous speed of the object that you calculate from your measurements the same as its average speed that you measure with a stopwatch and meter stick?

Have you shown that an object moving in a circle with a constant speed is always accelerating? Explain.

Compare the magnitude of the acceleration of the object that you calculate from your measurements to the "centripetal acceleration" that you can calculate from the speed and the radius of the object.

1301 LAB 4 PROBLEM 4: A VECTOR APPROACH TO CIRCULAR MOTION

You have a job supervising the construction of a highway. Safety requires that you know what the direction of a car's acceleration is when it moves at constant speed along curves. To check your prediction you decide to model, in the lab, curves that are arcs of circles.

Read: Mazur Section 11.1.

EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown below. You also have a stopwatch, meterstick and the video analysis equipment.

WARM UP

The following questions will help you to make your prediction and analyze your data. These questions assume that you have completed the predictions and warm up questions for the earlier problem **Acceleration and Circular Motion**. If you have not, you should do so before continuing.

1. Make a large (half-page) perpendicular coordinate system. Choose and label your axes. Draw the trajectory of the object moving along a circular road on this coordinate system. Show the positions of your object at equal time intervals around the circle. Choose several points along the trajectory (at least one per quadrant of the circle) and draw the position vector to each of these points. Write down the equations that describe the components of the object's position at each point.

2. From your position equations, calculate the components of the object's velocity at each point. Choose a scale that allows you to draw these components at each point. Add these components (as vectors) to draw the velocity vector at each point. What is the relationship between the velocity vector direction and the direction of the radial vector from the center of the circle?

3. From your velocity equations, calculate the components of the object's acceleration at each point. Choose a scale that allows you to draw these components at each point. Add these components (as vectors) to draw the acceleration vector at each point.   What is the relationship between the acceleration vector direction and the radius of the circle?

PREDICTION

What is the direction of the acceleration vector for an object moving at a constant speed along a circle's arc? Explain your reasoning.

EXPLORATION

Practice spinning the beam at different speeds. How many rotations does the beam make before it slows down appreciably? Use the stopwatch to determine which spin gives the closest approximation to constant speed. At that speed, how many video frames will you get for one rotation? Will this be enough to determine the characteristics of the motion?

Check to see if the spinning beam is level.

Move the apparatus to the floor and adjust the camera tripod so that the camera is directly above the middle of the spinning beam. Practice taking some videos. How will you make sure that you always click on the same position on the beam?

Decide how to calibrate your video.

MEASUREMENT

Take the position of a fixed point on the beam in enough frames of the video so that you have sufficient data to accomplish your analysis -- at least two complete rotations. Set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the object travels and total time to determine the maximum and minimum value for each axis before taking data.

ANALYSIS

Analyze your video by taking the position of a single point on the beam for at least two complete revolutions.

Choose a function to represent the graph of *horizontal position vs. time* and another for the graph of *vertical position vs. time*. How can you estimate the values of the constants in the functions? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell from the graph when a complete rotation occurred?

Choose a function to represent the *velocity vs. time graph* for each component of the velocity. How can you calculate the values of the constants of these functions from the functions representing the position vs. time graphs? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell when a complete rotation occurred from each graph?

Use the equations for the velocity components to calculate the speed of the object. Is the speed constant? How does it compare with your measurements using a stopwatch and meter stick?

Use the equations for the velocity components to calculate the equations that represent the components of the acceleration of the object. Use these components to calculate the magnitude of the total acceleration of the object as a function of time. Is the magnitude of the acceleration a constant? What is the relationship between the acceleration and the speed?

Use the procedure outlined in the Warm-up Questions to analyze your data to get the direction of the acceleration of the object in each quadrant of the circle.

CONCLUSION

How does the direction of the acceleration compare to your prediction? What are the limitations of your measurements and analysis?

What is the direction of the acceleration for a car moving with a constant speed along a curve that forms an arc of a circle? State your result in the most general terms supported by your analysis.

1301 LAB 4 PROBLEM 5: ACCELERATION AND ORBITS

You work with a research group investigating the possibility of extraterrestrial life. Your team is looking at the properties of newly discovered planets orbiting other stars. You have been assigned the task of determining the gravitational force between planets and stars. As a first step, you decide to calculate a planet's acceleration as a function of its orbital radius and period. You assume that it moves in a circle at a constant speed around the star. From previous measurements, you know the radius and period of the orbit.

Read: Mazur Section 11.1.

EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown below. You also have a stopwatch, meterstick and the video analysis equipment.

WARM UP

The following questions will help you to make your prediction and analyze your data. These questions assume that you have completed the predictions and warm up questions for the earlier problem **Acceleration and Circular Motion**. If you have not, you should do so before continuing.

1. Draw the trajectory of an object moving in a circle when its speed is not changing. Draw vectors describing the kinematic quantities of the object. Label the radius of the circle and the relevant kinematic quantities. Choose and label your coordinate axes.

2. Write down the kinematic equations that describe this type of motion. Your equations should include the definition of speed when the speed is constant and the relationship between acceleration and speed for uniform circular motion. You are now ready to plan your mathematical solution.

3. Select an equation identified in step 2, which gives the acceleration in terms of quantities you "know" and additional unknowns. In this problem, you know the radius and the period of the object's motion.

4. If you have additional unknowns, determine one of them by selecting a new equation, identified in step 2, relating that unknown to other quantities. Repeat this step until you have no additional unknowns.

PREDICTION

Calculate the acceleration of an object moving as the planet that you are investigating. Make two graphs: one showing acceleration as a function of radius (for a fixed period) and another showing acceleration as a function of period (for a fixed radius.)

EXPLORATION

Practice spinning the beam at different speeds. How many rotations does the beam make before it slows down appreciably? Use the stopwatch to determine which spin gives the closest approximation to constant speed. At that speed, how many video frames will you get for one rotation? Will this be enough to determine the characteristics of the motion?

Check to see if the spinning beam is level.

Move the apparatus to the floor and adjust the camera tripod so that the camera is directly above the middle of the spinning beam. Practice taking some videos. How will you make sure that you always click on the same position on the beam?

Decide how to calibrate your video.

Decide how you can measure objects at several different positions on the beam while holding the period of rotation constant. How many videos do you need to take for this measurement? Decide how you can measure objects at the same position on the beam for different periods of rotation. How many videos do you need to take for this measurement?

MEASUREMENT

Use your plan from the Exploration section to make your measurements.

Take the position of a fixed point on the beam in enough frames of the video so that you have sufficient data to accomplish your analysis -- at least two complete rotations. Set the scale for the axes of your graph so that you can see the data points as you take them. Use your measurements of total distance the object travels and total time to determine the maximum and minimum value for each axis before taking data.

Make several measurements at different radii and different periods in a range that will give your predictions the most stringent test.

ANALYSIS

Analyze your video by taking the position of a single point on the beam for at least two complete revolutions.

Choose a function to represent the graph of *horizontal position vs. time* and another for the graph of *vertical position vs. time*. How can you estimate the values of the constants in the functions? You can waste a lot of time if you just try to guess the constants. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell from the graph when a complete rotation occurred?

Choose a function to represent the *velocity vs. time graph* for each component of the velocity. How can you calculate the values of the constants of these functions from the functions representing the position vs. time graphs? Check how well this works. You can also estimate the values of the constants from the graph. Just trying to guess the constants can waste a lot of your time. What kinematic quantities do these constants represent? Which are the same for both components? How can you tell when a complete rotation occurred from each graph?

Use the equations for the velocity components to calculate the speed of the object. Is the speed constant? How does it compare with your measurements using a stopwatch and meter stick?

Use the equations for the velocity components to calculate the equations that represent the components of the acceleration of the object. Use these components to calculate the magnitude of the total acceleration of the object as a function of time. Is the magnitude of the acceleration a constant? What is the relationship between the acceleration and the speed?

You can also determine the radius of the object and its period from this data. Make a graph of acceleration as a function of radius for objects with the same period. Make a graph of acceleration as a function of period for objects with the same radius.

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CONCLUSION

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Are your measurements consistent with your predictions? Why or why not? What are the limitations of your measurements and analysis?

1301 LAB 5 PROBLEM 1: ANGULAR SPEED AND LINEAR SPEED

You are working with an engineering group testing equipment that might be used on a satellite. To equalize the heat load from the sun, the satellite will spin about its center. Your task is to determine the forces exerted on delicate measuring equipment when the satellite spins at a constant angular speed. You know that since any object traveling in a circular path must have exerted on it a non-zero net force, that object must be accelerating. As a first step in finding the net force, you decide to calculate the linear speed of any object in the satellite as a function of its distance from the center of the satellite and the satellite's angular speed. From the linear speed of the object in circular motion, you calculate its acceleration. You will test your calculations in a laboratory before launching the satellite.

Read: Mazur Sections 11.1 and 11.4.

EQUIPMENT

You have an apparatus that spins a horizontal platform. A top view of the device is shown to the right. You also have a stopwatch, meterstick and the video analysis equipment.

WARM UP

The following questions will help you to reach your prediction and the analysis of your data.

1. Draw the trajectory of a point on a beam rotating. Choose a coordinate system. Choose a point on that trajectory that is not on a coordinate axis. Draw vectors representing the position, velocity, and acceleration of that point.

2. Write equations for each component of the position vector, as a function of the distance of the point from the axis of rotation and the angle the vector makes with an axis of your coordinate system.   Next, calculate how that angle depends on time and the constant angular speed of the beam. Sketch three graphs, (one for each of these equations) as a function of time. Explain why one of the graphs increases monotonically with time, but the other two oscillate.

3. Using your equations for components of the position of the point, calculate an equation for each component of the velocity of the point. Graph these two equations as a function of time. Compare these graphs to those for the components of the position of the object (when one component of the position is at a maximum, for example, is the same component of the velocity at a maximum value?) Draw these components at the point you

have chosen in your drawing; verify that their vector sum gives the correct direction for the velocity of the point.

4. Use your equations for the point's velocity components to calculate its speed. Does the speed change with time? Should it?

5. Use the equations for the point's velocity components to calculate an equation for each component of the point's acceleration. Graph these two equations as functions of time, and compare to the velocity and position graphs. Verify that the vector sum of the components gives the correct direction for the acceleration of the point you have chosen in your drawing. Use the acceleration components to calculate the magnitude of the acceleration.

6. For comparison, write down the expression for the acceleration of the point as a function of its speed and its distance from the axis of rotation.

PREDICTION

What are you trying to calculate? Restate the problem to clearly identify your objective. Illustrate

EXPLORATION

Practice spinning the beam at different angular speeds. How many rotations does the beam make before it slows down appreciably? Select a range of angular speeds to use in your measurements.

Move the apparatus to the floor and adjust the camera tripod so that the camera is directly above the middle of the spinning beam. Make sure the beam is level. Practice taking some videos. Find the best distance and angle for your video. How will you make sure that you always measure the same position on the beam?

Plan how you will measure the perpendicular components of the velocity to calculate the speed of the point. How will you also use your video to measure the angular speed of the beam?

MEASUREMENT

Take a video of the spinning beam. Be sure you have more than two complete revolution of the beam. For best results, use the beam itself when calibrating your video.

Determine the time it takes for the beam to make two complete revolutions and the distance between the point of interest and the axis of rotation. Set the scale of your axes appropriately so you can see the data as it is taken.

Decide how many different points you will measure to test your prediction. How will you ensure that the angular speed is the same for all of these measurements? How many times will you repeat these measurements using different angular speeds?

ANALYSIS

Analyze your video by following a single point on the beam for at least two complete revolutions. Use the velocity components to determine the direction of the velocity vector. Is it in the expected direction?

Analyze enough different points in the same video to make a graph of speed of a point as a function of distance from the axis of rotation. What quantity does the slope of this graph represent?

Calculate the acceleration of each point and graph the acceleration as a function of the distance from the axis of rotation. What quantity does the slope of this graph represent?

CONCLUSION

How do your results compare to your predictions and the answers to the warm up questions? Did the measured acceleration match the acceleration predicted by your equation from Warm up question 5? Question 6? Explain.

Was the measured linear speed of each point on the beam a constant? Demonstrate this in terms of your fit equations for velocity.

1301 LAB 5 PROBLEM 2: ROTATION AND LINEAR MOTION AT CONSTANT SPEED

While helping a friend take apart a lawn mower engine, you notice the pull cord wraps around a heavy solid disk, "a flywheel," and that disk is attached to a shaft. You know that the flywheel must have at least a minimum angular speed to start the engine. Intrigued by this setup, you wonder how the angular speed of the flywheel is related to the speed of the handle at the end of the pull cord, and you make a prediction. To test your prediction, you make a laboratory model so that you can measure the speed of the cord, the speed of the point on the flywheel where the cord is attached, and the angular speed of the flywheel.

Read: Mazur Sections 11.1 and 11.4.

EQUIPMENT

You have an apparatus that spins a horizontal disk and ring, a track and cart. You also have a stopwatch, meterstick, track endstop, wooden blocks and the video analysis equipment. *Hint: if you turn the blocks at a diagonal while standing on end, the track will rest across them and be nearly level with the disk.*
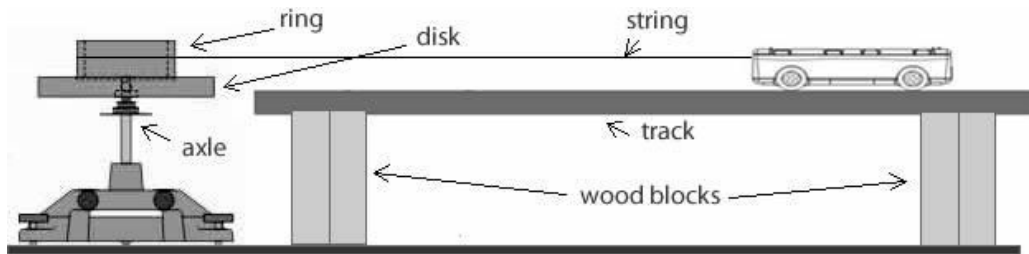
Together, the disk and the ring represent the flywheel. You attach one end of a string to the outside surface of the ring, allowing it to wrap around the ring. The other end of the string is connected to a cart that moves along a level track.

WARM UP

The following questions will help you to reach your prediction.

1. Draw a top view of the system. Draw the velocity and acceleration vectors of a point on the outside edge of the <u>ring</u>. Draw a vector representing the angular velocity of the ring. Draw the velocity and acceleration vectors of a point along the string. Draw the velocity and acceleration vectors of the cart. Write an equation for the relationship between the linear velocity of the point where the string is attached to the ring and the velocity of the cart (if the string is taut).

2. Choose a coordinate system useful for describing the motion of the point where the string is attached to the ring. Select a point on the outside edge of the ring. Write equations for the perpendicular components of the position vector as a function of the distance from the axis of rotation and the angle the vector makes with one axis of your coordinate system. Calculate how that angle depends on time and the constant angular speed of the ring. Sketch three graphs, (one for each of these equations) as a function of time.

3. Using your equations for the components of the position of the point, determine equations for the components of the velocity of the point. Graph these equations as a function of time. Compare these graphs to those representing the components of the position of the object.

4. Use your equations for the components of the velocity of the point to calculate its speed. Is the speed a function of time or is it constant?

5. Now write an equation for the cart's speed as a function of time, assuming the string is taut.

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PREDICTION

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Restate the problem. What are you trying to calculate? Which experimental parameters will be determined by the laboratory equipment, and which ones will you control?

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EXPLORATION

</div>

Try to make the cart move along the track with a constant velocity. (To account for friction, you may need to slant the track slightly. You might even use some quick video analysis to get this right.) Do this before you attach the string.

Try two different ways of having the string and the cart move with the same constant velocity so that the string remains taut. Try various speeds and pick the way that works most consistently for you. If the string goes slack during the measurement you must redo it.
  (1) Gently push the cart and let it go so that the string unwinds from the ring at a constant speed.
  (2) Gently spin the disk and let it go so that the string winds up on the ring at a constant speed.

Where will you place the camera to give the best recording looking down on the system? You will need to get data points for both the motion of the ring and the cart. Try some test runs.

Decide what measurements you need to make to determine the speed of the outer edge of the ring and the speed of the string from the same video.

Outline your measurement plan.

<div align="center">

MEASUREMENT

</div>

Make a video of the motion of the cart **and** the ring for several revolutions of the ring. Measure the radius of the ring. What are the uncertainties in your measurements? *(Review the appropriate appendix sections if you need help determining significant figures and uncertainties.)*

Analyze your video to determine the velocity of the cart and, because the string was taut throughout the measurement, the velocity of the string. Use your measurement of the distance

the cart goes and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it. If the velocity was not constant, adjust your equipment and repeat the measurement.

Analyze the same video to determine the velocity components of the edge of the ring.   Use your measurement of the diameter of the ring and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it.

In addition to finding the angular speed of the ring from the speed of the edge and the radius of the ring, also determine the angular speed directly (using its definition) from either position component of the edge of the ring versus time graph.

ANALYSIS

Use an analysis technique that makes the most efficient use of your data and your time.

Compare the measured speed of the edge of the ring with the measured speed of the cart and thus the string.   Calculate the angular speed of the ring from the measured speed of the edge of the ring and the distance of the edge of the ring from the axis of rotation. Compare that to the angular speed measured directly.

CONCLUSIONS

Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Explain why it is difficult to keep the string taut in this measurement, by considering the forces exerted on each end of the string? Determine the force of the string on the cart and the force of the cart on the string. Determine the force of the string on the ring and the force of the ring on the string. What is the string tension?

1301 LAB 5 PROBLEM 3: ANGULAR AND LINEAR ACCELERATION

You are working in a bioengineering laboratory when the building power fails. An ongoing experiment will be damaged if there is any temperature change. There is a gasoline powered generator on the roof for just such emergencies. You run upstairs and start the generator by pulling on a cord attached to a flywheel. It is such hard work that you begin to design a gravitational powered generator starter. The generator you design has its flywheel as a horizontal disk that is free to rotate about its center. One end of a rope is wound up on a horizontal ring attached to the center of the flywheel. The free end of the rope goes horizontally to the edge of the building roof, passes over a vertical pulley, and then hangs straight down. A heavy block is attached to the hanging end of the rope. When the power fails, the block is released; the rope unrolls from the ring giving the flywheel a large enough angular acceleration to start the generator. To see if this design is feasible you must determine the relationship between the angular acceleration of the flywheel, the downward acceleration of the block, and the radius of the ring. Before putting more effort in the design, you test your idea by building a laboratory model of the device.

Read: Mazur Chapter 11.

EQUIPMENT

You have an apparatus that spins a horizontal disk.   You also have a stopwatch, meterstick, pulley, table clamp, mass set and the video analysis equipment.

The disk represents the flywheel.   A string has one end wrapped around the plastic spool (under the disk) and the other end passing over a vertical pulley lined up with the tangent to the spool. A mass is hung from the free end of the string so it can fall.
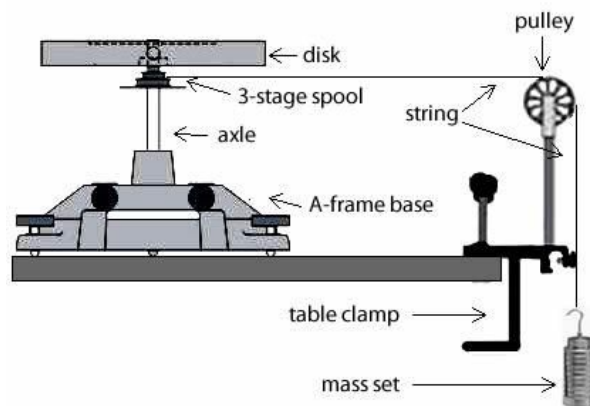
WARM UP

The following questions will help you to reach your prediction and the analysis of your data.

1. Draw a top view of the system. Draw the velocity and acceleration vectors of a point on the outside edge of the spool. Draw a vector representing the angular acceleration of the spool. Draw the velocity and acceleration vectors of a point along the string.

2. Draw a side view of the system. Draw the velocity and acceleration vectors of the hanging object. What is the relationship between the linear acceleration of the string and the acceleration of the hanging object if the string is taut? Do you expect the acceleration of the hanging object to be constant? Explain.

3. Choose a coordinate system useful to describe the motion of the spool. Select a point on the outside edge of the spool. Write equations giving the perpendicular components of the

point's position vector as a function of the distance from the axis of rotation and the angle the vector makes with one axis of your coordinate system. Assume the angular acceleration is constant and that the disk starts from rest. Determine how the angle between the position vector and the coordinate axis depends on time and the angular acceleration of the spool. Sketch three graphs, (one for each of these equations) as a function of time.

4. Using your equations for components of the position of the point, calculate the equations for the components of the velocity of the point. Is the *speed* of this point a function of time or is it constant? Graph these equations as a function of time.

5. Use your equations for the components of the velocity of the point on the edge of the spool to calculate the components of the *acceleration* of that point. From the components of the acceleration, calculate the *square of the total acceleration* of that point. It looks like a mess but it can be simplified to two terms if you can use: $\sin^2(z)+\cos^2(z) = 1$.

6. From step 5, the magnitude acceleration of the point on the edge of the spool has one term that depends on time and another term that does not. Identify the term that depends on time by using the relationship between the angular speed and the angular acceleration for a constant angular acceleration. If you still don't recognize this term, use the relationship among angular speed, linear speed and distance from the axis of rotation. Now identify the relationship between this time-dependent term and the centripetal acceleration.

7. We also can solve the acceleration vector of the point on the edge of the spool into two perpendicular components by another way. One component is the centripetal acceleration and the other component is the tangential acceleration. In step 6, we already identify the centripetal acceleration term from the total acceleration. So now you can recognize the tangential acceleration term. How is the tangential acceleration of the edge of the spool related to the angular acceleration of the spool and the radius of the spool? What is the relationship between the angular acceleration of the spool and the angular acceleration of the disk?

8. How is the tangential acceleration of the edge of the spool related to the acceleration of the string? How is the acceleration of the string related to the acceleration of the hanging object? Explain the relationship between the angular acceleration of the disk and the acceleration of the hanging object.

PREDICTION

Reformulate the problem in your own words to understand its target. What do you need to calculate?

EXPLORATION

Practice gently spinning the system by hand. How long does it take the disk to stop rotating about its central axis? What is the average angular acceleration caused by this friction? Make sure the angular acceleration you use in your measurements is much larger than the one caused by friction.

Find the best way to attach the string to the spool. How much string should you wrap around the spool? How should the pulley be adjusted to allow the string to unwind smoothly from the spool and pass over the pulley? Practice releasing the hanging object and the spool/disk system.

Determine the best mass to use for the hanging object. Try a large range. What mass will give you the smoothest motion? What is the highest angular acceleration? How many useful frames for a single video?

Where will you place the camera to give the best top view recording on the whole system? Since you can't get a video of the falling object and the top of the spinning spool/disk at the same time, attach a piece of tape to the string. The tape will have the same linear motion as the falling object.

Decide what measurements you need to make to determine the angular acceleration of the disk and the acceleration of the string from the same video.

Outline your measurement plan.

MEASUREMENT

Make a video of the motion of the tape on the string **and** the disk for several revolutions. Measure the radius of the spool. What are the uncertainties in your measurements? *(Review the appendix if you need help determining significant figures and uncertainties.)*

Analyze your video to determine the acceleration of the string and hanging object. Use your measurement of the distance and time that the hanging object falls to choose the scale of the graphs so that the data is visible when you take it. Check to see if the acceleration is constant.

Use a stopwatch and meter stick to directly determine the acceleration of the hanging object.

Analyze the same video to determine the velocity components of the edge of the disk. Use your measurement of the diameter of the disk and the time of the motion to choose the scale of the computer graphs so that the data is visible when you take it.

ANALYSIS

From the analysis of the video data for the tape on the string, determine the acceleration of the piece of tape on the string. Compare this acceleration to the hanging object's acceleration determined directly. Be sure to use an analysis technique that makes the most efficient use of your data and your time.

From your video data for the disk, determine if the angular speed of the disk is constant or changes with time.

Use the equations that describe the measured components of the velocity of a point at the edge of the disk to calculate the tangential acceleration of that point and use this tangential acceleration of the edge of the disk to calculate the angular acceleration of the disk (it is also the angular acceleration of spool). You can refer to the Warm up questions.

CONCLUSION

Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Explain why it is not difficult to keep the string taut in this measurement by considering the forces exerted on each end of the string? Determine the pull of the string on the hanging object and the pull of the hanging object on the string, in terms of the acceleration of the hanging object. Determine the force of the string on the spool and the force of the spool on the string. What is the string tension? Is it equal to, greater than, or less than the weight of the hanging object?

1301 LAB 6 PROBLEM 1: MOMENT OF INERTIA OF A COMPLEX SYSTEM

While examining the engine of your friend's snow blower you notice that the starter cord wraps around a cylindrical ring. This ring is fastened to the top of a heavy solid disk, "a flywheel," and that disk is attached to a shaft. You are intrigued by this configuration and decide to determine its moment of inertia. Your friend thinks you can add the moment of inertial by parts to get the moment of inertia of the system. To test this idea you decide to build a laboratory model described below to determine the moment of inertia of a similar system from the acceleration of the hanging weight.

Read: Mazur Chapter 12.

EQUIPMENT

You have an apparatus that spins a horizontal disk and ring. You also have a stopwatch, meterstick, pulley, table clamp, mass set and the video analysis equipment.

The disk and ring share the same rotational axis and represent the flywheel. A string has one end wrapped around the plastic spool (under the disk) and the other end passing over a vertical pulley lined up with the tangent to the spool. A mass is hung from the free end of the string so it can fall past the table, spinning the system.

WARM UP

The following questions will help you to reach your prediction and the analysis of your data.

1. Draw a side view of the equipment. Draw the velocity and acceleration vectors of the weight. Add the tangential velocity and tangential acceleration vectors of the outer edge of the spool. Also, show the angular acceleration of the spool. What are the relationships among the acceleration of the string, the acceleration of the weight, and the tangential acceleration of the outer edge of the spool if the string is taut?

2. To relate the moment of inertia of the system to the acceleration of the weight, you need to consider a dynamics approach (Newton's second law) especially considering the torques exerted on the system. The relationships between rotational and linear kinematics will also be involved.

3. Draw a free-body diagram for the ring/disk/shaft/spool system. Show the locations of the forces acting on that system. Label all the forces. Does this system accelerate? Is there an angular acceleration? Check to see if you have all the forces on your diagram. Which of these forces can exert a torque on the system? Identify the distance from the axis of rotation to the point where each force is exerted on the system. Write down an equation that gives the torque in terms of the distance and the force that causes it. Write down

Newton's second law in its rotational form for this system. Remember that the moment of inertia includes everything in the system that will rotate.

4. Draw a free-body diagram for the hanging weight. Label all the forces acting on it. Does this weight accelerate? Is there an angular acceleration? Check to see if you have included all the forces on your diagram. Write down Newton's second law for the hanging weight. Is the force of the string on the hanging weight equal to the weight of the hanging weight?

5. Can you use Newton's third law to relate pairs of forces shown in different force diagrams?

6. Is there a relationship between the angular acceleration of the ring/disk/shaft/spool system and the acceleration of the hanging weight? To decide, examine the accelerations that you labeled in your drawing of the equipment.

7. Solve your equations for the moment of inertia of the ring/disk/shaft/spool system as a function of the mass of the hanging weight, the acceleration of the hanging weight, and the radius of the spool. Start with the equation containing the quantity you want to know, the moment of inertia of the ring/disk/shaft/spool system. Identify the unknowns in that equation and select equations for each of them from those you have collected. If those equations generate additional unknowns, search your collection for equations that contain them. Continue this process until all unknowns are accounted for. Now solve those equations for your target unknown.

8. For comparison with your experimental results, calculate the moment of inertia of the ring/disk/shaft/spool system using your friend's idea.

PREDICTION

Restate your friend's idea as an equation.

What quantities will you measure in the lab? What relationships do you need to calculate in order to test your friend's ideas in the lab?

EXPLORATION

Practice gently spinning the ring/disk/shaft/spool system by hand. How long does it take the disk to stop rotating about its central axis? What is the average angular acceleration caused by this friction? Make sure the angular acceleration you use in your measurements is much larger than the one caused by friction so that it has a negligible effect on your results.

Find the best way to attach the string to the spool. How much string should you wrap around the spool? How should the pulley be adjusted to allow the string to unwind smoothly from the spool and pass over the pulley? Practice releasing the hanging weight and the ring/disk/shaft/spool system.

Determine the best mass to use for the hanging weight. Try a large range. What mass will give you the smoothest motion?

Decide what measurements you need to make to determine the moment of inertia of the system from your Prediction equation. If any major assumptions are involved in connecting your measurements to the acceleration of the weight, decide on the additional measurements that you need to make to justify them.

Outline your measurement plan. Make some rough measurements to make sure your plan will work.

MEASUREMENT

Follow your measurement plan. What are the uncertainties in your measurements? *(Review the appendix if you need help determining significant figures and uncertainties.)*

Don't forget to make the additional measurements required to determine the moment of inertia of the ring/disk/shaft/spool system from the sum of the moments of inertia of its components. What is the uncertainty in each of the measurements? What effects does the hole, the ball bearings, the groove, and the holes in the edges of the disk have on its moment of inertia? Explain your reasoning.

ANALYSIS

Determine the acceleration of the hanging weight. How does this acceleration compare to what its acceleration would be if you just dropped the weight without attaching it to the string? Explain whether or not this makes sense.

Using your Prediction equation and your measured acceleration, the radius of the spool and the mass of the hanging weight, calculate the moment of inertia (with uncertainty) of the disk/shaft/spool system.

Adding the moments of inertia of the components of the ring/disk/shaft/spool system, calculate the value (with uncertainty) of the moment of inertia of the system. What fraction of the moment of inertia of the system is due to the shaft? The disk? The ring? Explain whether or not this makes sense.

Compare the values of moment of inertia of the system from these two methods

CONCLUSION

Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

1301 LAB 6 PROBLEM 2: MOMENT OF INERTIA ABOUT DIFFERENT AXES

While spinning a coin on a table, you wonder if the coin's moment of inertia spinning on its edge is the same as if it were spinning about an axis through its center and perpendicular to its surface. You do a quick calculation to decide. To test your prediction, you build a laboratory model with a disk that can spin around two different axes, and find the moment of inertia in each configuration by measuring the acceleration of a hanging weight attached to the spinning system by a string.

Read: Mazur Chapter 12.

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EQUIPMENT

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You have an apparatus that spins a disk either about its central axis, or diameter (as shown). You also have a stopwatch, meterstick, pulley, table clamp, mass set and the video analysis equipment.

A string has one end wrapped around the plastic spool (under the disk) and the other end passing over a vertical pulley lined up with the tangent to the spool. A mass is hung from the free end of the string so it can fall past the table, spinning the system.

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WARM UP

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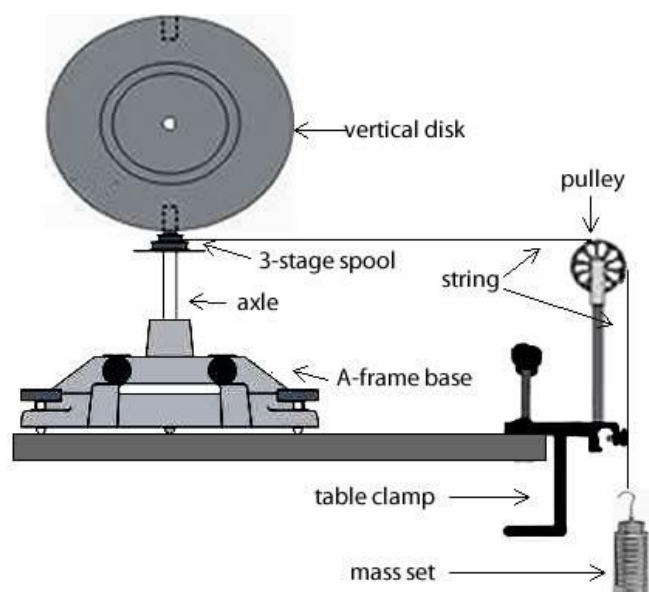
To figure out your prediction, you need to determine how to calculate the rotational inertia of the disk from the quantities you can measure in the laboratory. It is helpful to use a problem solving strategy such as the one outlined below:

If needed, a more detailed set of Warm-up questions are given in the earlier problem, **Moment of Inertia of a Complex System**.

1. Draw a side view of the equipment with all relevant kinematic quantities. Write down any relationships that exist between them. Label all the relevant forces.

2. Determine the basic principles of physics that you will use. Write down your assumptions and check to see if they are reasonable.

3. If you decide to use dynamics, draw a free-body diagram of all the relevant objects. Note the acceleration of the object as a check to see if you have drawn all the forces. Write down Newton's second law for each free-body diagram either in its linear form or its rotational form or both as necessary.

4. Use Newton's third law to relate the forces between two free-body diagrams. If forces are equal give them the same labels.

5. Identify the target quantity you wish to determine. Use the equations collected in steps 1 and 3 to plan a solution for the target.

6. For comparison with your experimental results, calculate the moment of inertia of the disk in each orientation.

PREDICTION

Restate the problem. What are you asked to predict? What relationships do you need to calculate to use the lab model?

EXPLORATION

Practice gently spinning the disk/shaft/spool system by hand. How long does it take the disk to stop rotating about its central axis? How long does it take the disk to stop rotating about its diameter? How will friction affect your measurements?

Find the best way to attach the string to the spool. How much string should you wrap around the spool? How much mass will you attach to the other end of the string? How should the pulley be adjusted to allow the string to unwind smoothly from the spool and pass over the pulley? Practice releasing the hanging weight and the disk/shaft/spool system.

Determine the best mass to use for the hanging weight. Try a large range. What mass will give you the smoothest motion?

Decide what measurements you need to make to determine the moment of inertia of the system from your Prediction equation. If any major assumptions are involved in connecting your measurements to the acceleration of the weight, decide on the additional measurements that you need to make to justify them. If you already have this data in your lab journal you don't need to redo it, just copy it.

Outline your measurement plan. Make some rough measurements to make sure your plan will work.

MEASUREMENT

Follow your measurement plan. What are the uncertainties in your measurements? *(Review the appendix if you need help determining significant figures and uncertainties.)*

Don't forget to make the additional measurements required to determine the moment of inertia of the disk/shaft/spool system by adding all of the moments of inertia of its components. What is the uncertainty of each of the measurements? What effects do the hole, the ball bearings, the groove, and the holes in the edges of the disk have on its moment of inertia? Explain your reasoning.

ANALYSIS

Determine the acceleration of the hanging weight. How does this acceleration compare to its acceleration if you just dropped the weight without attaching it to the string? Explain whether or not this makes sense.

Using your Prediction equation and your measured acceleration, the radius of the spool and the mass of the hanging weight, calculate the moment of inertia (with uncertainty) of the disk/shaft/spool system, for both orientations of the disk.

Adding the moments of inertia of the components of the disk/shaft/spool system, calculate the value (with uncertainty) of the moment of inertia of the system, for both orientations of the disk.

Compare the results from these two methods for both orientations of the disk.

CONCLUSION

How do the measured and predicted values of the disk's moment of inertia compare when the disk rotates about its central axis? When the disk rotates around its diameter?

Is the moment of inertia of a coin rotating around its central axis larger than, smaller than, or the same as its moment of inertia when it is rotating around its diameter? State your results in the most general terms supported by the data.

Did your measurements agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

1301 LAB 6 PROBLEM 3: MOMENT OF INERTIA WITH AN OFF-AXIS RING

You have been hired as a member of a team designing an energy efficient car. The brakes of a traditional car transform the kinetic energy of the car into internal energy of the brake material, resulting in an increased temperature of the brakes. That energy is lost in the sense that it cannot be recovered to power the car. Your task has been to evaluate a new braking system, which transforms the kinetic energy of the car into rotational energy of a flywheel system. The energy of the flywheel can then be used to drive the car. As designed, the flywheel consists of a heavy horizontal disk with an axis of rotation through its center. A metal ring is mounted on the disk but is not centered on the disk. You wonder what effect the off-center ring will have on the motion of the flywheel.

To answer this question, you decide to make a laboratory model to measure the moment of inertia of a ring/disk/shaft/spool system when the ring is off-axis and compare it to the moment of inertia for a system with a ring in the center.

Read: Mazur Chapter 12.

EQUIPMENT

You have an apparatus that spins a horizontal disk and ring. You also have a stopwatch, meterstick, pulley, table clamp, mass set and the video analysis equipment.

The ring is fixed (with tape) off-set from the axis of the disk and represents the flywheel. A string has one end wrapped around the plastic spool (under the disk) and the other end passing over a vertical pulley lined up with the tangent to the spool. A mass is hung from the free end of the string so it can fall past the table, spinning the system.
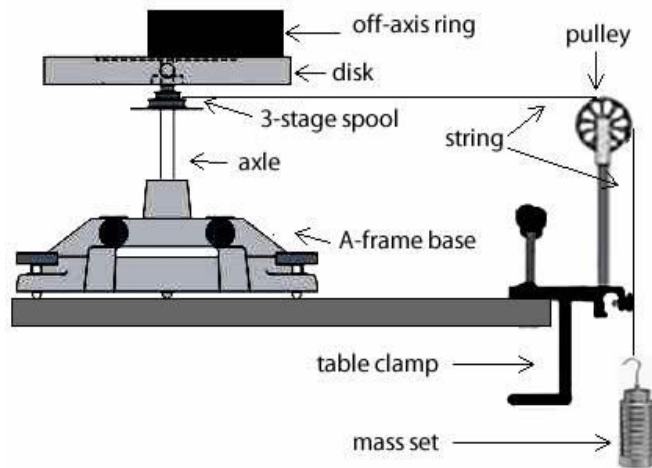
WARM UP

To figure out your prediction, you need to determine how to calculate the rotational inertia of the disk from the quantities you can measure in this problem. It is helpful to use a problem solving strategy such as the one outlined below:

If needed, a more detailed set of Warm-up questions are given in the earlier problem, **Moment of Inertia of a Complex System**.

1. Draw a side view of the equipment with all the relevant kinematics quantities. Write down any relationships that exist between them. Label all the relevant forces.

2. Determine the basic principles of physics that you will use. Write down your assumptions and check to see if they are reasonable.

3. If you decide to use dynamics, draw a free-body diagram of all the relevant objects. Note the acceleration of the object as a check to see if you have drawn all the forces. Write down Newton's second law for each free-body diagram either in its linear form or its rotational form or both as necessary.

4. Use Newton's third law to relate the forces between two free-body diagrams. If forces are equal give them the same labels.

5. Identify the target quantity you wish to determine. Use the equations collected in steps 1 and 3 to plan a solution for the target. If there are more unknowns than equations, reexamine the previous steps to see if there is additional information about the situation. If not, see if one of the unknowns will cancel out.

6. For comparison with your experimental results, calculate the moment of inertia of the disk/ring system in each configuration. The parallel-axis theorem should be helpful.

PREDICTIONS

Restate the problem. What are you asked to predict? What relationships do you need to calculate to use the lab model?

EXPLORATION

THE OFF-AXIS RING IS NOT STABLE BY ITSELF! Be sure to secure the ring to the disk, and be sure that the system is on a stable base.

Practice gently spinning the ring/disk/shaft/spool system by hand. How will friction affect your measurements?

Find the best way to attach the string to the spool. How much string should you wrap around the spool? How much mass will you attach to the other end of the string? How should the pulley be adjusted to allow the string to unwind smoothly from the spool and pass over the pulley? Practice releasing the mass and the ring/disk/shaft/spool system.

Determine the best mass to use for the hanging weight. Try a large range. What mass will give you the smoothest motion?

Decide what measurements you need to make to determine the moment of inertia of the system from your Prediction equation. If any major assumptions are involved in connecting your measurements to the acceleration of the weight, decide on the additional measurements that you need to make to justify them.

Outline your measurement plan. Make some rough measurements to make sure your plan will work.

MEASUREMENT

Follow your measurement plan. What are the uncertainties in your measurements? *(Review the appendix if you need help determining significant figures and uncertainties.)*

Don't forget to make the additional measurements required to determine the moment of inertia of the ring/disk/shaft/spool system from the moments of inertia of its components and the parallel axis theorem. What is the uncertainty in each of the measurements? What effects do the hole, the ball bearings, the groove, and the holes in the edges of the disk have on its moment of inertia? Explain your reasoning.

ANALYSIS

Determine the acceleration of the hanging weight. How does this acceleration compare to its acceleration if you just dropped the weight without attaching it to the string? Explain whether or not this makes sense.

Using your Prediction equation and your measured acceleration, the mass of the hanging weight and the radius of the spool, calculate the moment of inertia (with uncertainty) of the disk/shaft/spool system.

Adding the moments of inertia of the components of the disk/shaft/spool system and applying the parallel axis theorem, calculate the value (with uncertainty) of the moment of inertia of the system.

CONCLUSION

Compare the two values for the moment of inertia of the system *when the ring is off-axis*. Did your measurement agree with your predicted value? Why or why not?

Compare the moments of inertia of the system when the ring is centered on the disk, and when the ring is off-axis.

What effect does the off-center ring have on the moment of inertia of the ring/disk/shaft/spool system? Does the rotational inertia increase, decrease, or stay the same when the ring is moved off-axis?

State your result in the most general terms supported by your analysis. Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

1301 LAB 6 PROBLEM 4: FORCES, TORQUES, AND ENERGY

While examining the manual starter on a snow blower, you wonder why the manufacturer chose to wrap the starter cord around a smaller ring that is fastened to a spool under the flywheel instead of around the flywheel itself. When starting a snow blower, you know you need the starter system to spin as fast as possible when you pull the starter cord. Your friend suggests that the flywheel might spin faster, even if you do the same amount of work when you pull on the handle, if the cord is wrapped around a smaller diameter. You notice that the handle is not very light. To see whether this idea is correct, you decide to calculate the final angular speed of the flywheel after pulling on the handle for a fixed distance with a fixed force, as a function of the spool's radius. To test your calculation, you set up a laboratory model of the flywheel starter assembly. Unfortunately, it is difficult to keep the force on the handle consistent across trials, so in the lab you attach a hanging mass to one end of the cord.

Read: Mazur Chapter 12.

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EQUIPMENT

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You have an apparatus that spins a horizontal disk and ring. You also have a stopwatch, meterstick, pulley, table clamp, mass set and the video analysis equipment.

The disk and ring share the same rotational axis and represent the flywheel. A string has one end wrapped around the plastic spool (under the disk) and the other end passing over a vertical pulley lined up with the tangent to the spool. The spool has 3 different diameters to choose from, or the ring or disk can be used. A mass is hung from the free end of the string so it can fall past the table, spinning the system.

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WARM UP

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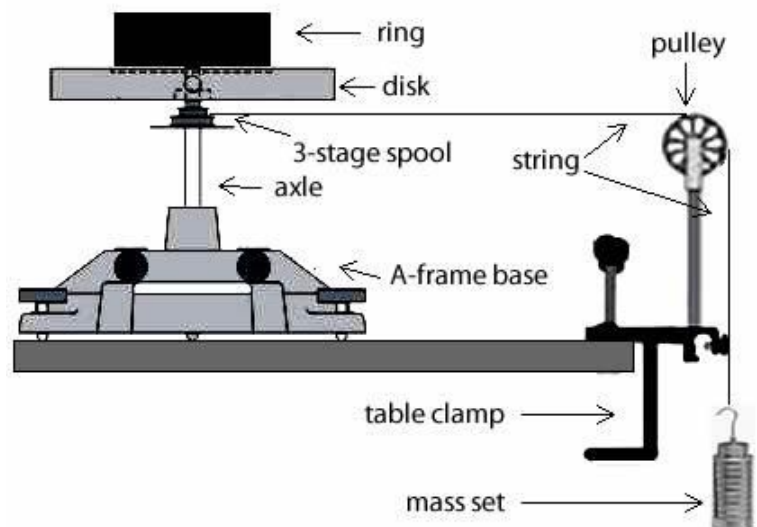
To figure out your prediction, it is useful to use a problem-solving strategy such as the one outlined below:

1. Make two side view drawings of the situation (similar to the diagram in the Equipment section), one just as the hanging mass is released, and one just as the hanging mass reaches the ground (but before it hits). Label all relevant kinematic quantities and write down the relationships that exist between them. What is the relationship between the velocity of the

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hanging weight and the angular velocity of the ring/disk/shaft/spool system? Label all the relevant forces.

2. Determine the basic principles of physics that you will use and how you will use them. Determine your system. Are any objects from outside your system interacting with your system? Write down your assumptions and check to see if they are reasonable. How will you ensure that your equipment always pulls the cord through the same length when it is wrapped around different diameters?

3. Use dynamics to determine what you must do to the hanging weight to get the force for each diameter around which the cord is wrapped. Draw a free-body diagram of all relevant objects. Note the acceleration of the object in the free-body diagram as a check to see if you have drawn all the forces. Write down Newton's second law for each free-body diagram either in its linear form or its rotational form or both as necessary. Use Newton's third law to relate the forces between two free-body diagrams. If forces are equal, give them the same symbol. Solve your equations for the force that the string exerts.

4. Use the conservation of energy to determine the final angular speed of the rotating objects. Define your system and write the conservation of energy equation for this situation:

   What is the energy of the system as the hanging weight is released? What is its energy just before the hanging weight hits the floor? Is any significant energy transferred to or from the system? If so, can you determine it or redefine your system so that there is no transfer? Is any significant energy changed into internal energy of the system? If so, can you determine it or redefine your system so that there is no internal energy change?

5. Identify the target quantity you wish to determine. Use the equations collected in steps 1, 3, and 4 to plan a solution for the target. If there are more unknowns than equations, re-examine the previous steps to see if there is additional information about the situation that can be expressed in an addition equation. If not, see if one of the unknowns will cancel out.

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PREDICTION

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Restate the problem. What quantities do you need to calculate to test your idea?

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EXPLORATION

</div>

Practice gently spinning the ring/disk/shaft/spool system by hand. How will friction affect your measurements?

Find the best way to attach the string to the spool, disk, or ring. How much string should you wrap around each? How should the pulley be adjusted to allow the string to unwind smoothly and pass over the pulley in each case? You may need to reposition the pulley when changing the position where the cord wraps. Practice releasing the weight and the ring/disk/shaft/spool system for each case.

Determine the best mass to use for the hanging weight. Remember this mass will be applied in every case. Try a large range. What mass range will give you the smoothest motion?

Is the time it takes the hanging weight to fall different for the different situations? How will you determine the time taken for it to fall? Determine a good setup for each case (string wrapped around the ring, the disk, or the spool).

Decide what measurements you need to make to check your prediction. If any major assumptions are used in your calculations, decide on the additional measurements that you need to make to justify them. If you already have this data in your lab journal you don't need to redo it, just copy it.

Outline your measurement plan. Make some rough measurements to be sure your plan will work.

MEASUREMENT

Follow your measurement plan. What are the uncertainties in your measurements?

ANALYSIS

Determine the final angular velocity of the ring/disk/shaft/spool system for each case after the weight hits the ground. How is this angular velocity related to the final velocity of the hanging weight? If your calculation incorporates any assumptions, make sure you justify these assumptions based on data that you have analyzed.

CONCLUSION

In each case, how do your measured and predicted values for the final angular velocity of the system compare?

Of the three places you attached the string, which produced the highest final angular velocity? Did your measurements agree with your initial prediction? Why or why not? What are the limitations on the accuracy of your measurements?

Given your results, how much does it matter where the starter cord is attached? Why do you think the manufacturer chose to wrap the cord around the ring? Explain your answers.

Can you make a qualitative argument, in terms of energy conservation, to support your conclusions?

1301 LAB 6 PROBLEM 5: CONSERVATION OF ANGULAR MOMENTUM

While driving around the city, your car is constantly shifting gears. You wonder how the gear shifting process works. Your friend tells you that there are gears in the transmission of your car that are rotating about the same axis. When the car shifts, one of these gear assemblies is brought into connection with another one that drives the car's wheels. Thinking about a car starting up, you decide to calculate how the angular speed of a spinning object changes when it is brought into contact with another object at rest. To keep your calculation simple, you decide to use a disk for the initially spinning object and a ring for the object initially at rest. Both objects will be able to rotate freely about the same axis, which is centered on both objects. To test your calculation you decide to build a laboratory model of the situation.

Read: Mazur Chapter 12.

EQUIPMENT

You have an apparatus that spins a horizontal disk and a ring to gently drop onto it.   You also have a stopwatch, meterstick and the video analysis equipment.

Take care not to drop the ring onto the disk from a measurable height**. The heavy ring should only be a COUPLE OF MILLIMETERS above the disk before it is released!**   Dropping it from greater separations has previously broken the plastic disk.
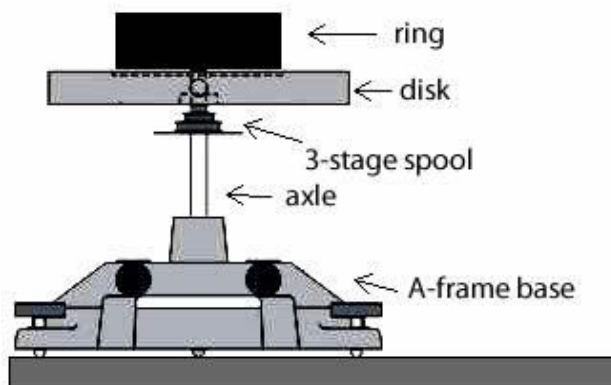
WARM UP

To figure out your prediction, it is useful to use a problem solving strategy such as the one outlined:

1. Make two side view drawings of the situation (similar to the diagram in the Equipment section), one just as the ring is released, and one after the ring lands on the disk. Label all relevant kinematic quantities and write down the relationships that exist between them. Label all relevant forces.

2. Determine the basic principles of physics that you will use and how you will use them. Determine your system. Are any objects from outside your system interacting with your system? Write down your assumptions and check to see if they are reasonable.

3. Use conservation of angular momentum to determine the final angular speed of the rotating objects. Why not use conservation of energy or conservation of momentum? Define your system and write the conservation of angular momentum equation for this situation:

   Is any significant angular momentum transferred to or from the system? If so, can you determine it or redefine your system so that there is no transfer?

4. Identify the target quantity you wish to determine. Use the equations collected in steps 1 and 3 to plan a solution for the target. If there are more unknowns than equations, reexamine the previous steps to see if there is additional information about the situation that can be expressed in an addition equation. If not, see if one of the unknowns will cancel out.

PREDICTION

Restate the problem. What quantities do you need to calculate to test your idea?

EXPLORATION

Practice dropping the ring into the groove on the disk as gently as possible to ensure the best data. What happens if the ring is dropped off-center? What happens if the disk does not fall smoothly into the groove? Explain your answers.

Decide what measurements you need to make to check your prediction. If any major assumptions are used in your calculations, decide on the additional measurements that you need to make to justify them.

Outline your measurement plan.

Make some rough measurements to be sure your plan will work.

MEASUREMENTS

Follow your measurement plan. What are the uncertainties in your measurements?

ANALYSIS

Determine the initial and final angular velocity of the disk from the data you collected. Using your prediction equation and your measured initial angular velocity, calculate the final angular velocity of the disk. If your calculation incorporates any assumptions, make sure you justify these assumptions based on data that you have analyzed.

CONCLUSION

Did your measurement of the final angular velocity agree with your calculated value by prediction? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Could you have easily measured enough information to use conservation of energy to predict the final angular velocity of this system? Why or why not? Use your data to check your answer.

1301 LAB 6 PROBLEM 6: DESIGNING A MOBILE

Your friend has asked you to help make a mobile for her daughter's room. You design a mobile using five pieces of string and two rods. The first rod hangs from the ceiling. One object hangs from one end of the rod and another rod hangs from the other end. That second rod has two objects hanging from each end. The project would be easier if your friend's daughter knew what she wanted to hang from the mobile, but she cannot make up her mind. One day it is dinosaurs, another day it is the Power Rangers, and another day it is famous women scientists. Frustrated, you decide to build a laboratory model to test the type of mobile you will build in order to make sure no matter what she decides to hang, the mobile can be easily assembled.

Read: Mazur Chapter 12.

EQUIPMENT

You have two wooden dowels, some string, and three mass sets. Your final mobile should use all these parts. A rod and table clamp is available to hang the mobile.
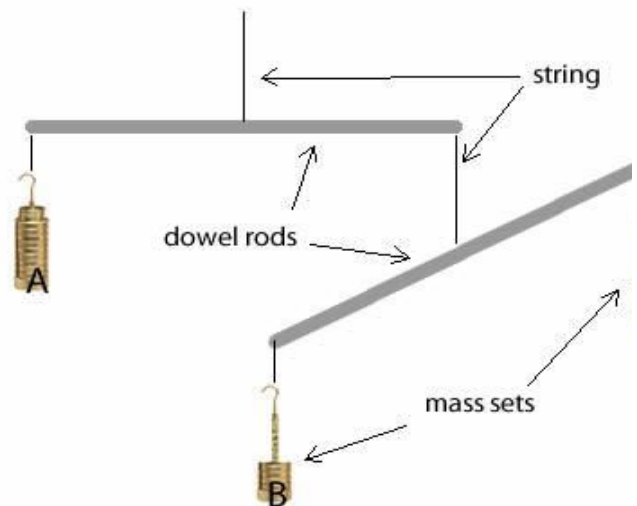
WARM UP

To figure out your prediction, it is useful to use a problem solving strategy such as the one outlined below:

1. Draw a mobile similar to the one in the Equipment section. Select your coordinate system. Identify and label the masses and lengths relevant to this problem. Draw and label all the relevant forces.

2. Draw a free-body diagram for each rod showing the location of the forces acting on the rods. Label these forces. Identify any forces related by Newton's third law. Choose the axis of rotation for each rod. Identify any torques on each rod.

3. For each free-body diagram, write the equation expressing Newton's second law for forces and another equation for torques. (Remember that your system is in equilibrium.) What are the total torque and the sum of forces on an object when it is in equilibrium?

4. Identify the target quantities you wish to determine. Use the equations collected in step 3 to plan a solution for the target. If there are more unknowns than equations, reexamine the previous steps to see if there is additional information about the situation that can be expressed in an additional equation. If not, see if one of the unknowns will cancel out.

PREDICTION

Restate the problem. What quantities do you need to calculate to test your design? What are the variables in the system?

EXPLORATION

Collect the necessary parts of your mobile. Find a convenient place to hang it. Decide on the easiest way to determine the position of the center of mass of each rod. Will the length of the strings for the hanging objects affect the balance of the mobile? Why or why not? Try it. Where does the heaviest object go? The lightest?

Decide what measurements you need to make to check your prediction. If any major assumptions are used in your calculations, decide on the additional measurements that you need to make to justify them. Outline your measurement plan.

MEASUREMENT

Measure and record the location of the center of mass of each rod. Determine the location on the top rod from which you will hang it. Determine the location on the second rod from which you will hang it. Also, measure and record the mass of each rod and the mass of the three hanging objects.

Is there another configuration of the three objects that also results in a stable mobile?

ANALYSIS

Using the values you measured and your prediction equations, calculate the locations (with uncertainties) of the two strings holding up the rods.

To test your prediction, build your mobile and then hang it. If your mobile did not balance, adjust the strings attached to the rods until it does balance and determine their new positions.

Is there another configuration of the three objects that also results in a stable mobile? Try it.

CONCLUSION

Did your mobile balance as designed? What corrections did you need to make to get it to balance? Were these corrections a result of some systematic error, or was there a mistake in your prediction?

Explain why the lengths of each string were or were not important in the mobile design.

1301 LAB 6 PROBLEM 7: EQUILIBRIUM

You have been hired to design new port facilities for Duluth. Your assignment is to evaluate a new crane for lifting containers from the hold of a ship. The crane is a boom (a steel bar of uniform thickness) with one end attached to the ground by a hinge that allows it to rotate in the vertical plane. Near the other end of the boom is a motor driven cable that lifts a container straight up at a constant speed. The boom is supported at an angle by another cable. One end of the support cable is attached to the boom and the other end goes over a pulley. That other end is attached to a counterweight that hangs straight down. The pulley is supported by a mechanism that adjusts its height so the support cable is always horizontal. Your task is to determine how the angle of the boom from the horizontal changes, as a function of the weight of the container being lifted. The mass of the boom, the mass of the counterweight, the attachment point of the support cable and the attachment point of the lifting cable have all been specified by the engineers.

You will test your calculations with a laboratory model of the crane.

Read: Mazur Chapter 12.

EQUIPMENT

WARNING: The equilibrium in this system is unstable; it is strongly recommended that you keep your hand next to the system while balancing. Do not let the system fall or fling equipment.

You have a channel of aluminum with a hinge on one end. A pulley, table clamp, two mass sets and string is available.

It helps to clamp the hinge under the pulley clamp to fix it in place. Make sure the aluminum channel can freely move up and down.
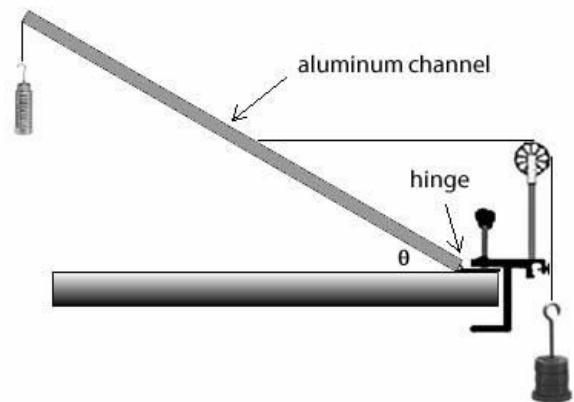
WARM UP

To figure out your prediction, it is useful to use a problem solving strategy such as the one outlined below:

1. Draw a crane similar to the one in the Equipment section. Select your coordinate system. Identify and label the masses and lengths relevant to this problem. Draw and label all the relevant forces.

2. Draw a free-body diagram for the bar showing the location of the forces acting on it. Label these forces. Choose the axis of rotation. Identify any torques on the rod.

3. Write the equation expressing Newton's second law for forces and another equation for torques. Remember that the bar is in equilibrium.

4. Identify the target quantities you wish to determine. Use the equations collected in step 3 to plan a solution for the target. If there are more unknowns than equations, reexamine the previous steps to see if there is additional information about the situation that can be expressed in an additional equation. If not, see if one of the unknowns will cancel out.

5. Make a graph of the bar's angle as a function of the weight of object A.

PREDICTION

Restate the problem. What quantities do you need to calculate to test your design? What parameters are set, and which one(s) will you vary?

EXPLORATION

Collect the necessary parts of your crane. Find a convenient place to build it.

Decide on the easiest way to determine where the center of mass is located on the bar.

Determine where to attach the lifting cable and the support cable so that the crane is in equilibrium for the weights you want to hang. Try several possibilities. If your crane tends to lean to one side or the other, try putting a vertical rod near the end of the crane to keep your crane from moving in that direction. If you do this, what effect will this vertical rod have on your calculations?

Do you think that the length of the strings for the hanging weights will affect the balance of the crane? Why or why not?

Outline your measurement plan.

MEASUREMENT

Build your crane.

Make all necessary measurements of the configuration. Every time only change the mass of object A and determine the angle of the bar when the system is in equilibrium. Remember to adjust the height of the pulley to keep the support string horizontal that hangs the object B for each case.

Is there another configuration of the three objects that also results in a stable configuration?

ANALYSIS

Make a graph of the bar's angle as a function of the weight of object A and compare it with your predicted graph.

What happens to that graph if you change the mass of object B   or the position of the attachment of the support cable to the bar?

CONCLUSION

Did your crane balance as designed? What corrections did you need to make to get it to balance? Were these corrections a result of some systematic error, or was there a mistake in your prediction? In your opinion, what is the best way to construct a crane that will allow you to quickly adjust the setup so as to meet the demands of carrying various loads? Justify your answer.

1301 LAB 7 PROBLEM 1: MEASURING SPRING CONSTANTS TWO WAYS

You are selecting springs for a large antique clock; to determine the forces they will exert in the clock, you need to know their spring constants. One book recommends a static approach: hang objects of different weights on the spring and measure the displacements. Another book suggests a dynamic approach: hang an object on the end of a spring and measure its oscillation frequency. You decide to compare the results of the two methods, in order to get the best precision possible for your characterization of the clock's springs. But first you have to figure out how to calculate spring constants from each type of measurement.

Read: Mazur Chapter 8, especially Sections 8.6 and 8.9, and Sections 15.1–15.6.

EQUIPMENT

You have springs, a table clamp, rod, meterstick, stopwatch, mass set and the video analysis equipment. You can hang the spring from a rod that is extended from a table clamp.

WARM UP

To figure out your predictions, it is useful to apply a problem-solving strategy such as the ones outlined below:

Method #1: Suppose you hang objects of several different masses on a spring and measure the vertical displacement of each object.

1. Make two sketches of the situation, one before you attach a mass to a spring, and one after a mass is suspended from the spring and is at rest. Draw a coordinate system and label the position where the spring is unstretched, the stretched position, the mass of the object, and the spring constant. Assume the springs are massless.

2. Draw a force diagram for the object hanging *at rest* from the end of the spring. Label the forces. Newton's second law gives the equation of motion for the hanging object. Solve this equation for the spring constant.

3. Use your equation to sketch the displacement (from the unstretched position) versus weight graph for the object hanging at rest from the spring. How is the slope of this graph related to the spring constant?

Method #2: Suppose you hang an object from the spring, start it oscillating, and measure the *period* of oscillation.

1. Make a sketch of the oscillating system at a time when the object is *below* its equilibrium position. Draw this sketch to the side of the two sketches drawn for method #1. Identify and label this new position on the same coordinate axis.

2. Draw a force diagram of the object at this new position. Label the forces.

3. Apply Newton's second law to write down the equation of motion for the object at each of the above positions.

   When the object is below its equilibrium position, how is the stretch of the spring from its unstretched position related to the position of the system's (spring & object) equilibrium position and its displacement from that equilibrium position to the position in your second sketch. Define these variables, and write an equation to show this relationship.

4. Solve your equations for acceleration of the object as a function of the mass of the suspended object, the spring constant, and the displacement of the spring/object system from its equilibrium position. Keep in mind that acceleration is second derivative of position with respect to time.

5. Try a periodic solution ($\sin(\omega \cdot t)$ or $\cos(\omega \cdot t)$) to your equation of motion (Newton's second law). Find the frequency ω that satisfies equation of motion for all times. How is the frequency of the system related to its period of oscillation?

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PREDICTION

</div>

Restate the problem. What two relationships must you calculate to prepare for your experiment?

<div align="center">

EXPLORATION

</div>

Method #1: Select a series of masses that give a usable range of displacements. The smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. **The largest mass should not pull the spring past its elastic limit (about 40 cm). Beyond that point you will damage the spring**. Decide on a procedure that allows you to measure the displacement of the spring-object system in a consistent manner. Decide how many measurements you will need to make a reliable determination of the spring constant.

Method #2: Secure one end of the spring safely to the metal rod and select a mass that gives a regular oscillation without excessive wobbling to the hanging end of the spring. Again, the largest mass should not pull the spring past its elastic limit and the smallest mass should be much greater than the mass of the spring. Practice starting the mass in vertical motion smoothly and consistently.

Practice making a video to record the motion of the spring-object system. Decide how to measure the period of oscillation of the spring-object system by video and stopwatch. How can you minimize the uncertainty introduced by your reaction time in starting and stopping the stopwatch? How many times should you measure the period to get a reliable value? How will you determine the uncertainty in the period?

MEASUREMENT

Method #1: Record the masses of different hanging objects and the corresponding displacements.

Method #2: For each hanging object, record the mass of the object. Use a stopwatch to roughly determine the period of the oscillation and then make a video of the motion of the hanging object. Repeat the same procedure for objects with different masses.

Analyze your data as you go along so you can decide how many measurements you need to make to determine the spring constant accurately and reliably.

ANALYSIS

Method #1: Make a graph of displacement versus weight for the object-spring system. From the slope of this graph, calculate the value of the spring constant, including the uncertainty.

Method #2: Determine the period of each oscillation from your videos. (Use the period by stopwatch as a predicted parameter in your fit equations.) Make a graph of period (or frequency) versus mass for the object-spring system. If this graph is not a straight line, make another graph of the period vs. some power of the mass that should produce a straight line. (Use your prediction equation to decide what that power should be.) From the slope of the straight-line graph, calculate the value of the spring constant, including the uncertainty.

CONCLUSION

How do the two values of the spring constant compare? Which method is faster? Which method gives you the best precision? Justify your answers in terms of your data and measurements.

Did your prediction equation for method #2 help you correctly identify a power of the mass that would produce a straight-line graph when you were working through the analysis? Explain why or why not.

How did you minimize the uncertainty involved in the timing for method #2? Did video analysis give you a better estimate of the period than the stopwatch?

1301 LAB 7 PROBLEM 2: THE EFFECTIVE SPRING CONSTANT

Your company has bought the prototype for a new flow regulator from a local inventor. Your job is to prepare the prototype for mass production. While studying the prototype, you notice the inventor used some rather innovative spring configurations to supply the tension needed for the regulator valve. In one location the inventor had fastened two different springs side-by-side, as in Figure A below. In another location the inventor attached two different springs end-to-end, as in Figure B below.

To decrease the cost and increase the reliability of the flow regulator for mass production, you need to replace each spring configuration with a single spring. These replacement springs must exert the same total forces when stretched the same amount as the original net displacement of a hanging object. The spring constant for a single spring that replaces a configuration of springs is called the configuration's *effective spring constant*.

Read: Mazur Chapter 8, especially Sections 8.6 and 8.9, and Sections 15.1-15.6.

EQUIPMENT

You have two different springs with the same unstretched length, but different spring constants k1 and k2. These springs can be hung vertically side-by-side (setup A) or end-to-end (setup B). You will also have a meterstick, stopwatch, rod, wooden dowel, table clamp, mass set and video analysis equipment.
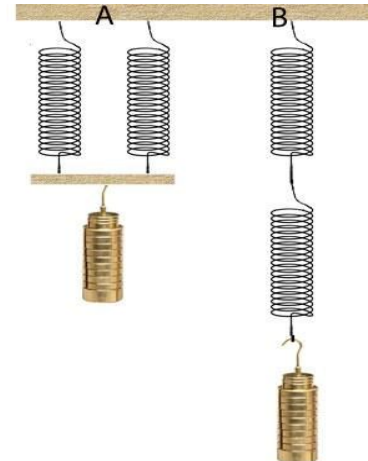
WARM UP

To figure out your predictions, it is useful to apply a problem-solving strategy such as the one outlined below. *Apply the strategy first to the side-by-side configuration, and then repeat for the end-to-end configuration:*

1. Make a sketch of the spring configuration similar to one of the drawings in the Equipment section. Draw a coordinate system and label the positions of each unstretched spring, the final stretched position of each spring, the two spring constants, and the mass of the object suspended. Assume that the springs are massless.

    For the side-by-side configuration, assume that the light bar attached to the springs remains horizontal (it does not twist).

    Now make a second sketch of a single (massless) spring with spring constant k' that has the same object suspended from it and the same total stretch as the combined springs. Label this second sketch with the appropriate quantities.

2. Draw force diagrams of the object suspended from the combined springs and the same object suspended from the single replacement spring. Label the forces. Use Newton's Third Law to identify forces on different diagrams that have the same magnitudes.

For the end-to-end configuration, draw an additional force diagram for the point at the connection of the two springs.

3. For each force diagram, write a Newton's Second Law equation to relate the net force on an object (or the point connecting the springs) to its acceleration.

Write an equation relating the total stretch of the combined springs related to the stretch of each of the springs? How does this compare to the stretch of the single replacement spring? How does the stretch of each spring relate to its spring constant and the force it exerts?

4. Re-write each Newton's second law equation in terms of the stretch of each spring.

For the end-to-end configuration: At the connection point of the two springs, what is the force of the top spring on the bottom spring? What is the force of the bottom spring on the top spring?

5. Solve your equations for the effective spring constant (k') of the single replacement spring, in terms of the two spring constants.

$$\boxed{\textsc{Prediction}}$$

Restate the problem. What ratios do you need to calculate for each spring configuration in the problem?

$$\boxed{\textsc{Exploration}}$$

To test your predictions, you must decide how to measure each spring constant of the two springs and the effective spring constants of the side-by-side and end-to-end configurations.

From your results of the earlier problem, **Measuring Spring Constants**, select the best method for measuring spring constants. Justify your choice. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 40 CM) OR YOU WILL DAMAGE THEM.**

Perform an exploration consistent with your selected method. If necessary, refer back to the appropriate Exploration section of the problem **Measuring Spring Constants**.

Remember, the smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. The largest mass should not pull the spring past its elastic limit.

Outline your measurement plan.

$$\boxed{\textsc{Measurement}}$$

Make the measurements that are consistent with your selected method. If necessary, refer back to the appropriate Measurement section of the problem **Measuring Spring Constants**. What are the uncertainties in your measurements?

ANALYSIS

Determine the effective spring constants (with uncertainties) of the side-by-side spring configuration and the end-to-end spring configuration. If necessary, refer back to the problem **Measuring Spring Constants** for the analysis technique consistent with your selected method.

Determine the spring constants of the two springs. Calculate the effective spring constants (with uncertainties) of the two configurations using your Prediction equations.

How do the measured and predicted values of the effective spring constants for the two configurations compare?

CONCLUSION

What are the effective spring constants of a side-by-side spring configuration and an end-to-end spring configuration? Did your measured values agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

Which configuration provides a larger effective spring constant?

1301 LAB 7 PROBLEM 3: OSCILLATION FREQUENCY WITH TWO SPRINGS

You have a summer job with a research group at the University. Your supervisor asks you to design equipment to measure earthquake aftershocks. The calibration sensor needs to be isolated from the earth movements, yet free to move. You decide to place the sensor on a track cart and attach a spring to both sides of the cart. You should now be able to measure the component of the aftershocks along the axis defined by the track. To make any quantitative measurements with the sensor you need to know the frequency of oscillation for the cart as a function of the spring constants and the mass of the cart.

Read: Mazur Chapter 15.

EQUIPMENT

You have a track, two track endstops, two oscillation springs, a meterstick, stopwatch, cart, cart masses and the video analysis equipment.

WARM UP

To figure out your prediction, it is useful to use a problem-solving strategy such as the one outlined below:

1. Make two sketches of the oscillating cart, one at its equilibrium position, and one at some other position and time while it is oscillating. On your sketches, show the direction of the velocity and acceleration of the cart. Identify and label the known (measurable) and unknown quantities.

2. Draw a force diagram of the oscillating cart away from its equilibrium position. Label the forces.

3. Apply Newton's laws as the equation of motion for the cart. Consider both cases when the cart is in the equilibrium and displaced from the equilibrium position.

    Solve your equation for the acceleration, simplifying the equation until it is similar to equation 15.21 (Mazur).

4. Try a periodic solution ($\sin(\omega \cdot t)$ or $\cos(\omega \cdot t)$) to your equation of motion (Newton's second law). Find the frequency ω that satisfies equation of motion for all times. How is the frequency of the system related to its period of oscillation? Calculate frequency of the system as a function of the mass of the cart and the two spring constants.

PREDICTION

Restate the problem. What quantities do you need to calculate to test your design?

EXPLORATION

Decide the best method to determine the spring constants based on your results of the problem **Measuring Spring Constants**. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 40 CM) OR YOU WILL DAMAGE THEM.**

Find the best place for the adjustable end stop on the track. *Do not stretch the springs past 40 cm*, but stretch them enough so they oscillate the cart smoothly.

Practice releasing the cart smoothly. You may notice the amplitude of oscillation decreases. What's the reason for it? Does this affect the period of oscillation?

MEASUREMENT

Determine the spring constants. Record these values. What is the uncertainty in these measurements?

Record the mass of the cart. Use a stopwatch to roughly determine the period of oscillation and then make a video of the motion of the oscillating cart. You should record at least 3 cycles.

ANALYSIS

Analyze your video to find the period of oscillation. Calculate the frequency (with uncertainty) of the oscillations from your measured period.

Calculate the frequency (with uncertainty) using your Prediction equation.

CONCLUSION

What is the frequency of the oscillating cart? Did your measured frequency agree with your predicted frequency? Why or why not? What are the limitations on the accuracy of your measurements and analysis? What is the effect of friction?

If you completed the earlier problem, **The Effective Spring Constant**: What is the effective spring constant of this configuration? How does it compare with the effective spring constants of the side-by-side and end-to-end configurations?

1301 LAB 7 PROBLEM 4: OSCILLATION FREQUENCY OF AN EXTENDED SYSTEM

You are the technical advisor for the next Bruce Willis action movie, *Die Even Harder*, which is to be filmed in Minnesota. The script calls for a spectacular stunt. Bruce Willis dangles over a cliff from a long rope whose other end is tied to the Bad Guy. The Bad Guy is on the ice-covered ledge of the cliff. The Bad Guy's elastic parachute line is tangled in a tree located several feet from the edge of the cliff. Bruce and the Bad Guy are in simple harmonic motion, and at the top of his motion, Bruce unsuccessfully tries to grab for the safety of the cliff edge while the Bad Guy reaches for his discarded knife. The script calls for Bad Guy to cut the rope just as Bruce reaches the top of his motion again.

The problem is that it is expensive to have Bruce hanging from the rope while the crew films close-ups of the Bad Guy, but the stunt double weighs at least 50 pounds more than Bruce. The director wants to know if the stunt double will have a different motion than Bruce, and if so whether the difference would be noticeable. Will he? You decide to test your prediction by modeling the situation with the equipment described below.

Read: Mazur Chapter 15.

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EQUIPMENT

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You have a track, endstop, pulley, table clamp, springs, cart, string, mass set, meterstick, stopwatch and the video analysis equipment.

The track represents the ice-covered ledge of the cliff, the end-stop represents the tree, the spring represents the elastic cord, the cart represents the Bad Guy, the string represents the rope and the hanging mass set represents Bruce or his stunt double.
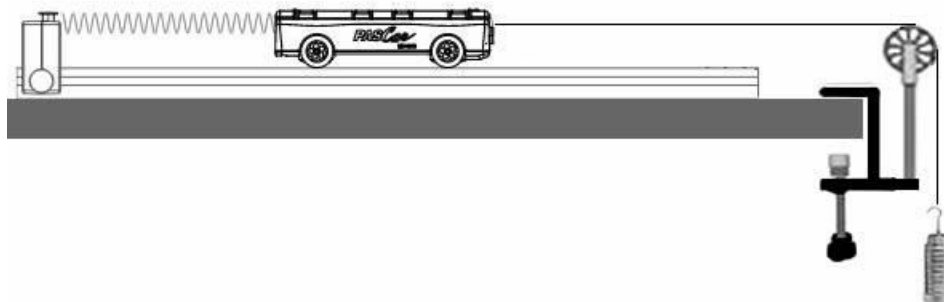
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WARM UP

</div>

To figure out your prediction, it is useful to use a problem-solving strategy such as the one below:

1. Make sketches of the situation when the cart and hanging object are at their equilibrium positions and at some other time while the system is oscillating. On your sketches, show the direction of the acceleration of the cart and hanging object. Identify and label the known (measurable) and unknown quantities.

2. Draw separate force diagrams of the oscillating cart and hanging object. Label each force. Are there any third-law pairs?

3. Independently apply Newton's laws to the cart and to the hanging object.

4. Solve your equations for the acceleration, simplifying the equation until it is similar to equation 15.21 (Mazur).

5. Try a periodic solution [$\sin(\omega \cdot t)$ or $\cos(\omega \cdot t)$] to your equation of motion (Newton's second law). Find the frequency ω that satisfies equation of motion for all times. How is the frequency of the system related to its period of oscillation? Calculate frequency of the system as a function of the mass of the cart, the mass of the hanging object, and the spring constant.

6. Use your equation to sketch the expected shape of a graph of the oscillation frequency versus hanging mass. *Will the frequency* **increase**, **decrease** or **stay the same** *as the hanging mass increases?*

7. *Now you can complete your prediction.* Use your equation to sketch the expected shape of the graph of oscillation frequency versus the hanging object's mass.

PREDICTION

Restate the problem. What quantities must you calculate to answer the director's question?

EXPLORATION

If you do not know the spring constant of your spring, you should decide the best way to determine the spring constant based on your results of the problem **Measuring Spring Constants**.

Find the best place for the adjustable end stop on the track. **DO NOT STRETCH THE SPRING PAST 40 CM OR YOU WILL DAMAGE IT,** but stretch it enough so the cart and hanging mass oscillate smoothly. Determine the best range of hanging masses to use.

Practice releasing the cart and hanging mass smoothly and consistently. You may notice the amplitude of oscillation decreases. What's the reason for it? Does this affect the period of oscillation?

MEASUREMENT

If necessary, determine the spring constant of your spring. What is the uncertainty in your measurement?

For each hanging object, record the masses of the cart and the hanging object. Use a stopwatch to roughly determine the period of oscillation and then make a video of the oscillating cart for each hanging object. You should record at least 3 cycles for each video.

Collect enough data to convince yourself and others of your conclusion about how the oscillation frequency depends on the hanging mass.

ANALYSIS

For each hanging object, digitize the video to get the period of oscillation and then calculate the oscillation frequency (with uncertainty) from your measured period.

Graph the frequency versus the hanging object's mass. On the same graph, show your predicted relationship.

What are the limitations on the accuracy of your measurements and analysis? Over what range of values does the measured graph match the predicted graph best? Do the two curves start to diverge from one another? If so, where? What does this tell you about the system?

CONCLUSION

Does the oscillation frequency increase, decrease or stay the same as the hanging object's mass increases? State your result in the most general terms supported by your analysis.

What will you tell the director? Do you think the motion of the actors in the stunt will change if the heavier stunt man is used instead of Bruce Willis? How much heavier would the stunt man have to be to produce a noticeable difference in the oscillation frequency of the actors? Explain your reasoning in terms the director would understand so you can collect your paycheck.

1301 LAB 7 PROBLEM 5: DRIVEN OSCILLATIONS

You have a summer job with a research group at the University. Your supervisor asks you to design equipment to measure earthquake aftershocks. To calibrate your seismic detector, you need to determine how the amplitude of the oscillations of the detector will vary with the frequency of the earthquake aftershocks. For that you decide to place the sensor on a track cart and attach a spring to both sides of the cart. The other side of the one of the springs is attached to an end stop. The second spring is attached to a device that moves the end of the spring back and forth, simulating the earth moving beneath the track. The device, called a mechanical oscillator, is designed so you can change its frequency of oscillation. You should now be able to measure the component of the aftershocks along the axis defined by the track.

Read: Mazur Chapter 15.

EQUIPMENT

You have a track, endstop, two springs, a meterstick, stopwatch, cart, mechanical oscillator, rod, table clamp, function generator, two banana cables and the video analysis equipment.

The oscillator is connected to a function generator which allows it to oscillate back and forth with adjustable frequencies.
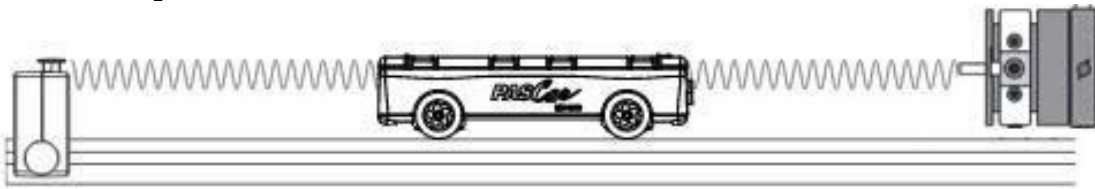
WARM UP

You should follow the Warm Up for the problem **Oscillation Frequency with Two Springs** if you have not already done so.

To qualitatively decide on the behavior of the system with the mechanical oscillator attached and turned on, think about an experience you have had putting energy into an oscillating system. For example, think about pushing someone on a swing. When is the best time to push to get the maximum height for the person on the swing? How does the frequency of your push compare to the natural frequency of the person on the swing? How does the maximum height of the swinger compare to the size of your push?

PREDICTION

Make your best-guess sketch of what you think a graph of the amplitude of the cart versus the frequency of the mechanical driver will look like. Assume the mechanical oscillator has constant amplitude of a few millimeters.

EXPLORATION

Examine the mechanical oscillator. Mount it at the end of the aluminum track, using the clamp and metal rod so its shaft is aligned with the cart's motion. Connect it to the function generator, using the output marked **Lo** (for ``low impedance''). Use middle or maximum amplitude to observe the oscillation of the cart at the lowest frequency possible.

Determine the accuracy of the digital display on the frequency generator by timing one of the lower frequencies. Devise a scheme to accurately determine the amplitude of a cart on the track, and practice the technique. For each new frequency, should you restart the cart at rest?

When the mechanical oscillator is at or near the un-driven frequency (natural frequency) of the cart-spring system, try to simultaneously observe the motion of the cart and the shaft of the mechanical oscillator. What is the relationship? What happens when the oscillator's frequency is twice as large as the natural frequency?

MEASUREMENT

If you do not know the natural frequency of your system when it is not driven, determine it using the technique used in the problem **Oscillation Frequency with Two Springs**. Collect enough cart amplitude and oscillator frequency data to test your prediction. Be sure to collect several data points near the natural frequency of the system.

ANALYSIS

Make a graph the oscillation amplitude of the cart versus oscillator frequency. Is this the graph you had anticipated? Where is it different? Why? What is the limitation on the accuracy of your measurements and analysis?

CONCLUSION

Can you explain your results? Is energy conserved? What will you tell your boss about your design for a seismic detector?

Recording Video and using MotionLab - Video Analysis of Motion

Analyzing pictures (movies or videos) is a powerful tool for understanding how objects move. This appendix will guide a person in the use of the app ProCam on an iPod to record videos and MotionLab to analyze motion. LabVIEW™ is a general-purpose data acquisition programming system. It is widely used in academic research and industry. Later you will use LabVIEW™ to acquire data from other instruments.

Using video to analyze motion is a two-step process. The first step is to record a video on the iPod and import it to the computer. The second step is to analyze the video to get a kinematic description of the recorded motion.

MAKING VIDEOS – USING PROCAM

Press the home button (the circle on the front) to unlock the iPod. The app you want use is ProCam. Press the home button to reach the home screen; the ProCam icon is at the bottom of the screen. With ProCam open, you should see a "live" video image. You can open the setup menu using the "< or >" button located adjacent to the red record button. With the menu open, your screen should look similar to the image below.
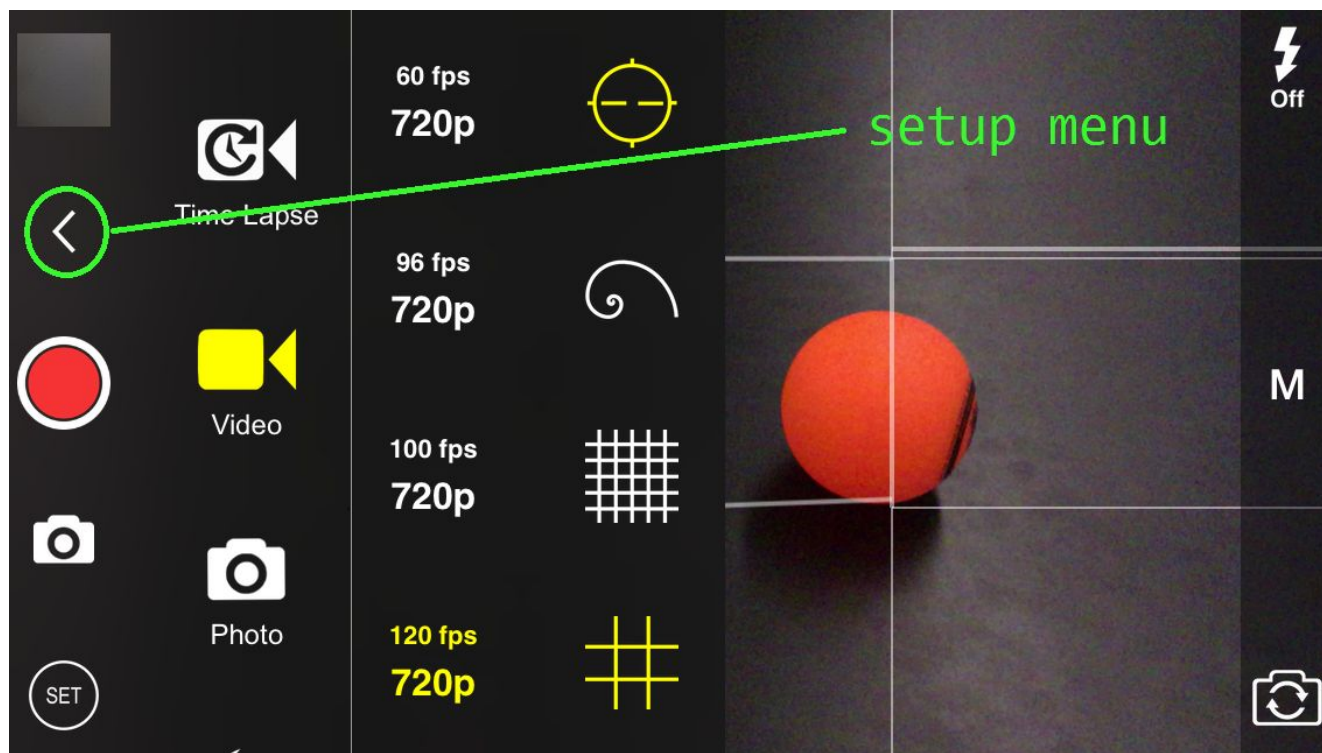
Make sure to select "Video"; when selected, it should be highlighted yellow. You need to use the 720p video format. You should select 120 fps – 720p for lab. Using other fps settings is not currently suggested.

In video mode, select the "M" button on the opposite side of the screen to open the manual controls. The screenshot below shows the manual settings menu options. The settings you will adjust are ISO and shutter speed; you select ISO or shutter speed, then adjust using the vertical scroll bar. The ISO determines how sensitive the camera is to light and the shutter speed controls how long the shutter is open. An ISO of 200 and shutter speed around 1/360 should work for most purposes. Adjust the values as necessary to get a clear image where the moving object is discrete in each frame. It is okay if the video appears dark, you will have the option to adjust the contrast after you import the video.
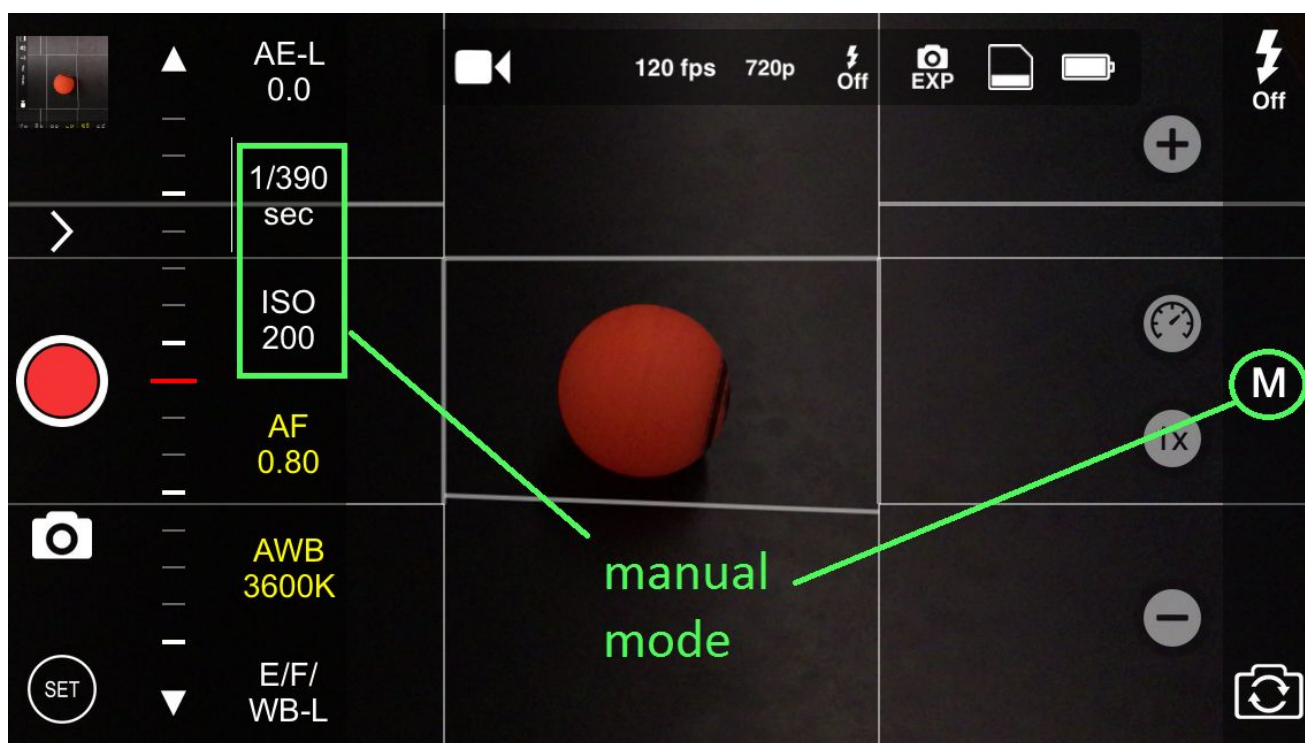
To import videos to the computer, connect the iPod to the computer. Click "allow" to access to the device, and then browse to your videos. The path is This PC\Apple iPod\Internal Storage\DCIM\100APPLE.

Copy the files from the iPod to the computer. Once you have copied the videos to the computer, you should delete them off the iPod to avoid cluttering. If there are multiple videos on the iPod you can sort them by date created to find the most recent.

ANALYSIS BASICS – USING MOTIONLAB

Open the video analysis application *MotionLAB*. You should take a moment to identify several elements of the program. As a whole the application looks complex; once it is broken down it is easy to use.

The application will prompt you to open a movie (or previously saved session) as shown here.
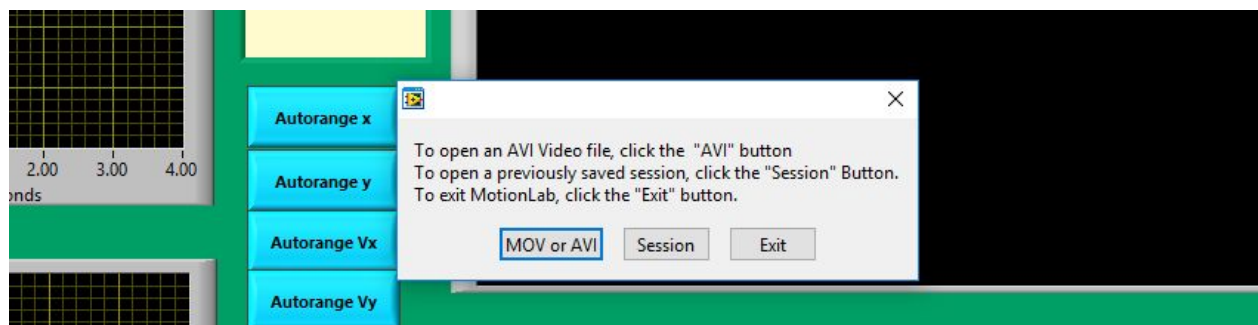
When you open your video, a dialog box will appear where you will be able to rotate the movie and change the contrast before as shown in the screenshot below. Before accepting the movie to be imported use the frame slider to make sure you do not see significant blurring of the motion. If you do, ask your instructor if the range of motion is good enough for analysis. If not, you need to change the shutter speed in ProCam.
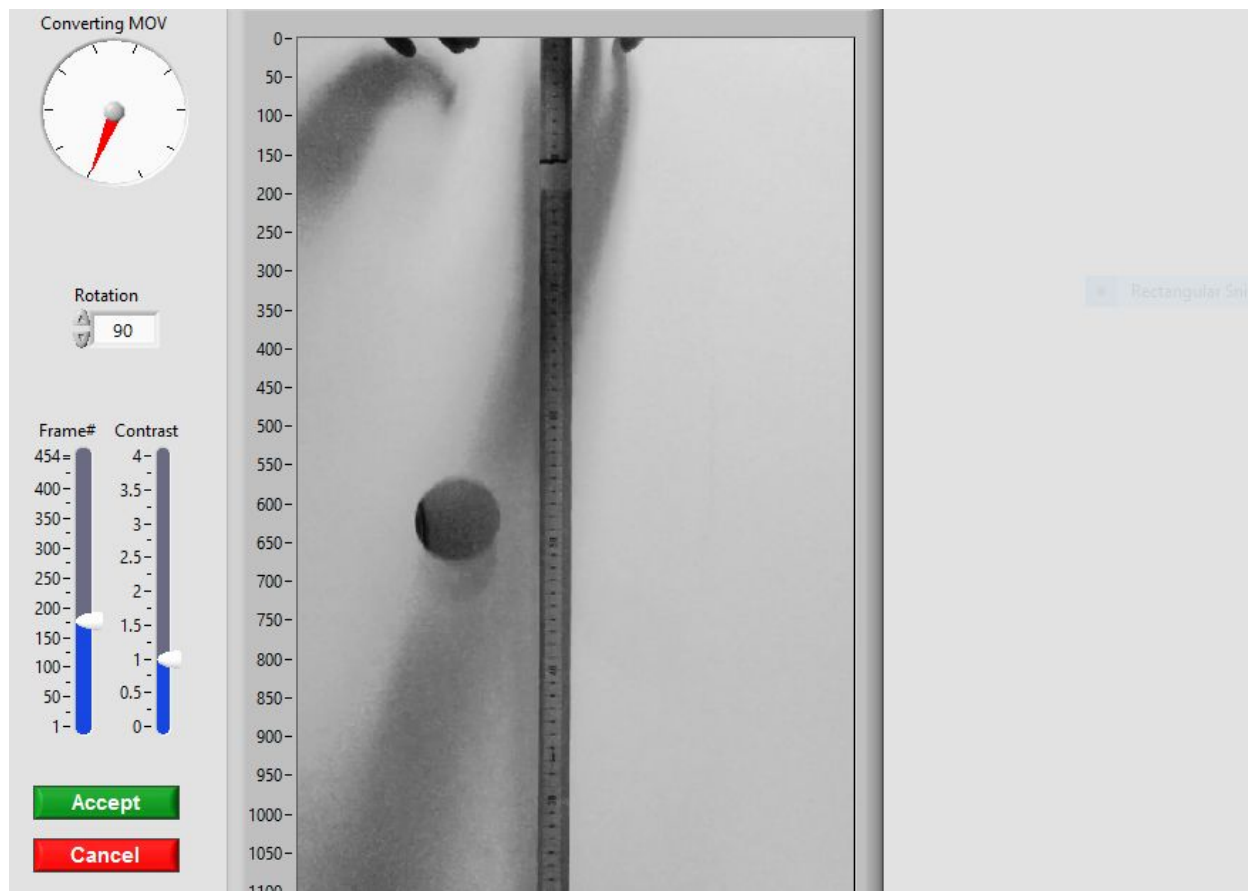
The lower right corner displays a dialog box with instructions for each step during movie analysis. To the left of the video screen is the progress indicator. It will highlight the step you are currently performing.
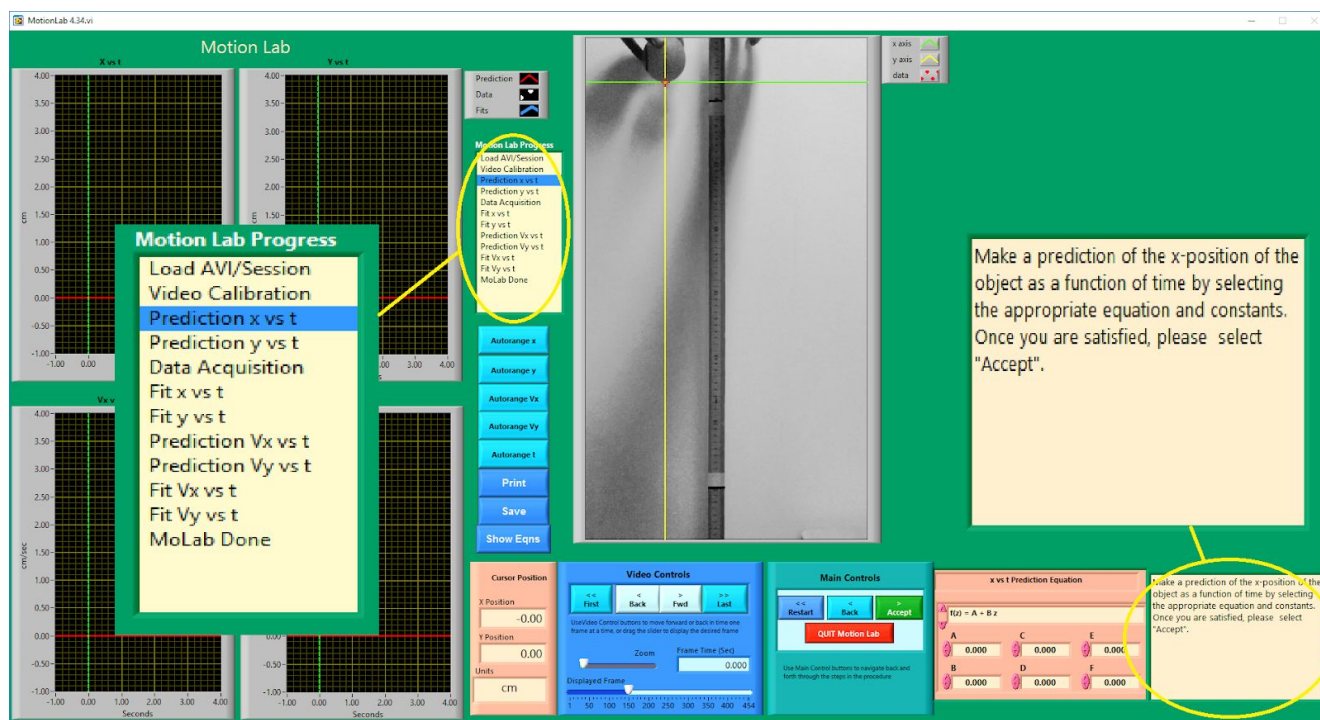
Below the video display is the Video Controls for moving within the video. The slider bar indicating the displayed frame can select any frame within the movie. The zoom slider allows you to zoom within a frame when calibrating and collecting data. Directly to the right of the Video Controls is the Main Controls. Use the Main Control buttons to navigate back and forth through the steps shown in the progress box. The red Quit Motion Lab button closes the program.
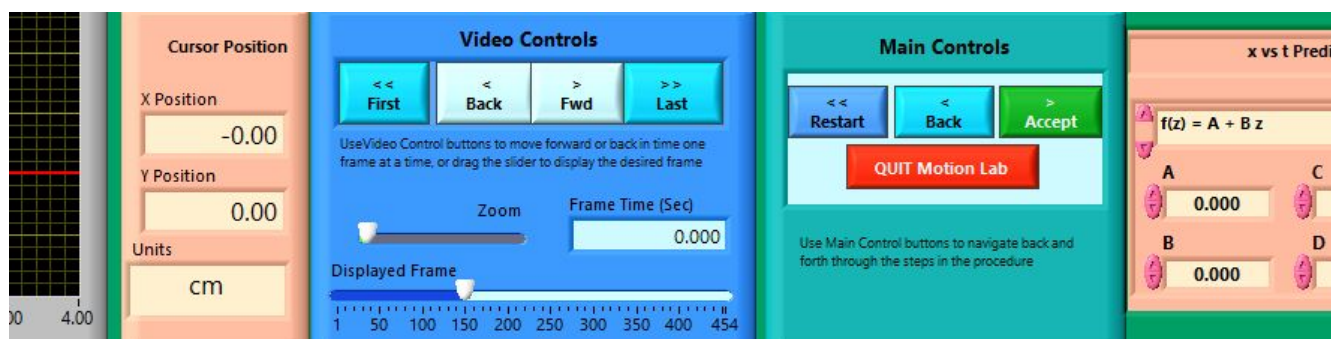
During the course of using MotionLAB, larger resolution screens pop up to allow you to calibrate your movie and take data as accurately as possible. The calibration screen has an instructions box to the left of the video with Main Controls and Video Controls directly below. The calibration screen automatically opens once your movie is loaded.
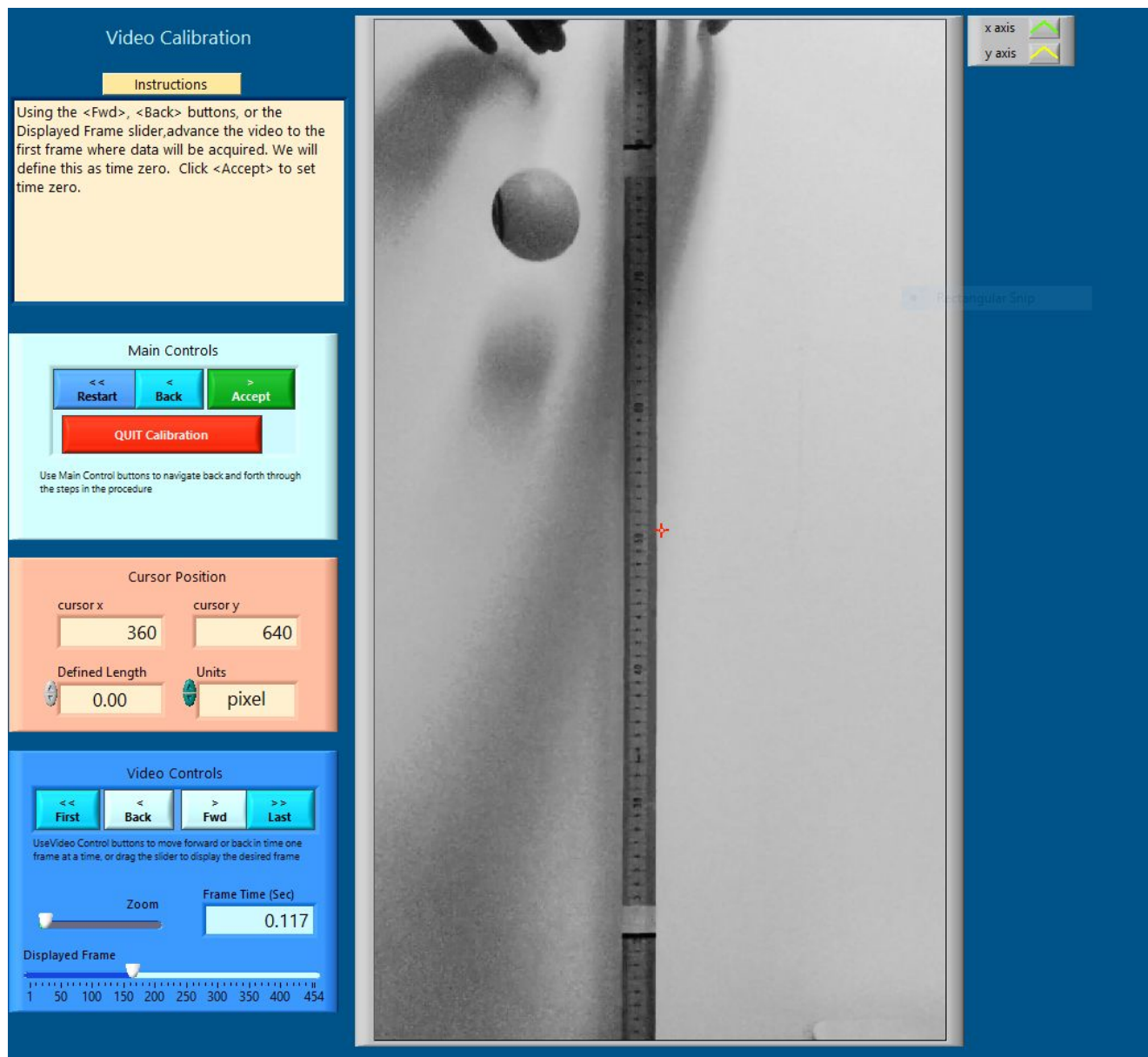
The data acquisition screen appears only after you enter predictions (the progress indicator will display which step you are at.) More will be said about predictions in a bit. The data acquisition screen has the same instructions box and Video Controls, along with a Data Acquisition Control box. The Data Acquisition controls allow you to take and remove data points. The red Quit Data Acq button exits the data collection subroutine and returns to the main screen once your data has been collected. The red cursor is moved around to take position data from each frame using your mouse. The zoom slider centers the zoomed image on the red cursor and will re-center on the red cursor for each datapoint added.
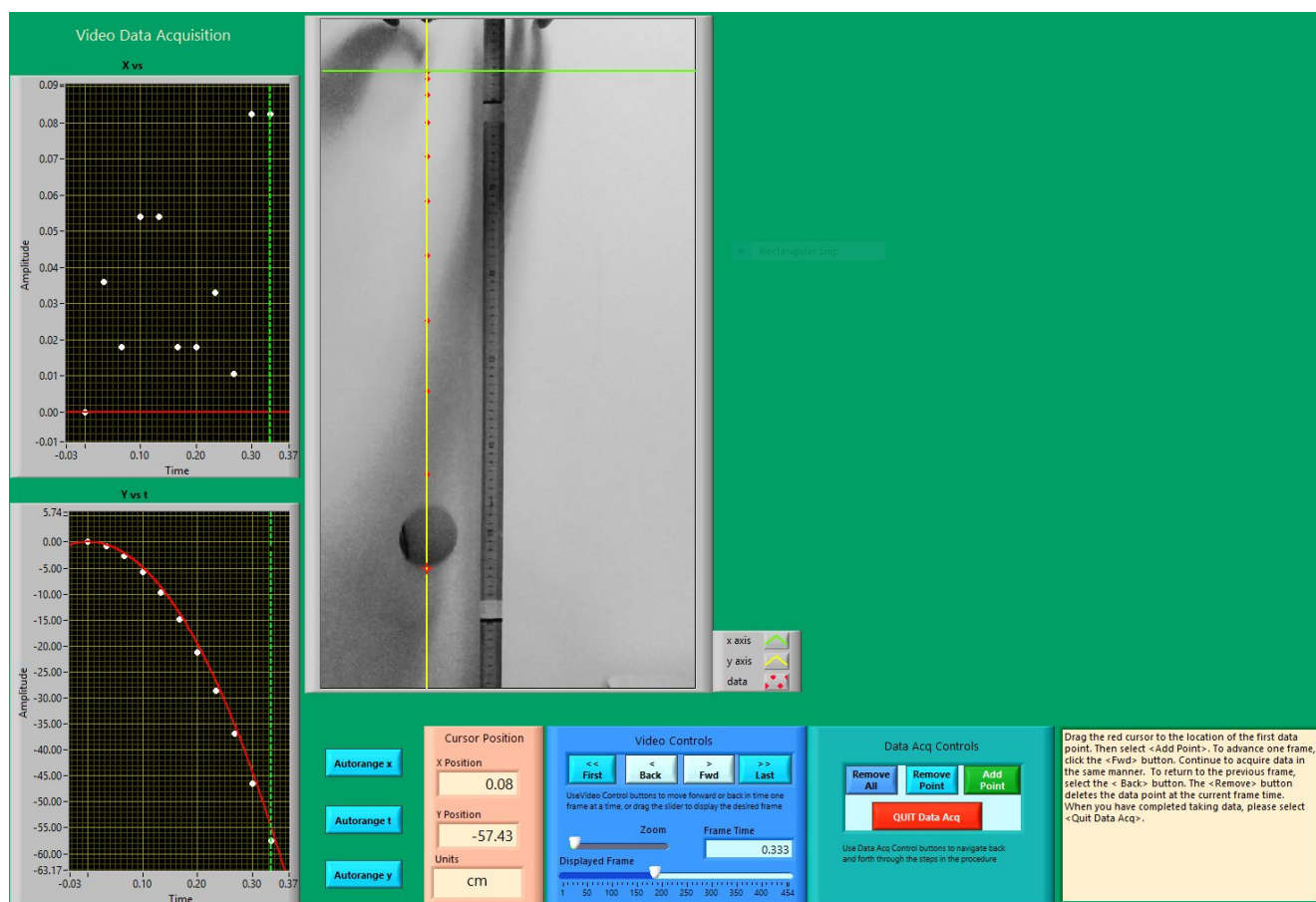
| | **Be careful not to quit without printing and saving your data!** You will have to go back and analyze the data again if you fail to select *Print Results* or *Save* before selecting *Quit*. |

There are just a few more items to point out before getting into calibration, making predictions, taking data and matching your data in more detail. To the right of the main controls shows the equation box for entering predictions and matching data. To the left of the image and below the progress indicator you have controls for automatically setting the range of the graph data based on the data points acquired. You can manually change the range by clicking on the maximum or minimum values on the graphs. Directly below that you have controls for printing and saving. Printing generates a PDF file with the graphs and the equations used for fitting. Saving generates a text file of your data points which can be

further analysed using spreadsheet programs. The graphs that display your collected data are shown below. Your predictions are displayed with red lines; fits are displayed with blue lines.

CALIBRATION

While the computer is a very handy tool, it is not smart enough to identify objects or the sizes of those objects in the videos that you take and analyze. For this reason, you will need to enter this information into the computer. If you are not careful in the calibration process, your analysis will not make any sense.

After you open the video that you wish to analyze the calibration screen will open automatically. Advance the video to a frame where the first data point will be taken. To advance the video to where you want time t=0 to be, you need to use the video control buttons. This action is equivalent to starting a stopwatch.

When you are ready to continue with the calibration, locate the object you wish to use to calibrate the size of the video. You must do your best to use an object that is in the plane of motion of your object being analyzed. At times the object under motion can be used, but often placing an additional object in the plane of motion is required.

Follow the direction in the *Instructions* box and define the length of an object that you have measured for the computer. Zooming in can help with clearly finding the boundaries. Once this is completed, input the scale length with proper units. Read the directions in the *Instructions* box carefully.

Lastly, decide if you want to rotate your coordinate axes. If you choose not to rotate the axes, the computer will use the first calibration point as the origin with positive x to the right and positive y up. If you choose to rotate your axis, follow the directions in the *Instructions* box very carefully. Your chosen axes will appear on the screen once the process is complete. This option may also be used to reposition the origin of the coordinate system, should you require it, however it might be best to start completely over.

Once you have completed this process, select Quit Calibration.

ANALYSIS PREDICTIONS

This video analysis relies on your graphical skills to interpret the data from the videos. Before doing your analysis, you should be familiar with the **Review of Graphs** and **Accuracy, Precision and Uncertainties** appendices..

Before analyzing the data, enter your prediction of how you expect the data to behave. This pattern of making predictions before obtaining results is the only reliable way to take data. How else can you know if something has gone wrong? This happens so often that it is given a name (Murphy's Law). It is also a good way to make sure you have learned something, but only if you stop to think about the discrepancies or similarities between your prediction and the results.

In order to enter your prediction into the computer, you first need to decide on your coordinate axes, origin, and scale (units) for your motion. Record these in your lab journal.

Next you will need to select the generic equation, f(z), which describes the graph you expect for the motion along your x-axis seen in your video. You must choose the appropriate function that matches the predicted curve. The analysis program is equipped with several equations, which are accessible using the pull-down menu on the equation line. The available equations are shown to the right.	$\checkmark$ f(z) = A + B z f(z) = A + B z + C z^2 f(z) = A + B z + C z^2 + D z^3 f(z) = A + B z + C z^2 + D z^3 + E z^4 f(z) = A + B sin (C z + D) f(z) = A + B cos (C z + D) f(z) = A + B sin (C + D z + E z^2) f(z) = A + B cos (C + D z + E z^2) f(z) = A + B exp (- C z) f(z) = A + B (1 - exp (- C z)) f(z) = A + (B z + C) sin (D + E z + F z^2) f(z) = A + (B z + C) cos (D + E z + F z^2)

You can change the equation to one you would like to use by clicking on the arrows to the left of the equation

After selecting your generic equation, you next need to enter your best approximation for the parameters A and B and C and D where you need them. Take this time to think of the physical meaning of the parameters and what units these constants are in. Note that in the equations z stands for time. If you took good notes of these values during the filming of your video, inputting these values should be straightforward.

Once you are satisfied that the equation you selected for your motion and the values of the constants are correct, click "*Accept*" in the *Main Controls*.   Your prediction equation will then show up on the graph on the computer screen. If you wish to change your prediction simply repeat the above procedure. Repeat this procedure for the Y direction.

DATA COLLECTION

To collect data, you first need to identify a very specific point on the object whose motion you are analyzing. Next, move the cursor over this point and click the green *ADD Data Point* button in Data Acquisition control box. The computer records this position and time. The computer will automatically advance the video to the next frame, leaving a mark on the point you have just selected. Then move the cursor back to the same place on the object and click *ADD Data Point* button again. As long as you always use the same point on the object, you will get reliable data from your analysis. It is helpful to zoom in on the object using the zoom slider under Video Controls, which will make it easier to click on the same point on the object. The data will automatically appear on the graph on your computer screen each time you accept a data point. If you don't see the data on the graph, you will need to change the scale of the axes. If you are satisfied with your data, choose *Quit Data Acq* from the *controls*

FITTING YOUR DATA

Deciding which equation best represents your data is the most important part of your data analysis. The actual mechanics of choosing the equation and constants is similar to what you did for your predictions.

First you must find your data on your graphs. Usually, you can find your full data set by using the Autorange buttons to the left of the graphs.

Secondly, after you find your data, you need to determine the best possible equation to describe this data. After you have decided on the appropriate equation, you need to determine the constants of this equation so that it best fits the data. Although this can be done by trial and error, it is much more efficient to think of how the behavior of the equation you have chosen depends on each parameter. Calculus can be a great help here.

Lastly, you need to estimate the uncertainty in your fit by deciding the range of other lines that *could* also fit your data. This method of estimating your uncertainty is described in the appendix **Accuracy, Precision and Uncertainty**. Slightly changing the values for each constant in turn will allow you to do this quickly. For example, the X-motion plots below show both the predicted line (down) and two other lines that also fit the data (near the circles).
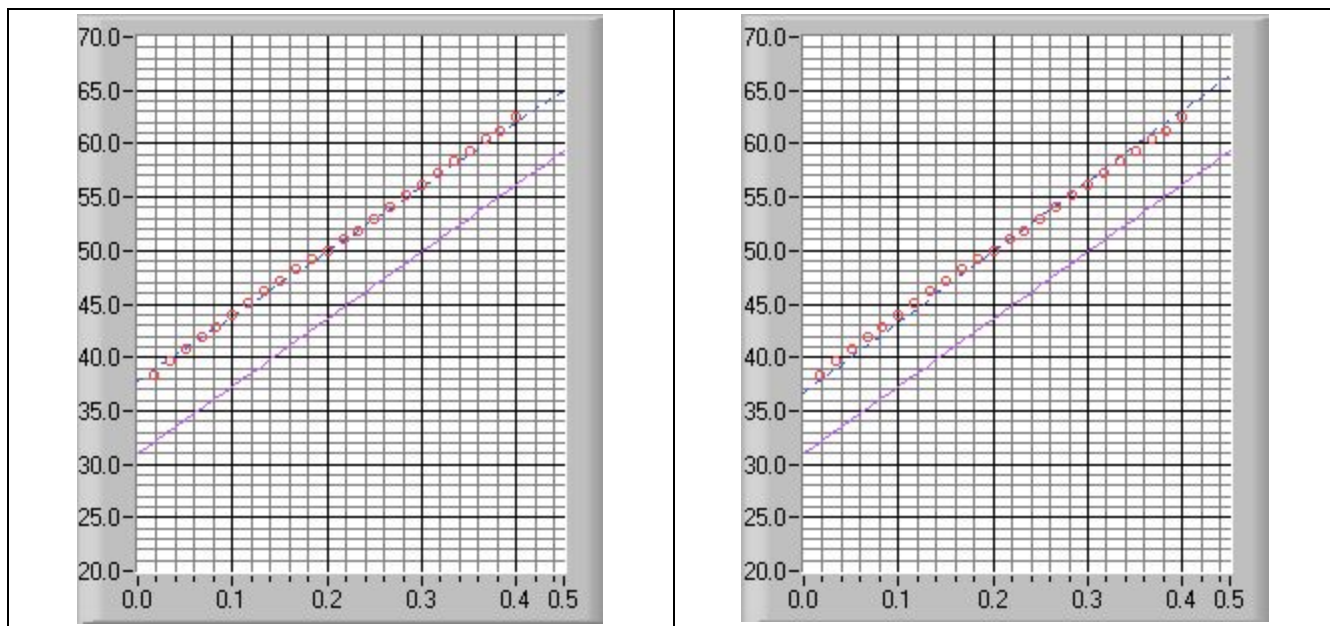
After you have found the uncertainties in your constants, return to your best-fit line and use it as your fit by selecting *Accept x- (or y-) fit* in the *Program Controls* panel.

LAST WORDS

These directions are not meant to be exhaustive. You will discover more features of the video analysis program as you use it. Be sure to record these features in your lab journal.

Appendix: **Significant Figures**

Calculators make it possible to get an answer with a huge number of figures. Unfortunately, many of them are meaningless. For instance, if you needed to split $1.00 among three people, you could never give them each exactly $0.333333… The same is true for measurements. If you use a meter stick with millimeter markings to measure the length of a key, as in Figure 1, you could not measure more precisely than a quarter or half or a third of a mm. Reporting a number like 5.37142712 cm would not only be meaningless, it would be misleading.

Figure 1
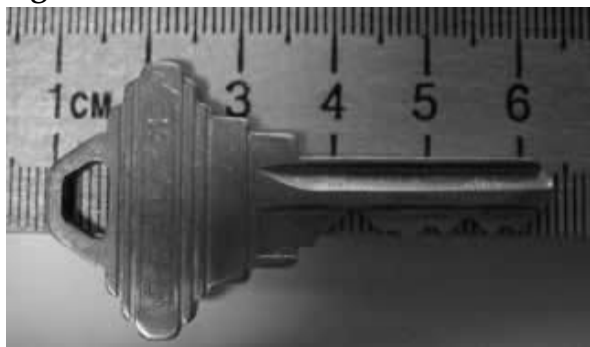
In your measurement, you can precisely determine the distance down to the nearest millimeter and then improve your precision by estimating the next figure. It is always assumed that the last figure in the number recorded is uncertain. So, you would report the length of the key as 5.37 cm. Since you estimated the 7, it is the uncertain figure. If you don't like estimating, you might be tempted to just give the number that you know best, namely 5.3 cm, but it is clear that 5.37 cm is a better report of the measurement. An estimate is always necessary to report the most precise measurement. When you quote a measurement, the reader will always assume that the last figure is an estimate. Quantifying that estimate is known as **estimating uncertainties**. Appendix C will illustrate how you might use those estimates to determine the uncertainties in your measurements.

What are significant figures?

The number of significant figures tells the reader the precision of a measurement. Table 1 gives some examples.

Table 1

Length (centimeters)	Number of Significant Figures
12.74	4
11.5	3
1.50	3
1.5	2
12.25345	7
0.8	1
0.05	1

One of the things that this table illustrates is that not all zeros are significant. For example, the zero in 0.8 is not significant, while the zero in 1.50 is significant. Only the zeros that appear after the first non-zero digit are significant.

A good rule is to always express your values in scientific notation. If you say that your friend lives 143 m from you, you are saying that you are sure of that distance to within a few meters (3 significant figures). What if you really only know the distance to a few tens of meters (2 significant figures)? Then you need to express the distance in scientific notation 1.4×10^2 m.

Is it always better to have more figures?

Consider the measurement of the length of the key shown in Figure 1. If we have a scale with ten etchings to every millimeter, we could use a microscope to

measure the spacing to the nearest tenth of a millimeter and guess at the one hundredth millimeter. Our measurement could be 5.814 cm with the uncertainty in the last figure, four significant figures instead of three. This is because our improved scale allowed our estimate to be more precise. This added precision is shown by more significant figures. The more significant figures a number has, the more precise it is.

How do I use significant figures in calculations?

When using significant figures in calculations, you need to keep track of how the uncertainty propagates. There are mathematical procedures for doing this estimate in the most precise manner. This type of estimate depends on knowing the statistical distribution of your measurements. With a lot less effort, you can do a cruder estimate of the uncertainties in a calculated result. This crude method gives an overestimate of the uncertainty but it is a good place to start. For this course this simplified uncertainty estimate (described in Appendix C and below) will be good enough.

Addition and subtraction

When adding or subtracting numbers, the number of decimal places must be taken into account.

*The result should be given to as many decimal places as the term in the sum that is given to the **smallest** number of decimal places.*

Examples:

Addition	Subtraction
6.24**2**	5.87**5**
+4.2**3**	-3.3**4**
+0.01**3**	2.5**35**
10.4**85**	

10.4**9**	2.5**4**

The uncertain figures in each number are shown in **bold-faced** type.

Multiplication and division

When multiplying or dividing numbers, the number of significant figures must be taken into account.

*The result should be given to as many significant figures as the term in the product that is given to the **smallest** number of significant figures.*

The basis behind this rule is that the least accurately known term in the product will dominate the accuracy of the answer.

As shown in the examples, this does not always work, though it is the quickest and best rule to use. When in doubt, you can keep track of the significant figures in the calculation as is done in the examples.

Examples:

Multiplication

15.8**4**	17.2**7**
x 2.**5**	x 4.**0**
7920	69.**080**
3168	
3**9.600**	
40	69

Division

11**7**	2**5**
23)269**1**	75)187**5**
23	150
39	375
2**3**	375
1**61**	
1**61**	
1.**2** x 10²	2.**5** x 10¹

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PRACTICE EXERCISES

1. Determine the number of significant figures of the quantities in the following table:

Length (centimeters)	Number of Significant Figures
17.87	
0.4730	
17.9	
0.473	
18	
0.47	
1.34×10^2	
2.567×10^5	
2.0×10^{10}	
1.001	
1.000	
1	
1000	
1001	

2. Add: 121.3 to 6.7×10^2:

3. Multiply: 34.2 and 1.5×10^4

How tall are you? How old are you? When you answered these everyday questions, you probably did it in round numbers such as "five foot, six inches" or "nineteen years, three months." But how true are these answers? Are you exactly 5' 6" tall? Probably not. You estimated your height at 5' 6" and just reported two significant figures. Typically, you round your height to the nearest inch, so that your actual height falls somewhere between 5' 5½" and 5' 6½" tall, or 5' 6" ± ½". This ± ½" is the **uncertainty,** and it informs the reader of the precision of the **value** 5' 6".

What is uncertainty?

Whenever you measure something, there is always some uncertainty.   There are two categories of uncertainty: **systematic** and **random**.

(1)   **Systematic uncertainties** are those that consistently cause the value to be too large or too small. Systematic uncertainties include such things as reaction time, inaccurate meter sticks, optical parallax and miscalibrated balances. In principle, systematic uncertainties can be eliminated if you know they exist.

(2)   **Random uncertainties** are variations in the measurements that occur without a predictable pattern. If you make precise measurements, these uncertainties arise from the estimated part of the measurement. Random uncertainty can be reduced, but never eliminated. We need a technique to report the contribution of this uncertainty to the measured value.

Uncertainties cause every measurement you make to be **distributed**. For example, the key in Figure 2 is approximately 5.37cm long. For the sake of argument, pretend that it is exactly 5.37cm long. If you measure its length many times, you expect that most of the measurements will be close to, but not exactly, 5.37cm, and that there will be a few measurements much more than or much less than 5.37cm. This effect is due to random uncertainty. You can never know how accurate any single measurement is, but you expect that many measurements will cluster around the real length, so you can take the average as the "real" length, and more measurements will give you a better answer; see Figure 1.

Figure 1
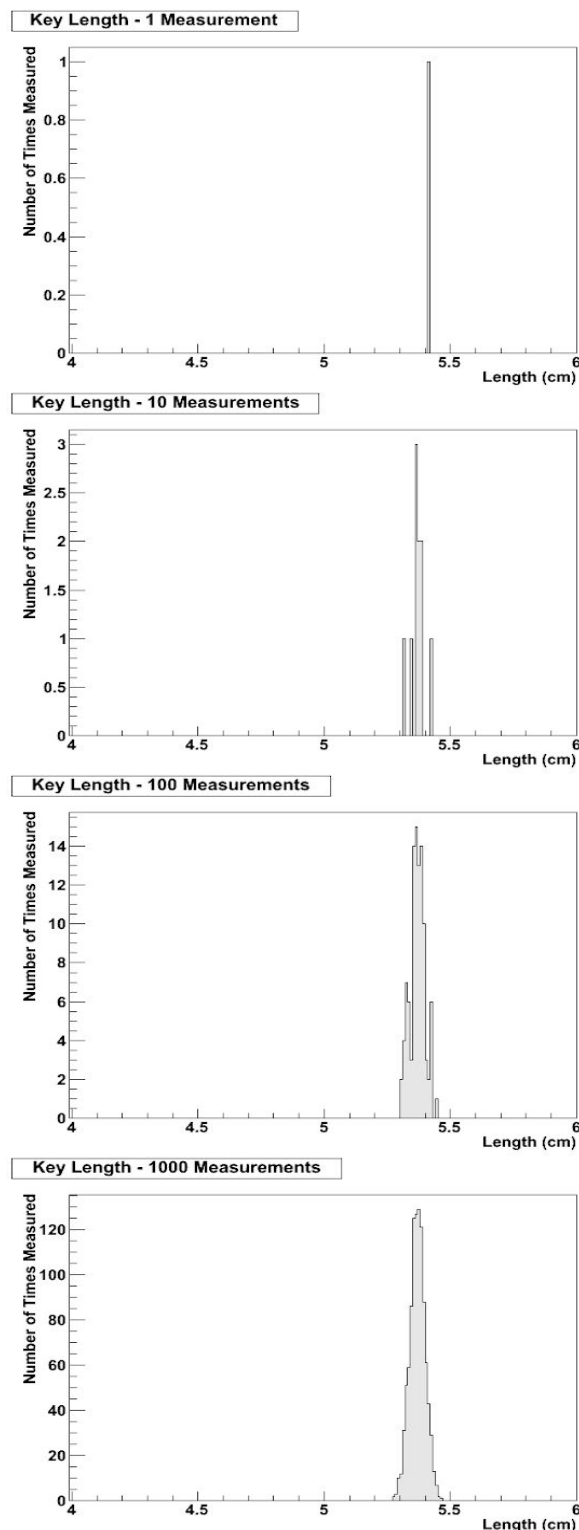
You must be very careful to estimate or eliminate (by other means) systematic uncertainties well because they cannot be eliminated in this way; they would just shift the distributions in Figure 1 left or right.

Roughly speaking, the average or "center" of the distribution is the "measurement," and the width or "deviation" of the distribution is the random uncertainty.

How do I determine the uncertainty?

This Appendix will discuss three basic techniques for determining the uncertainty: **estimating the uncertainty,** measuring the **average deviation**, and finding the **uncertainty in a linear fit**. Which one you choose will depend on your situation, your available means of measurement, and your need for precision. If you need a precise determination of some value, and you are measuring it directly (e.g., with a ruler or thermometer), the best technique is to measure that value several times and use the average deviation as the uncertainty. Examples of finding the average deviation are given below.

How do I estimate uncertainties?

If time or experimental constraints make repeated measurements impossible, then you will need to estimate the uncertainty. When you estimate uncertainties you are trying to account for anything that might cause the measured value to be different if you were to take the measurement again. For example, suppose you were trying to measure the length of a key, as in Figure 2.

Figure 2
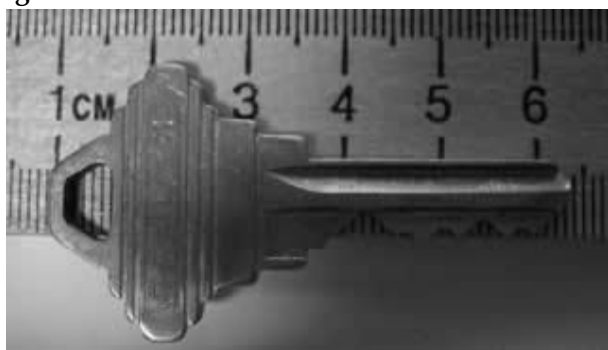
If the true value were not as important as the magnitude of the value, you could say that the key's length was 5cm, give or take 1cm. This is a crude estimate, but it may be acceptable. A better estimate of the key's length, as you saw in Appendix A, would be 5.37cm. This tells us that the worst our measurement could be off is a fraction of a mm. To be more precise, we can estimate it to be about a third

of a mm, so we can say that the length of the key is 5.37 ± 0.03 cm.

Another time you may need to estimate uncertainty is when you analyze video data. Figures 3 and 4 show a ball rolling off the edge of a table. These are two consecutive frames, separated in time by 1/30 of a second.

Figure 3
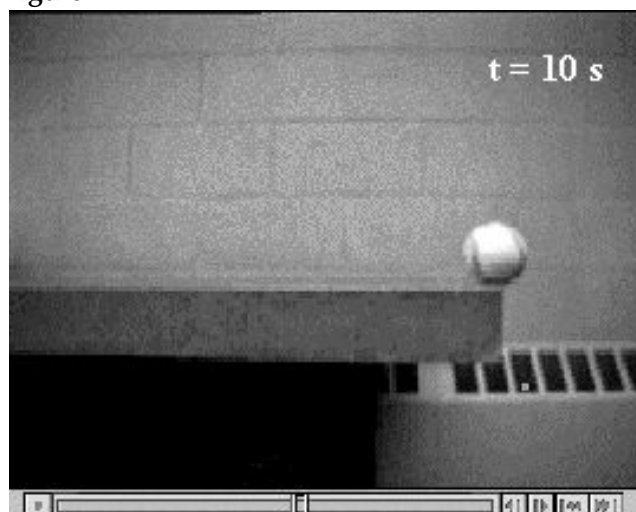
Figure 4
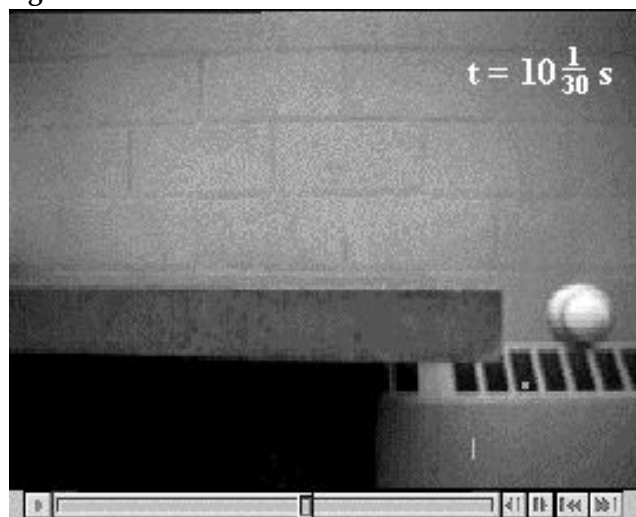
The exact moment the ball left the table lies somewhere between these frames. We can estimate that this moment occurs midway between them ($t = 10\frac{1}{60} s$). Since it must occur at some point between them, the worst our estimate could be off by

is $\frac{1}{60}s$. We can therefore say the time the ball leaves

the table is $t = 10\frac{1}{60} \pm \frac{1}{60}s.$

How do I find the average deviation?

If estimating the uncertainty is not good enough for your situation, you can experimentally determine the un-certainty by making several measurements and calculating the average deviation of those measurements. To find the average deviation: (1) Find the average of all your measurements; (2) Find the absolute value of the difference of each measurement from the average (its deviation); (3) Find the average of all the deviations by adding them up and dividing by the number of measurements. Of course you need to take enough measurements to get a distribution for which the average has some meaning.

In example 1, a class of six students was asked to find the mass of the same penny using the same balance. In example 2, another class measured a different penny using six different balances. Their results are listed below:

Class 1: Penny A massed by six different students on the same balance.

Mass (grams)
3.110
3.125
3.120
3.126
3.122
<u>3.120</u>
3.121 average.
The deviations are: 0.011g, 0.004g, 0.001g, 0.005g, 0.001g, 0.001g
Sum of deviations: 0.023g
Average deviation: (0.023g)/6 = 0.004g
Mass of penny A: 3.121 ± 0.004g

Class 2: Penny B massed by six different students on six different balances

Mass (grams)
3.140
3.133
3.144
3.118
3.126
<u>3.125</u>
3.131 average
The deviations are: 0.009g, 0.002g, 0.013g, 0.013g, 0.005g, 0.006g
Sum of deviations: 0.048g
Average deviation: (0.048g)/6= 0.008g
Mass of penny B: 3.131 ± 0.008g

Finding the Uncertainty in a Linear Fit

Sometimes, you will need to find the uncertainty in a linear fit to a large number of measurements. The most common situation like this that you will encounter is fitting position or velocity with respect to time from MotionLab.

When you fit a line to a graph, you will be looking for the "best fit" line that "goes through the middle" of the data; see the appendix about graphs for more about this procedure. To find the uncertainty, draw the lines with the greatest and least slopes that still roughly go through the data. These will be the upper and lower limits of the uncertainty in the slope. These lines should also have lesser and greater y-intercepts than the "best fit" line, and they define the lower and upper limits of the uncertainty in the y-intercept.

Note that when you do this, the uncertainties above and below your "best fit" values will, in general, **not** be the same; this is different than the other two methods we have presented.

For example, in Figure 5, the y-intercept is 4.25 +2.75/-2.00, and the slope is 0.90 +0.20/-0.25.

Figure 5a

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Figure 5b

Greatest Slope, Least y-Intercept

y = 2 + 1.1x

Figure 5c

Least Slope, Greatest y-Intercept

y = 7 + 0.65x
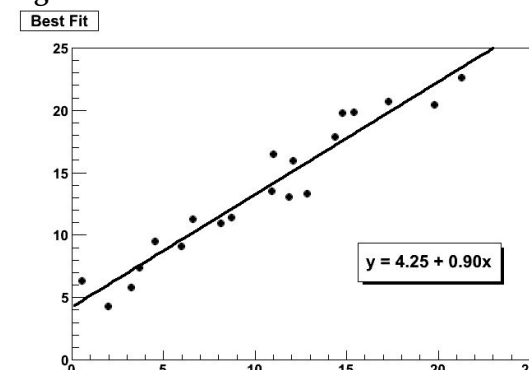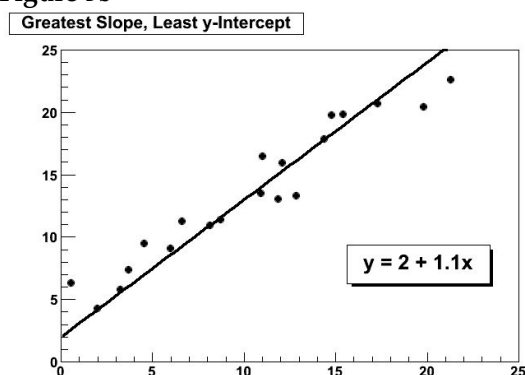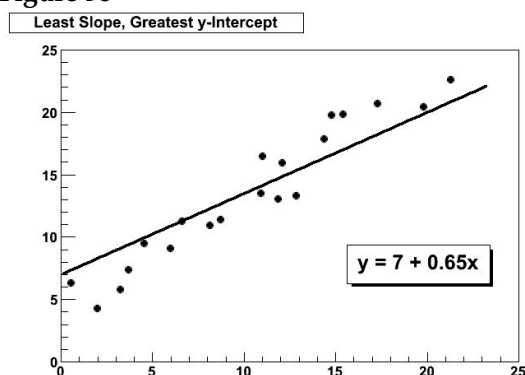
However you choose to determine the uncertainty, you should always state your method clearly in your report.

How do I know if two values are the same?

Go back to the pennies. If we compare only the average masses of the two pennies we see that they are different. But now include the uncertainty in the masses. For penny A, the most likely mass is somewhere between 3.117g and 3.125g. For penny B, the most likely mass is somewhere between 3.123g and 3.139g. If you compare the ranges of the masses for the two pennies, as shown in Figure 6, they just overlap. Given the uncertainty in the masses, we are able to conclude that the masses of the two pennies could be the same. If the range of the masses did not overlap, then we ought to conclude that the masses are probably different.

Figure 6
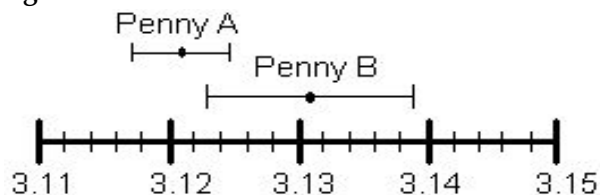
Mass of pennies (in grams) with uncertainties

An important application of this is determining agreement between experimental and theoretical values. If you use a formula to generate a theoretical value of some quantity and use the method below to generate the uncertainty in the calculation, and if you generate an experimental value of the same quantity by measuring it and use the method above to generate the uncertainty in the measurement, you can compare the two values in this way. If the ranges overlap, then the theoretical and experimental values agree. If the ranges do not overlap, then the theoretical and experimental values do not agree.

What are R^2, X^2, and p?

It sometimes happens in statistical analysis that instead of determining whether two numbers agree, you need to determine whether a function ("theoretical value") and some data ("experimental value") agree. Our method of comparing two numbers with uncertainties is too primitive for this task. R^2 (the Pearson correlation), X^2 (Greek letter "Chi," not Roman X), and p are numbers that describe how well these things agree. They are too sophisticated for this appendix, but you may see them from time to time. If you feel comfortable with some basic statistics, you can look them up. You should never need to calculate them by hand; let your fitting software do it for you if your analysis gets that sophisticated. The most you might encounter in this class is that spreadsheet programs will give you R^2 if you use them to fit data; for your purposes, you can consider your fit "good" if $R^2 \geq 0.95$.

Which result is more precise?

Suppose you use a meter stick to measure the length of a table and the width of a hair, each with an uncertainty of 1 mm. Clearly you know more about the length of the table than the width of the hair. Your measurement of the table is very precise but your measurement of the width of the hair is rather crude. To express this sense of precision, you need to calculate the percentage uncertainty. To do this, divide the uncertainty in the measurement by the value of the measurement itself, and then multiply by 100%. For example, we can calculate the precision in the measurements made by class 1 and class 2 as follows:

Precision of Class 1's value:
(0.004 g ÷ 3.121 g) x 100% = 0.1 %
Precision of Class 2's value:
(0.008 g ÷ 3.131 g) x 100% = 0.3 %

Class 1's results are more precise. This should not be surprising since class 2 introduced more uncertainty in their results by using six different balances instead of only one.

Which result is more accurate?

Accuracy is a measure of how your measured value compares with the real value. Imagine that class 2 made the measurement again using only one balance. Unfortunately, they chose a balance that was poorly calibrated. They analyzed their results and found the mass of penny B to be 3.556 ± 0.004 g. This number is more precise than their previous result since the uncertainty is smaller, but the new measured value of mass is very different from their previous value. We might conclude that this new value for the mass of penny B is different, since the range of the new value does not overlap the range of the previous value. However, that conclusion would be **wrong** since our uncertainty has not taken into account the inaccuracy of the balance. To determine the accuracy of the measurement, we should check by measuring something that is known. This procedure is called calibration, and it is absolutely necessary for making accurate measurements.

Be cautious! It is possible to make measurements that are extremely precise and, at the same time, grossly inaccurate.

How can I do calculations with values that have uncertainty?

When you do calculations with values that have uncertainties, you will need to estimate (by calculation) the uncertainty in the result. There are mathematical techniques for doing this, which depend on the statistical properties of your measurements.

The following are rules that can all be derived from the Gaussian equation for normally distributed errors. You will occasionally need to use each of the rules below, often in combination with each other. You are not expected to derive them, but you should be prepared to use them.

These techniques help you estimate the random uncertainty that always occurs in measurements. They will not help account for mistakes or poor measurement procedures. There is no substitute for taking data with the utmost of care. A little forethought about the possible sources of uncertainty can go a long way in ensuring precise and accurate data.

Addition or subtraction: If you have measured values for the quantities X, Y, and Z, with uncertainties δX, δY, and δZ, and your final result, R, is the sum or difference of these quantities, then the uncertainty δR is:

$$R = X + Y - Z \quad \delta R = \sqrt{(\delta X)^2 + (\delta Y)^2 + (\delta Z)^2}$$

Example: Suppose you measure the length of a car and get 4.50 ± 0.03 m. This means that L = 4.50 m and δL = 0.03 m. You know the car has a steering wheel which is a length l = 2.27 ± 0.04 m from the edge of the front bumper. You are asked to find the distance from the steering wheel to the edge of the rear bumper, this is R = L − l = 2.23 m. What is the uncertainty in R? Using the above equation,

$$\delta R = \sqrt{(\delta L)^2 + (\delta l)^2}$$
$$= \sqrt{(0.03\ m)^2 + (0.04\ m)^2}$$
$$= \sqrt{0.0009\ m^2 + 0.0016\ m^2}$$
$$= \sqrt{0.0025\ m^2} = .05\ m$$

So for this example, R = 2.23 ± .05 m.

Multiplication or division: If you have measured values for the quantities X, Y, and Z, with uncertainties δX, δY, and δZ, and your final result, R, is the product of these quantities through multiplication and division, then the uncertainty δR is:

$$R = \frac{X \cdot Y}{Z} \qquad \delta R = |R|\sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2 + \left(\frac{\delta Z}{Z}\right)^2}$$

Example: Suppose a Frisbee is tossed a distance d = 100 ± 3 m during a time of flight t = 15.0 ± 1.2 s. The average speed of the Frisbee is v = d/t = 6.0 m/s. What is the uncertainty of v? Using the above equation,

$$\delta v = |v|\sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta t}{t}\right)^2}$$
$$= |v|\sqrt{\left(\frac{3\,m}{100\,m}\right)^2 + \left(\frac{1.2\,s}{15.0\,s}\right)^2}$$
$$= |v|\sqrt{(3\%)^2 + (8\%)^2}$$
$$= |v|\sqrt{.0009 + .006} = |v|(8.5\%)$$
$$= |v|(8.5\%) = (6\tfrac{m}{s})(8.5\%) = 0.51 \; m/s$$

So for this example, v = 6.0 ± .5 m/s.

Multiplication with a constant: If you have measured value for the quantity X with uncertainties δX and you need to multiply it with a constant that is exactly known, your final result, R, is the product of these quantities and the uncertainty δR is:

$$R = c \cdot X \qquad \delta R = |c|\delta X$$

Example: Suppose you measure the diameter of a sphere to be d = 2.00 ± 0.08 cm and you wish to calculate the radius with uncertainty. The radius is simply r = d/2 = 1.00cm. Using the above equation for multiplication,

$$\delta r = |r|\sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{0}{c}\right)^2}$$
$$= |r|\sqrt{\left(\frac{.08\,cm}{2\,cm}\right)^2 + (0)^2}$$
$$= |r|\sqrt{(4\%)^2} = |r|(4\%) = 0.04\text{cm}$$

Using the multiplication with a constant rule,
$$\delta r = |c|\delta d = \tfrac{1}{2}(0.08 \text{ cm}) = 0.04\text{cm}$$

So for this example, r = 1.00 ± 0.04 cm.

Polynomial functions: If you have measured value for the quantity X with uncertainties δX and n is an exact number and you need to find the uncertainty δR, the following rule applies where R is a polynomial function of one variable X.

$$R = X^n \qquad \delta R = |n| \cdot \frac{\delta X}{|X|} \cdot |R|$$

Example: Suppose you have the formula L = A$^{-1/2}$ where A = 44 ± 2 m^2. Calculating we get L = 6.63 m. Using the equation for polynomial uncertainty propagation we get,

$$\delta L = |n| \cdot \frac{\delta A}{|A|} \cdot |L|$$
$$\delta L = |1/2| \cdot \frac{2\,m^2}{|44\,m^2|} \cdot |6.63 \; m| = 0.15 \; m$$

So for this example, L = 6.6 ± 0.2 m.

A special case exists where n = -1. For this case, the rule states that the uncertainty is unchanged if you take the reciprocal of a quantity.

Example: Suppose the period of oscillation is measured to be T = 0.10 ± 0.01 s. The frequency is f = 1/T = 10 Hz. What is the uncertainty in f?

$$\delta f = |n| \cdot \tfrac{\delta T}{|T|} \cdot |f|$$
$$\delta f = |1| \cdot \tfrac{0.01\ s}{|.10|} \cdot |10\ hz| = 1\ hz$$

So for this example, f = 10 ± 1 hz

For more complicated formulas: At times you will need to propagate uncertainties through more complicated formula. The above rules can generally be used in combination to solve most error propagation problems.

Example: Suppose a ball is tossed straight up with initial speed of v_o = 4.0 ± 0.2 m/s. After a time t = 0.60 ± 0.06 s, the height of the ball is $h = v_o t - \tfrac{1}{2} g t^2 = 0.636\ m$. What is the uncertainty of h? Assume g = 9.80 m/s² with no uncertainty.

You can break up the problem. Let $y = v_o t = $ 2.4 m and $z = \tfrac{1}{2} g t^2 = 1.764\ m$. Then, using the multiplication rule, we can get the uncertainty in y:

$$\delta y = |y| \sqrt{\left(\tfrac{\delta v_o}{v_o}\right)^2 + \left(\tfrac{\delta t}{t}\right)^2}$$
$$= |y| \sqrt{\left(\tfrac{0.2\ m}{4\ m}\right)^2 + \left(\tfrac{0.06\ s}{0.60\ s}\right)^2}$$
$$= v_o t \sqrt{(0.05)^2 + (0.10)^2}$$
$$= (2.4\ m)\,(0.112) = 2.7\ m$$

For δ z, we can use the power rule:

$$\delta z = |n| \cdot \tfrac{\delta t}{|t|} \cdot |z|$$
$$= |2| \cdot \tfrac{0.06\ s}{|0.60\ s|} \cdot |1.764\ m| = 0.35\ m$$

Finally, h = y + z, so we get δ h from the sum rule:

$$\delta h = \sqrt{(\delta y)^2 + (\delta z)^2}$$
$$= \sqrt{(2.7 m)^2 + (0.35\ m)^2}$$
$$= 0.44\ m$$

So for this example, h = 0.6 ± 0.4 m.

General functions: If you have R that is a function of X and Y, the uncertainty in R is obtained by taking the partial derivatives of R with respect to each variable, multiplying with the uncertainty in that variable and adding the individual terms.

$$R = R(X, Y, \ldots) \quad \delta R = \sqrt{\left(\tfrac{\partial R}{\partial X} \cdot \delta X\right)^2 + \left(\tfrac{\partial R}{\partial Y} \cdot \delta Y\right)^2 + \ldots}$$

PRACTICE EXERCISES:

B-1. Consider the following results for different experiments. Determine if they agree with the accepted result listed to the right. Also calculate the precision for each result.∂

a) $g = 10.4 \pm 1.1$ m/s^2 $g = 9.8$ m/s^2

b) $T = 1.5 \pm 0.1$ sec $T = 1.1$ sec

c) $k = 1368 \pm 45$ N/m $k = 1300 \pm 50$ N/m

B-2.   The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be 5.37 ± 0.05 cm. The width was found to be 3.42 ± 0.02 cm. What is the area and the deviation using the formula for multiplying?

B-3.   Each member of your lab group weighs the cart and two mass sets twice. The following table shows this data. Calculate the total mass of the cart with each set of masses and for the two sets of masses combined.

Cart (grams)	Mass set 1 (grams)	Mass set 2 (grams)
201.3	98.7	95.6
201.5	98.8	95.3
202.3	96.9	96.4
202.1	97.1	96.2
199.8	98.4	95.8
200.0	98.6	95.6

Graphs are visual tools used to represent relationships (or the lack thereof) among numerical quantities in mathematics. In particular, we are interested in the graphs of functions.

What is a graph?

In this course, we will be dealing almost exclusively with graphs of functions. When we graph a quantity A with respect to a quantity B, we mean to put B on the horizontal axis and A on the vertical axis of a two-dimensional region and then to draw a set of points or curve showing the relationship between them. We do not mean to graph any other quantity from which A or B can be determined. For example, a plot of acceleration versus time has acceleration itself, $a(t)$, on the vertical axis, not the corresponding velocity $v(t)$; the time t, of course, goes on the horizontal axis. See Figure 1.

(a)   (b)
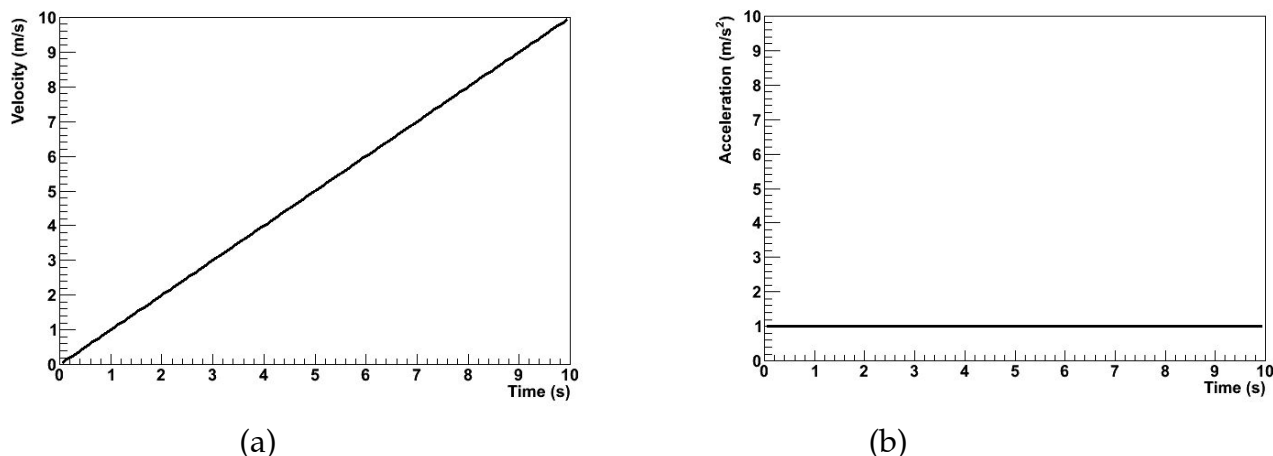
Figure 1: Graphs of acceleration a and velocity v for an object in 1-dimensional motion with constant acceleration.

Traditionally, we call the vertical axis the "y-" axis; the horizontal axis, the "x-" axis. Please note that there is nothing special about these variables. They are not fixed, and they have no special meaning. If we are graphing, say, a velocity function $v(t)$ with respect to time t, then we do not bother trying to identify $v(t)$ with y or t with x; in that case, we just forget about y and x. This can be particularly important when representing position with the variable x, as we often do in physics. In that case, graphing $x(t)$ with respect to t would give us an x on both the vertical and horizontal axes, which would be extremely confusing. We can even imagine a scenario wherein we should graph a function x of a variable y such that y would be on the horizontal axis and $x(y)$ would be on the vertical axis. In particular, in MotionLab, the variable z, not x, is always used for the horizontal axis; it represents time. Both x and y are plotted on vertical axes as functions of the time z.

There are graphs which are not graphs of functions, e.g. pie graphs. These are not of

relevance to this course, but much of what is contained in this document still applies.

Data, Uncertainties, and Fits

When we plot empirical data, it typically comes as a set of ordered pairs (x, y). Instead of plotting a curve, we just draw dots or some other kind of marker at each ordered pair.

Empirical data also typically comes with some uncertainty in the independent and dependent variables of each ordered pair. We need to show these uncertainties on our graph; this helps us to interpret the region of the plane in which the true value represented by a data point might lie. To do this, we attach error bars to our data points. Error bars are line segments passing through a point and representing some confidence interval about it.

After we have plotted data, we often need to try to describe that data with a functional relationship. We call this process "fitting a function to the data" or, more simply, "fitting the data." There are long, involved statistical algorithms for finding the functions that best fit data, but we won't go into them here. The basic idea is that we choose a functional form, vary the parameters to make it look like the experimental data, and then see how it turns out. If we can find a set of
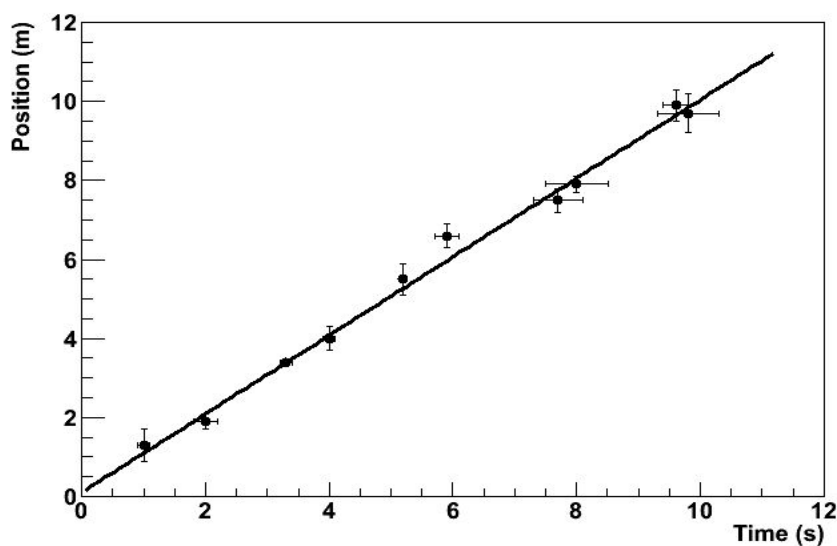
Figure 2: An empirical data set with associated uncertainties and a best-fit line.

parameters that make the function lie very close to most of the data, then we probably chose the right functional form. If not, then we go back and try again. In this class, we will be almost exclusively fitting lines because this is easiest kind of fit to perform by eye. Quite simply, we draw the line through the data points that best models the set of data points in question. The line is not a "line graph;" we do not just connect the dots (That would almost never be a line, anyway, but just a series of line segments.). The line does not actually need to

pass through any of the data points. It usually has about half of the points above it and half of the points below it, but this is not a strict requirement. It should pass through the confidence intervals around most of the data points, but it does not need to pass through all of them, particularly if the number of data points is large. Many computer programs capable of producing graphs have built-in algorithms to find the best possible fits of lines and other functions to data sets; it is a good idea to learn how to use a high-quality one.

Making Graphs Say Something

So we now know what a graph is and how to plot it; great. Our graph still doesn't say much; take the graph in Figure 4(a). What does it mean? Something called q apparently varies quadratically with something called τ, but that is only a mathematical statement, not a physical one. We still need to attach physical meaning to the mathematical relationship that the graph communicates. This is where labels come into play.

Graphs should always have labels on both the horizontal and vertical axes. The labels should be terse but sufficiently descriptive to be unambiguous. Let's say that q is position and τ is time in Figure 4. If the problem is one-dimensional, then the label "Position" is probably sufficient for the vertical axis (q). If the problem is two-dimensional, then we probably need another qualifier. Let's say that the object in question is moving in a plane and that q is the vertical component of its position; then "Vertical Position" will probably do the trick. There's still a problem with our axis labels. Look more closely; where is the object at $\tau = 6s$? Who knows? We don't know if the ticks represent seconds, minutes, centuries, femtoseconds, or even some nonlinear measure of time, like humans born. Even if we did, the vertical axis has no units, either. We need for the units of each axis to be clearly indicated if our graph is really to say something. We can tell from Figure 4(b) that the object is at $q = 36m$ at $\tau = 6s$. A grain of salt: our prediction graphs will not always need units. For example, if we are asked to draw a graph predicting the relationship of, say, the acceleration due to gravity of an object with respect to its mass, the label "Mass" will do just fine for our horizontal axis. This is because we are not expected to give the precise functional dependence in this situation, only the overall behavior. We don't know exactly what the acceleration will be at a mass of $10g$, and we don't care. We just need to show whether the variation is increasing, decreasing, constant, linear, quadratic, etc. In this specific case, it might be to our advantage to include units on the vertical axis, though; we can probably predict a specific value of the acceleration, and that value will be meaningless without them.
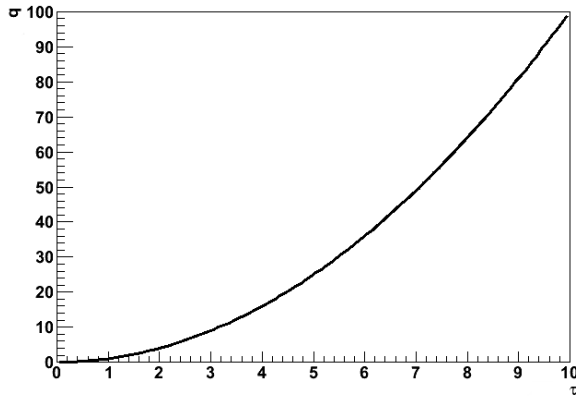
(a)
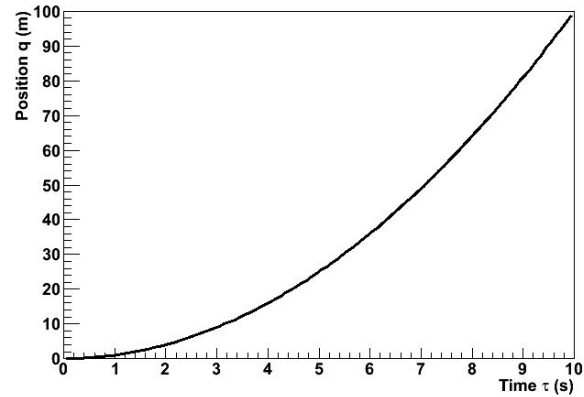
(b) Position q with respect to time τ for a mass of 3kg. The acceleration is constant.

Figure 4: Poorly- versus well-labeled and -captioned graphs. The labels and caption make the second graph much easier to interpret.

Every graph we make should also have some sort of title or caption. This helps the reader quickly to interpret the meaning of the graph without having to wonder what it's trying to say. It particularly helps in documents with lots of graphs. Typically, captions are more useful than just titles. If we have some commentary about a graph, then it is appropriate to put this in a caption, but not a title. Moreover, the first sentence in every caption should serve the same role as a title: to tell the reader what information the graph is trying to show. In fact, if we have an idea for the title of a graph, we can usually just put a period after it and let that be the first "sentence" in a caption. For this reason, it is typically redundant to include both a title and a caption. After the opening statement, the caption should add any information important to the interpretation of a graph that the graph itself does not communicate; this might be an approximation involved, an indication of the value of some quantity not depicted in the graph, the functional form of a fit line, a statement about the errors, etc. Lastly, it is also good explicitly to state any important conclusion that the graph is supposed to support but does not obviously demonstrate. For example, let's look at Figure 4 again. If we are trying to demonstrate that the acceleration is constant, then we would not need to point this out for a graph of the object's acceleration with respect to time. Since we did not do that, but apparently had some reason to plot position with respect to time instead, we wrote, "The acceleration is constant."

Lastly, we should choose the ranges of our axes so that our meaning is clear. Our axes do not always need to include the origin; this may just make the graph more difficult to interpret. Our data should typically occupy most of the graph to make it easier to interpret; see Figure 5. However, if we are trying to demonstrate a functional form, some extra space beyond any statistical error helps to prove our point; in Figure 5(c), the variation of the dependent with respect to the independent variable is obscured by the random variation of the data. We must

be careful not to abuse the power that comes from freedom in
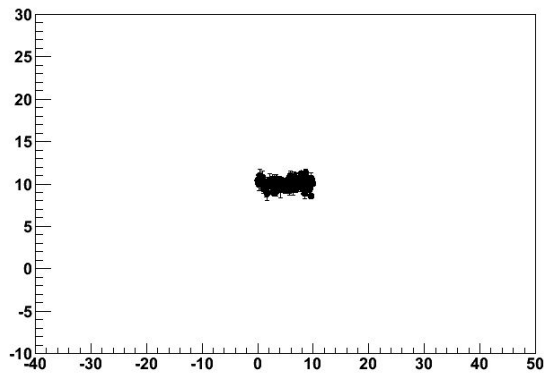
(a)
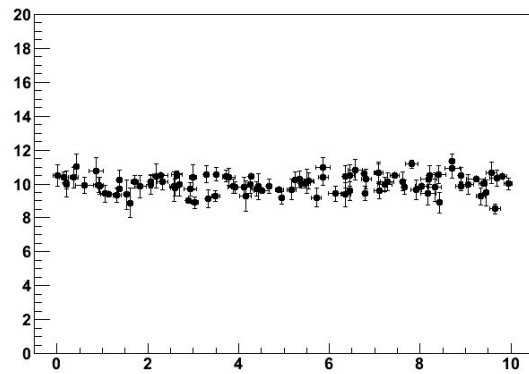
(b)
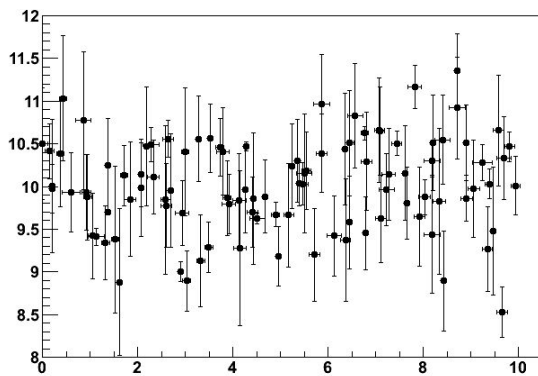
(c)

Figure 5: Graphs with too much (a), just enough (b), and too little space (c) to be easy to interpret.

plotting our data, however. Graphs can be and frequently are drawn in ways intended to manipulate the perceptions of the audience, and this is a violation of scientific ethics. For example, consider Figure 6. It appears that Candidate B has double the approval of Candidate A, but a quick look at the vertical axis shows that the lead is actually less than one part in seventy. The moral of the story is that our graphs should always be designed to communicate our point, but not to create our point.
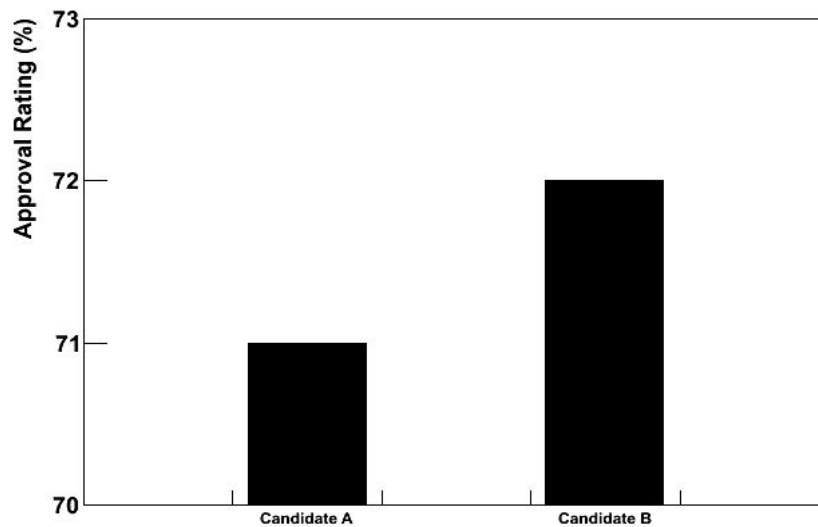
Figure 6: Approval ratings for two candidates in a mayoral race. This graph is designed to mislead the reader into believing that Candidate B has a much higher approval rating than Candidate A.

Using Linear Relationships to Make Graphs Clear

The easiest kind of graph to interpret is often a line. Our minds are very good at interpreting lines. Unfortunately, data often follow nonlinear relationships, and our minds are not nearly as good at interpreting those. It is sometimes to our advantage to force data to be linear on our graph. There are two ways that we might want to do this in this class; one is with calculus, and the other is by cleverly choosing what quantities to graph.

The "calculus" method is the simpler of the two. Don't let its name fool you: it doesn't actually require any calculus. Let's say that we want to compare the constant accelerations of two objects, and we have data about their positions and velocities with respect to time. If the accelerations are very similar, then it might be difficult to decide the relationship from the position graphs because we have a hard time detecting fine variations in curvature. It is much easier to compare the accelerations from the velocity graphs because we then just have to look at the slopes of lines; see Figure 7. We call this the "calculus" method because velocity is the first derivative with respect to time of position; we have effectively chosen to plot the derivative of position rather than position itself. We can sometimes use these calculus-based relationships to graph more meaningful quantities than the obvious ones.

(a)

(b)

(c)

(d)
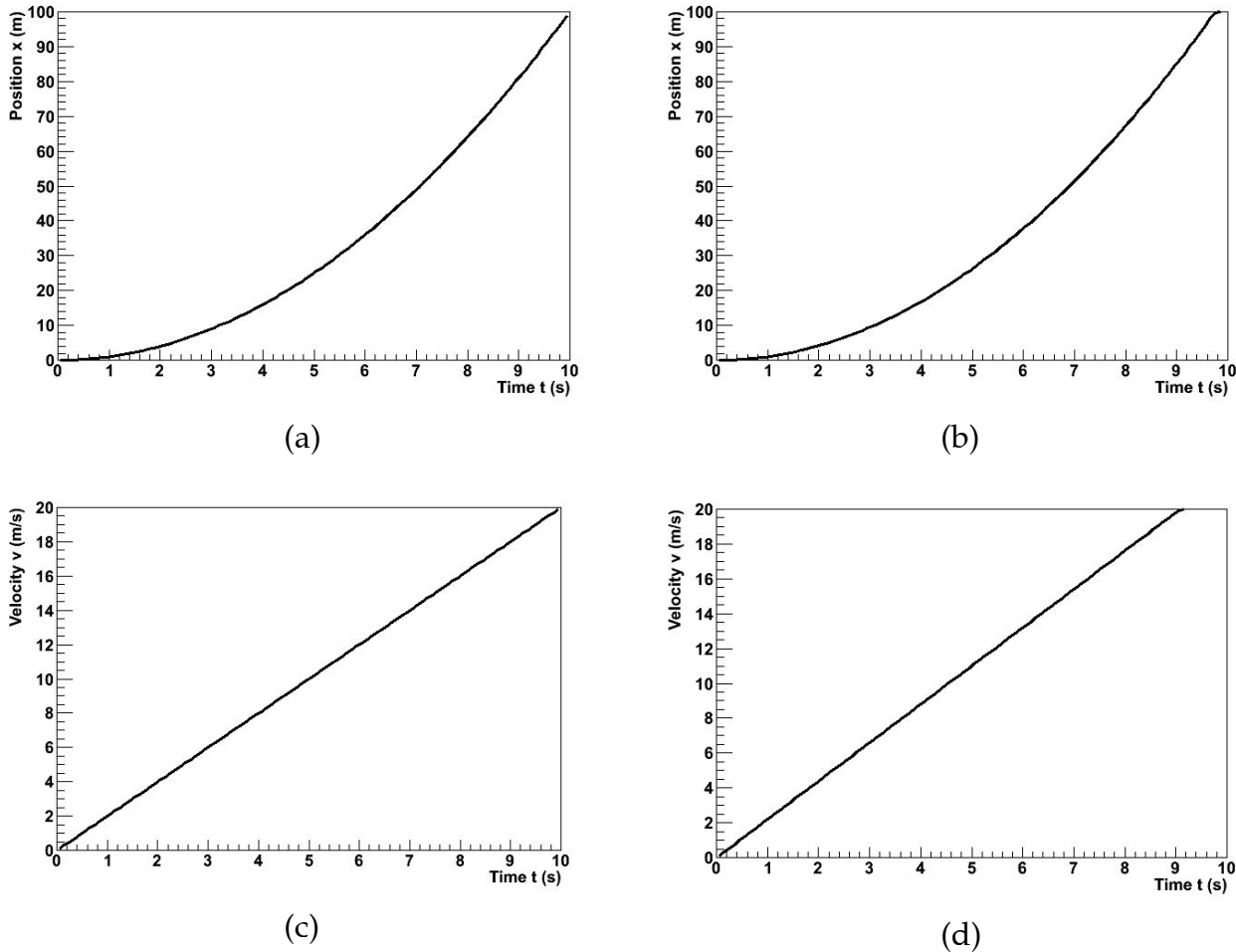
Figure 7: Position and velocity with respect to time for an objects with slightly different accelerations. The difference is easier to see in the velocity graphs.

The other method is creatively named "linearization." Essentially, it amounts to choosing non-obvious quantities for the independent and/or dependent variables in a graph in such a way that the result graph will be a line. An easy example of this is, once again, an object moving with a constant acceleration, like one of those in Figure 7. Instead of taking the derivative and plotting the velocity, we might have chosen to graph the position with respect to $\frac{t^2}{2}$; because the initial velocity for this object happened to be 0, this would also have produced a graph with a constant slope.

The Bottom Line

Ultimately, graphs exist to communicate information. This is the objective that we should have in mind when we create them. If our graph can effectively communicate our point to our readers, then it has accomplished its purpose.
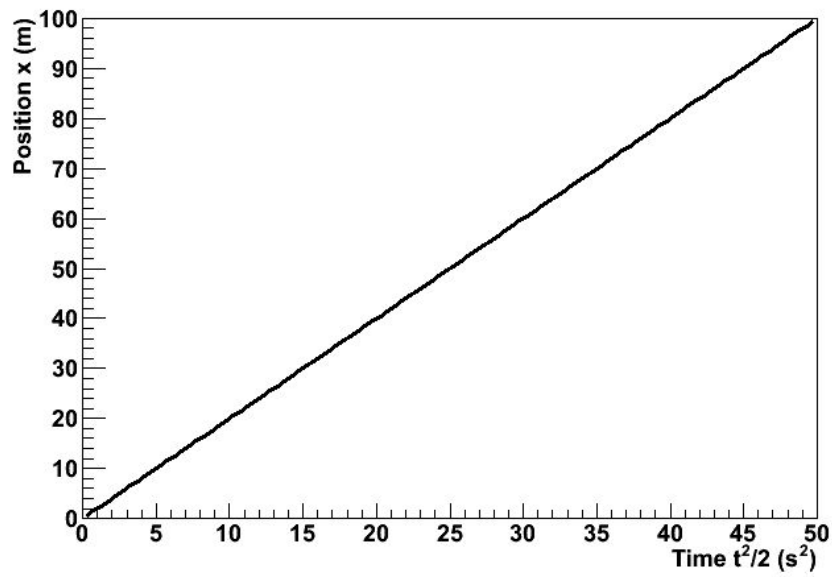
Figure 8: The position of the first object from Figure 7 plotted with respect to $\frac{t^2}{2}$. The relationship has been linearized.

Excel - MAKING GRAPHS

You will find that numerous exercises in this manual will require graphs. Microsoft Excel is a spreadsheet program that can create fourteen types of graphs, each of which have from two to ten different formats. This results in a maze of possibilities. There are help screens in Excel; however, this overview is covers the type of graph you should include in your lab reports. This is meant to be a brief introduction to the use of Microsoft Excel for graphing scientific data. If you are acquainted with Excel already, you should still skim through this appendix to learn about the type of graph to include in reports.

Step 1. Input your measurements and highlight the data using your cursor.
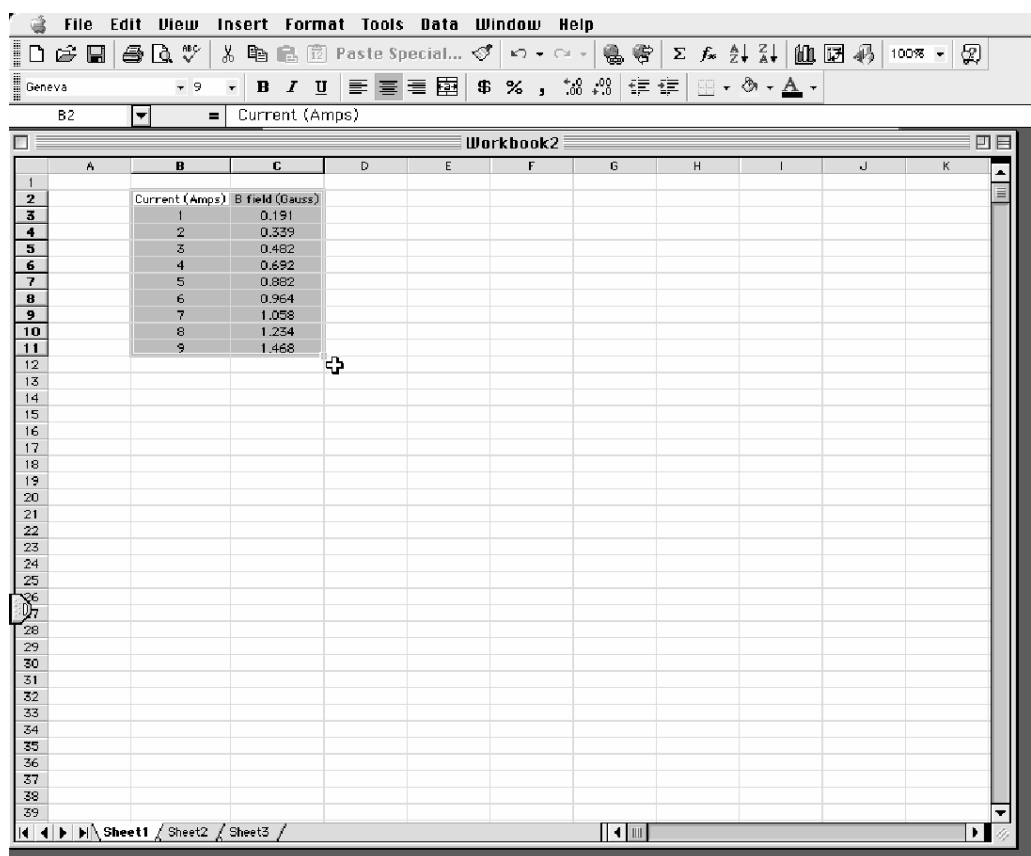
Step 2. Click on the "Chart Wizard" on the toolbar.
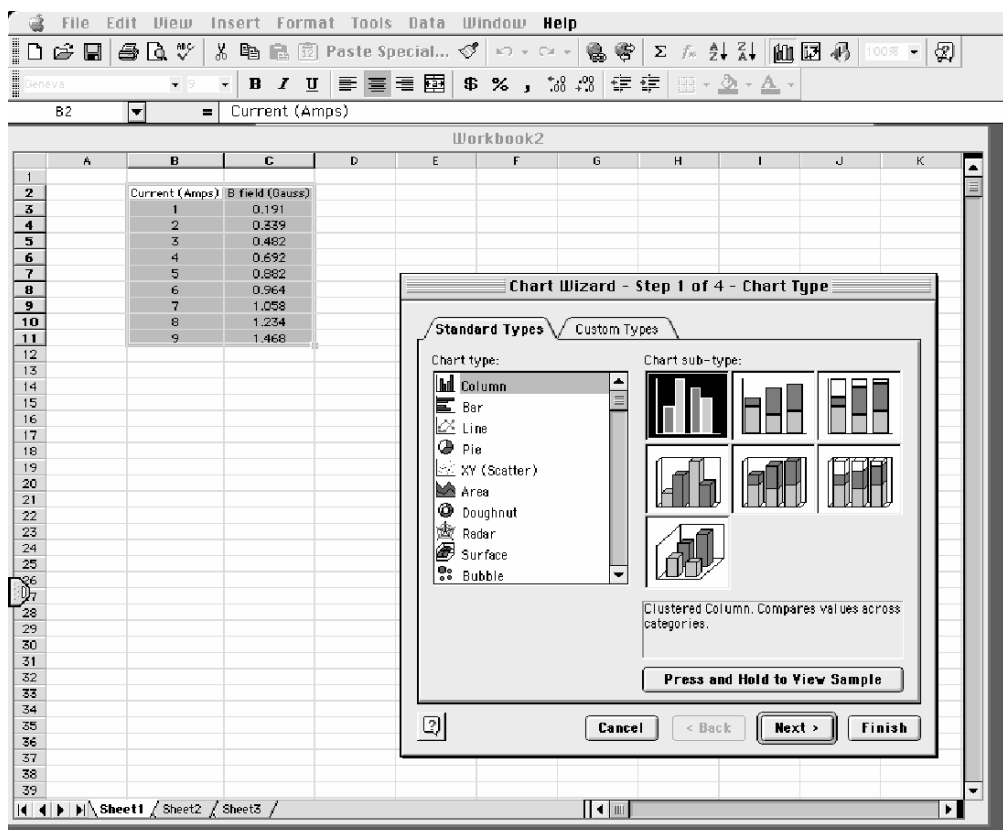
Step 3. Choose XY Scatter, not Line, from the list and click the "Next" button.
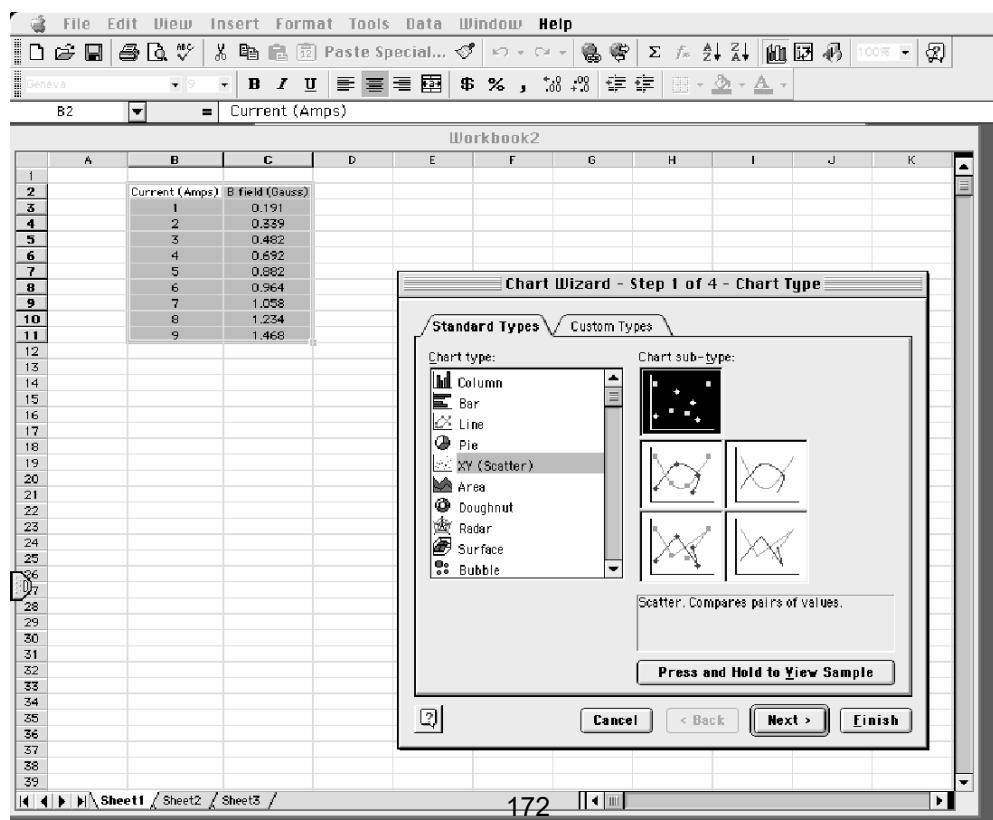
Step 4. Select the "Series in: Columns" option and click the "Next" button.
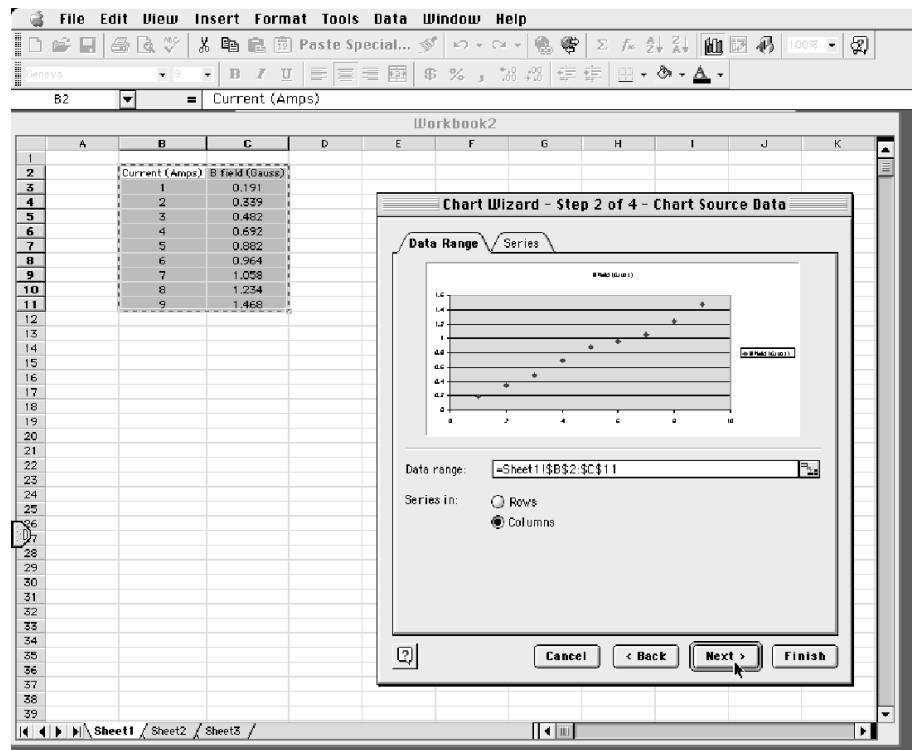
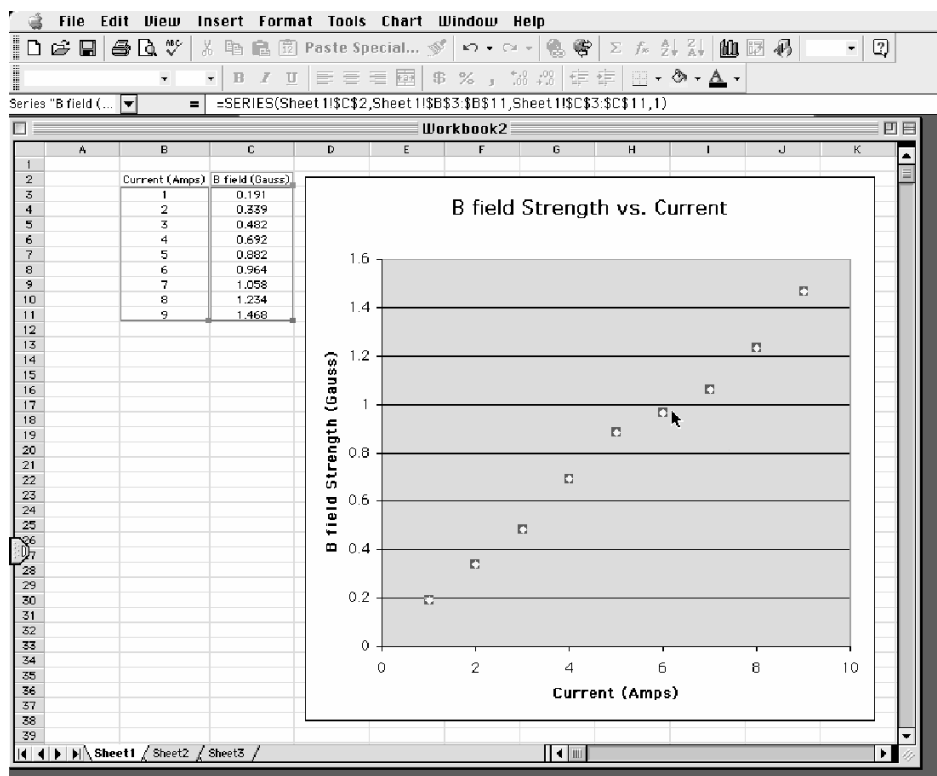**Step 5. Fill in the chart title and axis labels, and click the "Next" button.**
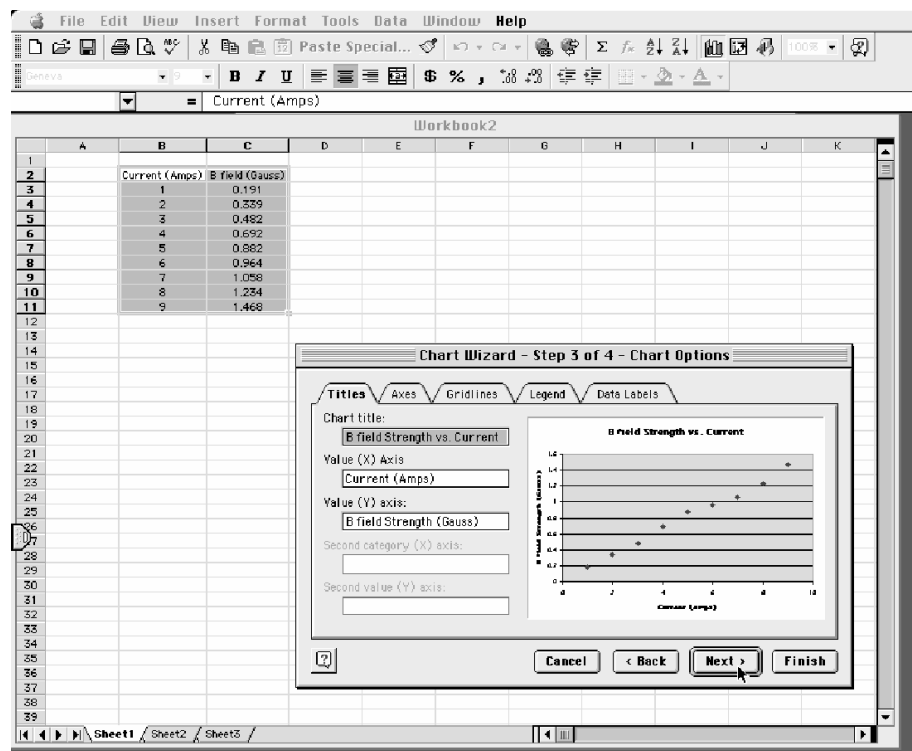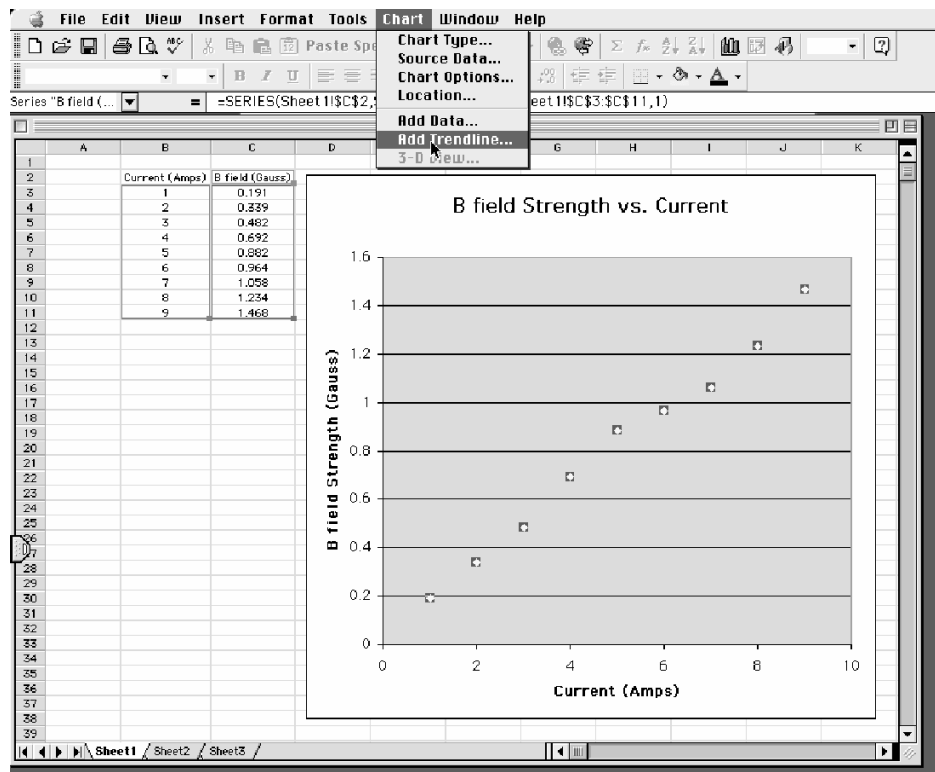
Step 6. Click the "Finish" button.
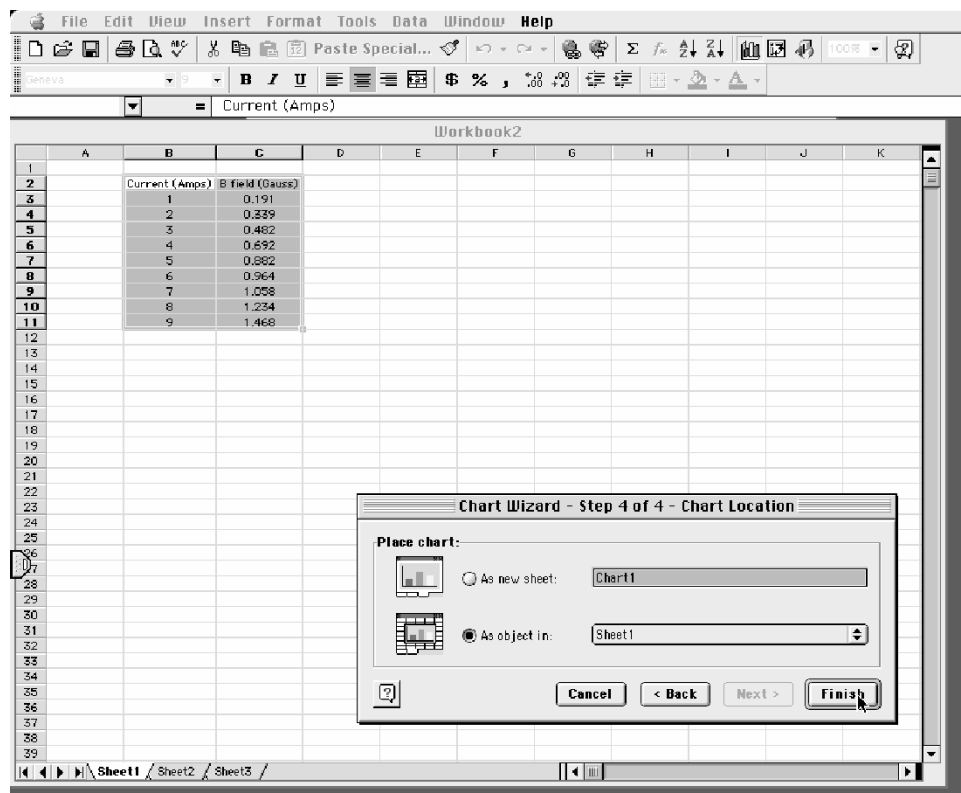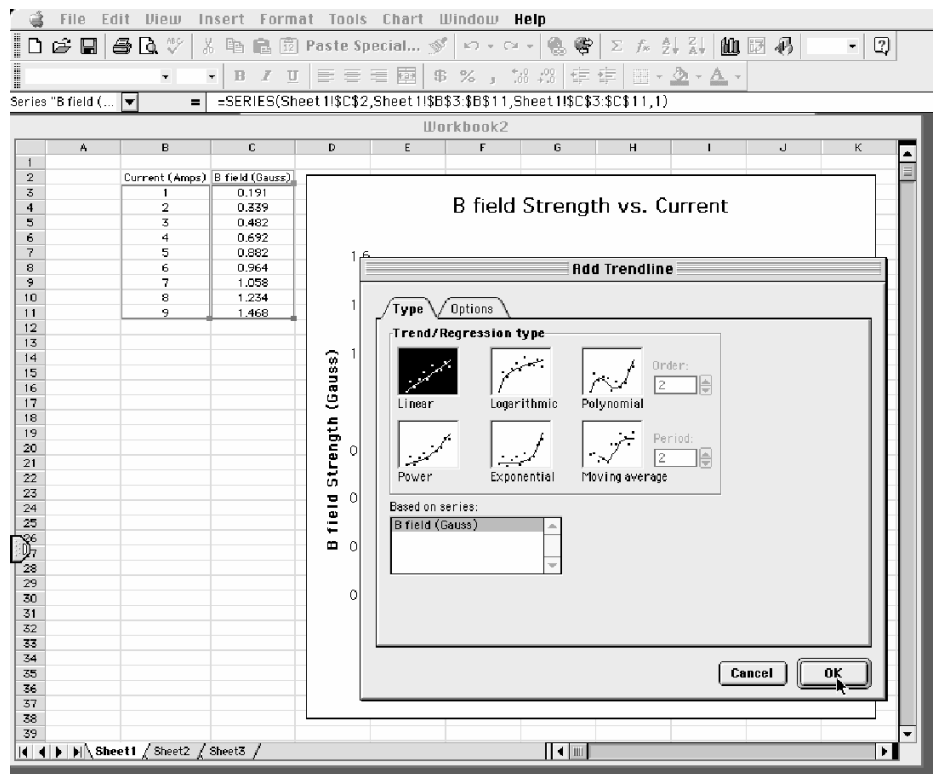

Step 7. Your graph will appear on the worksheet.
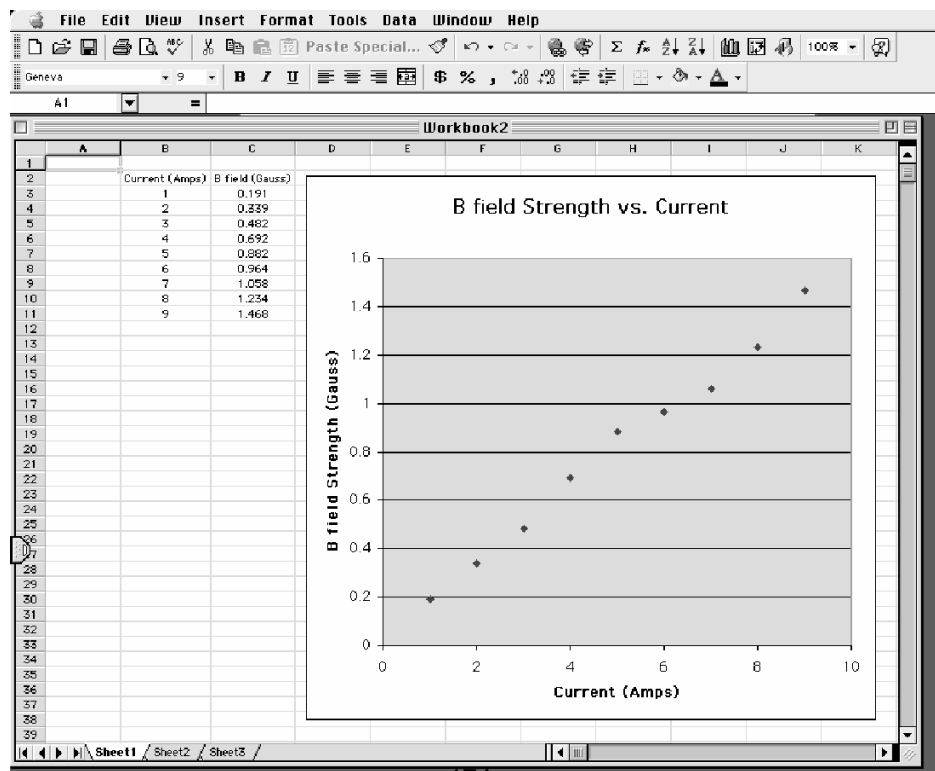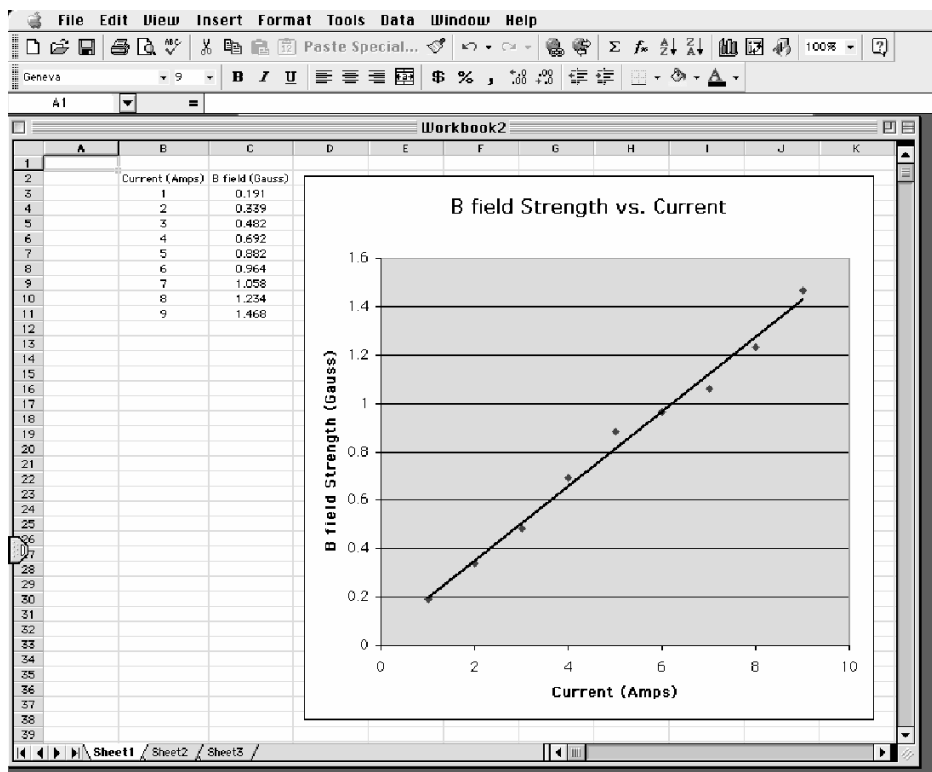
Step 8. Click on the data points to highlight them.

Step 9. Select "Add a Trendline" from the "Chart" menu.
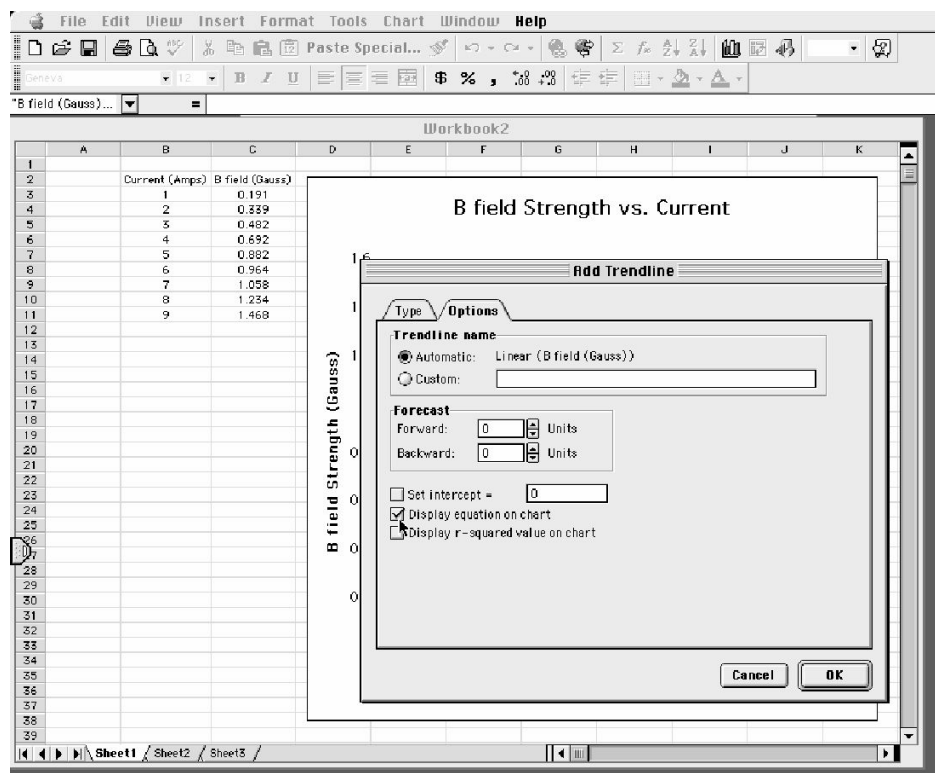
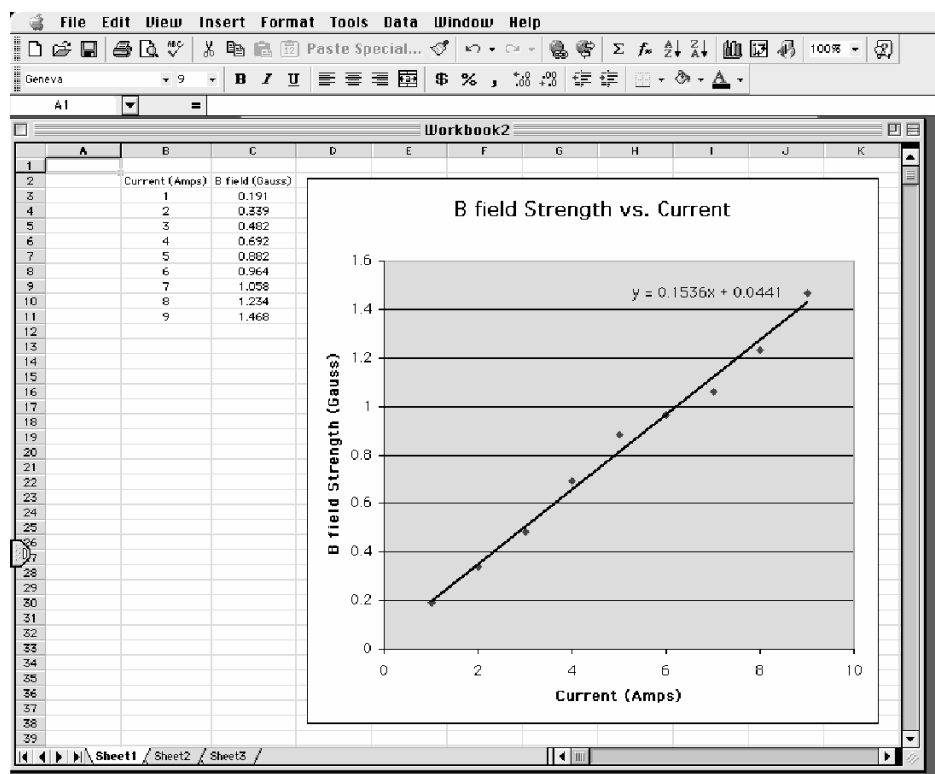**Step 10. Choose the best type of trend line for your data.**

Step 11. The trend line will appear – is it a good fit to your data?

Step 12. If the equation of the line is needed, choose "Display equation on chart."

Step 13. The equation of the trend line should appear on your graph.

Appendix: **Guide to Writing Lab Reports** (130x)

Many students have a great deal of trouble writing lab reports. They don't know what a lab report is; they don't know how to write one; they don't know what to put in one. This document seeks to resolve those problems. We will address them in that order.

This manual includes examples of a good and of a bad lab report; examine them in conjunction with this document to aid your understanding.

What Is a Lab Report?

Everyone seems to understand that a lab report is a written document about an experiment performed in lab. Beyond that, a lab report's identity is less obvious and more disputed. Let's save ourselves some misery by first listing some things that a lab report is not. A lab report is not

- … a worksheet; you may not simply use the example like a template, substituting what is relevant for your experiment.

- … the story of your experiment; although a description of the experimental procedure is necessary and very story-like, this is only one part of the much greater analytical document that is the report.

- … rigid; what is appropriate for a report about one experiment may not be appropriate for another.

- … a set of independent sections; a lab report should be logically divided, but its structure should be natural, and its prose should flow.

So what, then, is a lab report? A lab report is a document beginning with the proposal of a question and then proceeding, using your experiment, to answer that question. It explains not only what was done, but why it was done and what it means. To try to specify the content in much more detail than this is too constraining; you must simply do whatever is necessary to accomplish these goals. However, a lab report usually accomplishes them in four phases. First, it introduces the experiment by placing it in context, usually the motivation for performing it and some question that it seeks to answer. The context used in the report should be along the lines of the context laid out in the first section of each lab problem. Second, it describes the methods of the experiment. Third, it analyzes the data to yield some scientifically meaningful result. Fourth, it discusses the result, answering the original question posed in the context and explaining what the result means.

There are, of course, other senses of what a lab report is — it is quantitative, it is persuasive, etcetera — but we will come to those along the way.

How Do I Write a Lab Report?

Now that we have a vague idea of what a lab report is, let's discuss how to write it. By this, we do not mean its content, but its audience, style, etcetera.

Making an Argument

We already mentioned that a lab report uses an experiment to answer a question, but merely answering it is not enough; your report must convince the reader that the answer is correct, hopefully by answering the context. This makes a lab report a persuasive document. Your persuasive argument is the single most important part of any lab report. You must be able to communicate and demonstrate a clear point. If you can do this well, your report will be a success; if you cannot, it will be a failure.

At some point, you have certainly written a traditional, five-paragraph essay. The first paragraph introduces a thesis, the second through fourth defend the thesis, and the fifth paragraph concludes by restating the thesis. This is a little too simple for a lab report, but the basic idea is the same; keep it in mind. This structure is typically implemented in science in four basic sections: introduction, methodology, results, and discussion. This is sometimes called the "IMRD method." Begin by stating your thesis, along with enough background information to explain it and a brief preview of how you intend to support it, in your introduction. Defend your thesis in the methodology and results sections. Restate your thesis, this time with a little more critical evaluation, in your discussion. However, keep in mind that IMRD can be a rule or a guideline. In this class, we shall not have exactly four sections with these titles; we shall divide the report more finely (See below.). Roughly speaking, "Introduction" will become the Introduction and Prediction sections, "Methodology" and "Results" will become the Procedure, Data, and Analysis sections, and "Discussion" will become the Conclusion section. Introduce the context-based question and state your prediction in the Introduction and Prediction sections; test your prediction in the Procedure, Data, and Analysis sections; and restate and critically evaluate both your prediction and your result in your Conclusion section. You should always attempt to revisit the context of the lab experiment in the Conclusion section.

Audience

If you are successfully to persuade your audience, you must know something about her. What sorts of things does she know about physics, and what sorts of things does she find convincing? For your lab report, she is an arbitrary scientifically-literate person. She is not quite your professor, not quite your TA, and not quite your labmates, but she is this same sort of person. The biggest difference is that she doesn't know what your experiment is, why you are doing it, or what you hope to prove until you tell her. Use physics and mathematics freely in your report, but explain your experiment and analysis in detail.

Technical Style

A lab report is a technical document. This means that it is stylistically quite different from other documents you may have written. What characterizes technical writing, at least as far as your lab report is concerned? Here are some of the most prominent features, but for a general idea, read the sample good lab report included in this manual.

A lab report does not entertain. When you read the sample reports, you may find them boring; that's OK. The science in your report should be able to stand for itself. If your report needs to be entertaining, then its science is lacking.

A lab report is a persuasive document, but it does not express opinions. Your prediction should be expressed as an objective hypothesis, and your experiment and analysis should be a disinterested effort to confirm or deny it. Your result may or may not coincide with your prediction, and your report should support that result objectively.

A lab report is divided into sections. Each section should clearly communicate one aspect of your experiment or analysis.

A lab report may use either the active or the passive voice. Use whichever feels natural and accomplishes your intent, but you should be consistent.

A lab report presents much of its information with media other than prose. Tables, graphs, diagrams, and equations frequently can communicate far more effectively than can words. Integrate them smoothly into your report.

A lab report is quantitative. If you don't have numbers to support what you say, you may as well not say it at all.

Some of these points are important and sophisticated enough to merit sections of their own, so let's discuss them some more.

Nonverbal Media

A picture is worth a thousand words. Take this old sentiment to heart when you write your lab report, but do not limit yourself to pictures. Make your point as clearly and tersely as possible; if a graph will do this better than words will, use a graph.

When you incorporate these media, you must do so well, in a way that serves the fundamental purpose of clear communication. Label them "Figure 1" and "Table 2." Give them meaningful captions that inform the reader what information they are presenting. Give them context in the prose of your report. They need to be functional parts of your document's argument, and they need to be well-integrated into the discussion.

Students sometimes think that they are graded "for the graphs," and TAs sometimes over-emphasize the importance of these media. Avoid these pitfalls by keeping in mind that the purpose of these things is communication. If you can make your point more elegantly with these tools, then use them. If you cannot, then stick to tried-and-true prose. Use your best judgment.

Quantitativeness

A lab report is quantitative. Quantitativeness is the power of scientific analysis. It is objective. It holds a special power lacking in all other forms of human endeavor: it allows us to know precisely how well we know something. Your report is scientifically valid only insofar as it is quantitative.

Give numbers for everything, and give the numerical errors in those numbers. If you find yourself using words like "big," "small," "close," "similar," etcetera, then you are probably not being sufficiently quantitative. Replace vague statements like these with precise, quantitative ones.

If there is a single "most important part" to quantitativeness, it is error analysis. This lab manual contains an appendix about error analysis; read it, understand it, and take it to heart.

What Should I Put in My Lab Report?

Structure your report like this.

Abstract

Think of the abstract as your report in miniature. Make it only a few sentences long. State the question you are trying to answer, the method you used to answer it, and your results. It is not an introduction. Your report should make sense in its absence. You do not need to include your prediction here.

Introduction

Do three things in your introduction. First, provide enough context so that your audience can understand the question that your report tries to answer. This typically involves a brief discussion of the hypothetical real-world scenario from the lab manual. Second, clearly state the question. Third, provide a brief statement of how you intend to answer it.

It can sometimes help students to think of the introduction as the part justifying your report to your company or funding agency. Leave your reader with an understanding of what your experiment is and why it is important.

Predictions

Include the same predictions in your report that you made prior to the beginning of the experiment. They do not need to be correct. You will do the same amount of work whether they are correct or incorrect, and you will receive far more credit for an incorrect, well-refuted prediction than for a correct, poorly-supported one.

Your prediction will often be an equation or a graph. If so, discuss it in prose.

Procedure

Explain what your actual experimental methodology was in the procedure section. Discuss the apparatus and techniques that you used to make your measurements.

Exercise a little conservatism and wisdom when deciding what to include in this section. Include all of the information necessary for someone else to repeat the experiment, but only in the important ways. It is important that you measured the time for a cart to roll down a ramp through a length of one meter; it is not important who released the cart, how you chose to coordinate the person releasing it with the person timing it, or which one meter of the

ramp you used. Omit any obvious steps. If you performed an experiment using some apparatus, it is obvious that you gathered the apparatus at some point. If you measured the current through a circuit, it is obvious that you hooked up the wires. One aspect of this which is frequently problematic for students is that a step is not necessarily important or non-obvious just because they find it difficult or time-consuming. Decide what is scientifically important, and then include only that in your report.

Students approach this section in more incorrect ways than any other. Do not provide a bulleted list of the equipment. Do not present the procedure as a series of numbered steps. Do not use the second person or the imperative mood. Do not treat this section as though it is more important than the rest of the report. You should rarely make this the longest, most involved section.

Data

This should be your easiest section. Record your empirical measurements here: times, voltages, fits from MotionLab, etcetera.

Do not use this as the report's dumping ground for your raw data. Think about which measurements are important to your experiment and which ones are not. Only include data in processed form. Use tables, graphs, and etcetera, with helpful captions. Do not use long lists of measurements without logical grouping or order.

Give the units and uncertainties in all of your measurements.

This section is a bit of an exception to the "smoothly integrate figures and tables" rule. Include little to no prose here; most of the discussion belongs in the Analysis section. The distinction between the Data and Analysis sections exists mostly for your TA.

Analysis

Do the heavy lifting of your lab report in the Analysis section. Take the data from the Data section, scientifically analyze it, and finally answer the question you posed in your Introduction. Do this quantitatively.

Your analysis will almost always amount to quantifying the errors in your measurements and in any theoretical calculations that you made in the Predictions section. Decide whether the error intervals in your measurements and predictions are compatible. This manual contains an appendix about error analysis; read it for a description of how to do this.

If your prediction turns out to be incorrect, then show that as the first part of your analysis. Propose the correct result and show that it is correct as the second part of your analysis.

Finally, discuss any shortcomings of your procedure or analysis, such as sources of systematic error for which you did not account, approximations that are not necessarily valid, etcetera. Decide how badly these shortcomings affect your result. If you cannot confirm your prediction, then estimate which are the most important.

Conclusion

Consider your conclusion the wrapping paper and bow tie of your report. At this point, you should already have said most of the important things, but this is where you collect them in one place. Remind your audience what you did, what your result was, and how it compares to your prediction. Tell her what it means in regards to the context posed. Leave her with a sense of closure.

Quote your result from the Analysis section and interpret it in the context of the hypothetical scenario from the Introduction. If you determined that there were any major shortcomings in your experiment, you might also propose future work to overcome them.

If the Introduction was your attempt to justify your past funding, then the Conclusion is your attempt to justify your future funding.

What Now?

Read the sample reports included in this manual. There are two; one is an example of these instructions implemented well, and the other is an example of these instructions implemented poorly. Then, talk to your TA. He can answer any remaining questions that you might have.

There is a lot of information here, so using it and actually writing your lab report might seem a little overwhelming. A good technique for getting started is this: complete your analysis and answer your question before you ever sit down to write your report. At that point, the hard part of the writing should be done: you already know what the question was, what you did to answer it, and what the answer was. Then just put that down on paper.

GOOD SAMPLE LAB

Lab I, Problem 1b: Measurement and Uncertainty Using Video Analysis

Pat Kline

August 7th, 2017

Physics 1301W, Professor: Y. Ryoma, TA: P. Hearn

Abstract

The use of video analysis software to model the motion of an object and the importance of properly calibrating the video were studied. A video of a constant velocity electric buggy was made and the motion was analyzed. With a camera placed 94.0 cm from the plane of motion, it was found that a calibration object 10.0 cm in front of the plane of motion led to an error of $9.4 \pm 4.3\,\%$ in the measured buggy velocity compared to having the calibration object in the plane of motion. Additionally, it was found that the random uncertainty of the measured velocity using video analysis was less than that from a previous hand measurement made using a dowel and stopwatch.

Introduction

BuggyMagic wants to use video analysis as a method of quality control to provide buggies of constant velocity. The quality-control team have been analyzing videos of buggy motion and have gotten different velocity values, even for the same video. The team suspects that reaching a different conclusion using the same video may be due to miscalibration. Therefore, to demonstrate the effects of miscalibration, a buggy's motion will be analyzed when a calibration object is in the plane of motion compared to when the calibration object lies in front of the plane.

Prediction

Using simple geometry, the effects of miscalibration can be estimated. As seen in Figure 1, the whole setup can be modelled by a triangle ABC with A representing the point-like camera and the side BC representing the plane of motion, or a part of it. If a calibration object, signified by the line DE, is added in front of the plane of motion, the camera thinks that the plane of motion is shorter than it actually is.
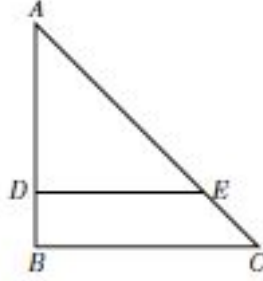
Figure 1: Sketch of a setup with miscalibration. The corner A is the camera, the side BC is the plane of motion and the side DE is the calibration object. This setup makes the camera think that the length $|BC|$ is equal to the length $|DE|$.

The percentage error in distances due to the miscalibration is then predicted to be

$$\%E = \frac{|BC|-|DE|}{|BC|} = 1 - \frac{|DE|}{|BC|} \tag{1}$$

Since the length $|BC|$ does not correspond to any real object and the experiment could also be done with a different calibration object of random length, a different way of presenting the prediction is preferred. Using simple geometry of similar triangles, the distances between the plane of motion, the camera, and the calibration object can be related through

$$\frac{|DE|}{|BC|} = \frac{|AD|}{|AB|} \tag{2}$$

and, along with the relation $|AD| + |DB| = |AB|$, Eq.1 can be rewritten as

$$\%E = \frac{|DB|}{|AB|} \tag{3}$$

Thus, the effect of miscalibration can be predicted based on the distances of the camera and the calibration object from the plane of motion.

Procedure

Two videos of a buggy's motion were made. The camera was located 94.0 cm away from the plane of motion. A wooden dowel of length 22.0 cm was used as a calibration object for both videos: one in which it was aligned with the plane of motion and another where it was placed 10.0 cm in front of and parallel to the plane of motion. The buggy was released to move with a constant velocity in the plane of motion for a distance of about 5 buggy lengths to accumulate enough data points. MotionLab analysis software was used to generate (horizontal position, time) pairs at each frame in the trajectory and, by linear interpolation, (horizontal velocity, time) pairs between each pair of consecutive frames in the trajectory. To analyze the motion in detail, data was exported

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into Excel and plots were generated of the buggy's position and velocity as functions of time.

Data

The distance from the camera to the plane of motion was
$$|AB| = 94.0 \pm 0.2 \; cm$$
The distance the dowel was placed in front of the plane of motion was
$$|DB| = 10.0 \pm 0.2 \; cm$$
Then the expected percentage error in the buggy velocity from the miscalibration is
$$\%E = 10.6 \pm 0.2 \; \%$$

As seen in Figures 2 and 3, the data was exported into Excel to produce graphs of the data.
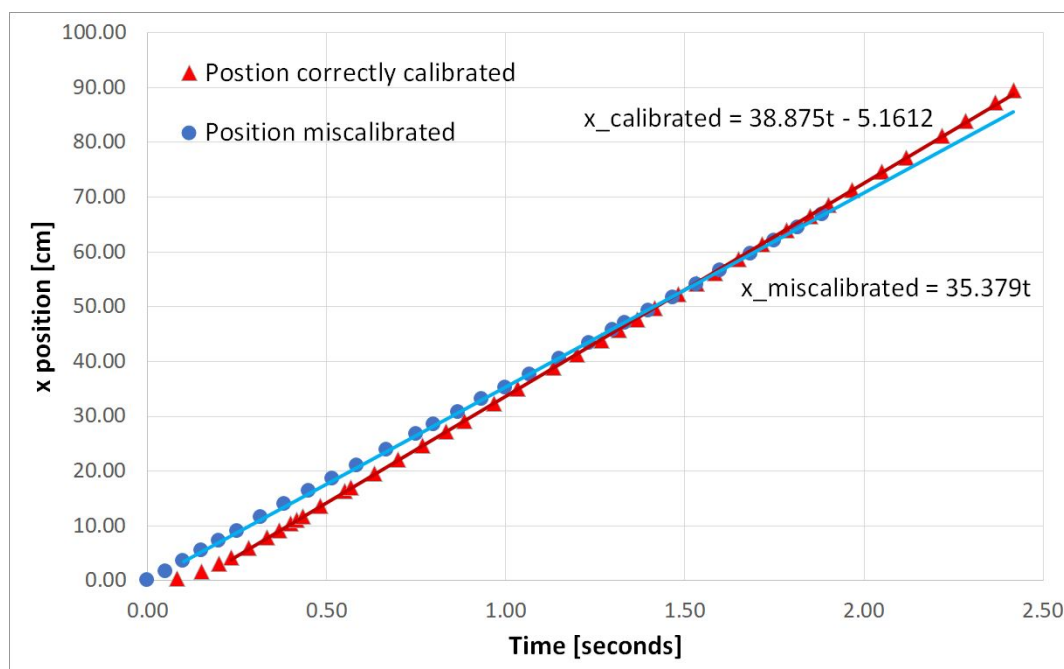

Figure 2. The measured position of the buggy as a function of time. Equations of best fit lines through the data show the velocity as the slope value. Error bars are not shown because the uncertainty of individual point values was small compared to the systematic error.
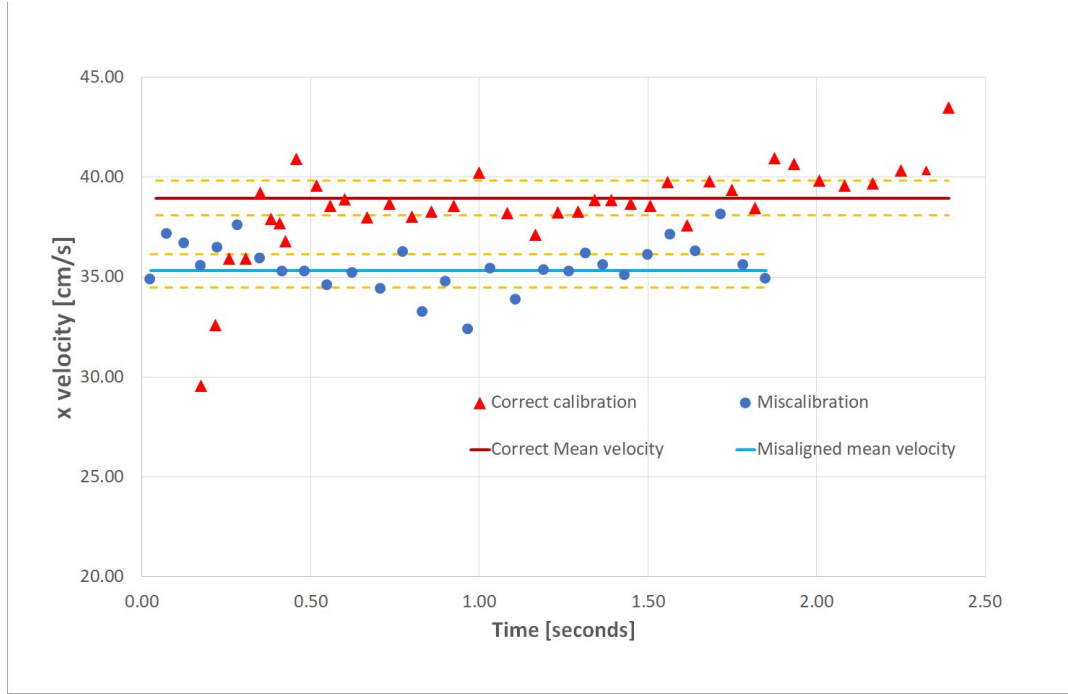
Figure 3. The measured velocity of the buggy as a function of time for the correctly calibrated (triangles) and miscalibrated (circles) videos, along with mean velocities and uncertainty ranges: $v_c = 38.96 \pm 0.87$ *cm/s* **,** $v_m = 35.31 \pm 0.82$ *cm/s* **. Error bars are not shown because the uncertainty of individual point values was small compared to the systematic error.**

Analysis

The results of the buggy's motion are shown in Figure 2 (position versus time) and Figure 3 (velocity versus time). The first thing to note is the early velocity values for the proper calibration in Figure 3. This large deviation from the mean is due to taking data points as soon as the buggy was started. Pressure from the hand holding the buggy slowed it down until it was fully released. This error can also be seen in Figure 2 as a slight curve in the beginning. All analysis of the data excluded those early points. This is not seen on the miscalibrated data as care was taken to start data acquisition after the buggy was fully released.

Based on our prediction we expect the velocity percentage error from the miscalibrated data to be $10.6 \pm 0.2\,\%$. From Figure 3, we found the velocity and the average deviation for the proper calibration case to be $v_c = 38.96 \pm 0.87$ *cm/s,* while the velocity for the miscalibration case turned out to be $v_m = 35.31 \pm 0.82$ *cm/s .* As expected, in a case with the calibration object placed in front of the plane of motion, the buggy appears to move slower. From those values the percentage error of those velocities is calculated to be $\%E = 9.4 \pm 4.3\%$. This is slightly less than expected but comparable when the uncertainty range from the propagation of error is taken into account.

From earlier hand measurements in Problem 1a, the buggy velocity was found to be $v_h = 40.2 \pm 3.2$ cm/s; therefore, the result achieved by the properly calibrated video does fall within the expected uncertainty of v_h.

In comparison with the hand measurements, the random measurement uncertainty is smaller for the video analysis technique. Considering the uncertainty of a few pixels in distance measurements and some small fraction of a second in time measurements, it can be said that the random uncertainty in the velocity value found in MotionLab is negligible, particularly compared to the larger error values that a possible miscalibration procedure might introduce. Thus, video analysis is a more effective technique for investigating motion, provided that the proper calibration is used.

Several potential sources of error might explain the difference between the predicted and the measured percentage error values. One is the distortion effect of the camera; data points that do not come from the center-most portion of the field of view might be discarded to limit this effect. Another is the slightly curved trajectory of the buggy. This effect was minimized by modifying the initial direction of the motion. Moreover, to prevent the buggy's wheels from hitting the dowel, the calibration object could not be placed exactly on the plane of motion, which might have caused some minor error.

Conclusion

It was shown that a significant error can appear while using video-analysis software if the video is not properly calibrated. The measured velocity of a buggy when the video was properly calibrated with the calibration object located in the plane of motion was compared to the measured velocity when it was miscalibrated with the calibration object in front of the plane of motion.

The effect of miscalibration was modelled using simple geometry of triangles. Even this simple model was able to predict the percentage error in the velocity caused by miscalibration, $10.6 \pm 0.2\,\%$, within the uncertainty value of the measured percentage error, $9.4 \pm 4.3\%$.

This result implies that it is feasible for BuggyMagic to utilize a video analysis technique for quality control, provided that the quality control team properly calibrates their videos of the buggies' motion.

BAD SAMPLE LAB

Lab I, Problem 1b

Comte de Rochefort

August 7, 2017

Introduction

We seek to determine how miscalibration effects the results of a video analysis in MotionLab. To do this, we used a buggy that moves with a constant velocity. We used iPad to record videos of it moving on a straight line, being as careful as possible to simulate the constant velocity motion accurately and to minimize errors. We analyzed the videos with MotionLab, taking several data points for each case: with a proper calibration and a miscalibration.

Prediction
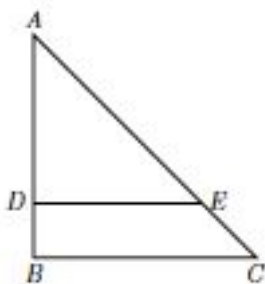
$$\%E = \frac{|DB|}{|AB|}$$

Procedure

We performed this experiment by a scientific procedure. We first made a prediction; then, we performed the experiment; then, we analyzed the data; then, we drew a conclusion.

We began by gathering the materials. They included:
- meter stick
- dowel
- buggy that can move with a constant velocity
- iPad on tripod
- computer
- tape

We put the dowel right next to the wheels for the proper calibration and 10 cm in front of the wheels for the miscalibration of MotionLab. We faced the camera toward the buggy.

We released the buggy after turning it on and recorded a video using the iPad. We then analyzed the motion using MotionLab. This began with the proper calibration. We first set time zero at the exact time when we released the buggy. We then had to calibrate the length. We put the dowel in the frame of the video, so we used the dowel for the calibration. We then defined our coordinate system so that the motion of the buggy would be straight to the left.

We then made predictions about the motion. We predicted that the y position would not change and that the x position would be a straight line with a slope B=38cm/s. The predicted equations were y(z)=0 and x(z)=-38z.

We then had to acquire data. We measured the position of the buggy at each frame in the video, starting at t=0. We put the red point at the center of the front wheel each time for consistency. This was important to keep from measuring a length that changed from frame to frame based on where we put the data point on the buggy. We also did not use some of the frames at the end of the video, where the buggy was at the edge where the camera is susceptible to the fisheye effect.

When this was finished, we fit functions to the data points. The functions did not fit the points exactly, but they were acceptably close. We fit y(z)=0 for the y position and x(z)=-37.9z for the x position. These were close to our predictions.

It then came time to make predictions of the velocity graphs. We predicted that the Vy graph would be a straight line with Vy(z)=0 and that the Vx graph would be a linear line with Vx(z)=-37.9.

Next, we fit the functions to the data points for the velocity graphs. We got the predictions exactly right.

We then printed our data for the constant velocity buggy and closed MotionLab.

We repeated this process for the miscalibration case. It was mostly the same, with some exceptions. The x(z) fit was x(z)=-35z instead of y(z)=-37.9z. The Vx(z) prediction was Vx(z)=-35 instead of Vx(z)=-37.9. These were also exactly right, so the Vx(z) fit was the same.
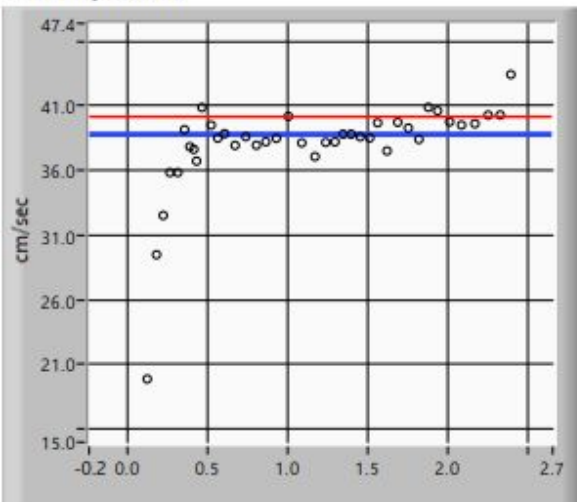
Data

X Position vs. Time
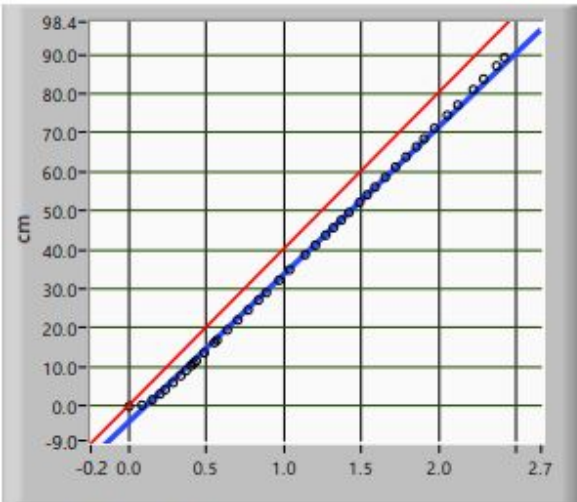
x Prediction Equation

f(z) = 0 + 40.17 z

x Fit Equation

f(z) = -4.3 + 37.9 z

Y Position vs. Time

Fit

y Prediction Equation

f(z) = 0 + 0 z

y Fit Equation
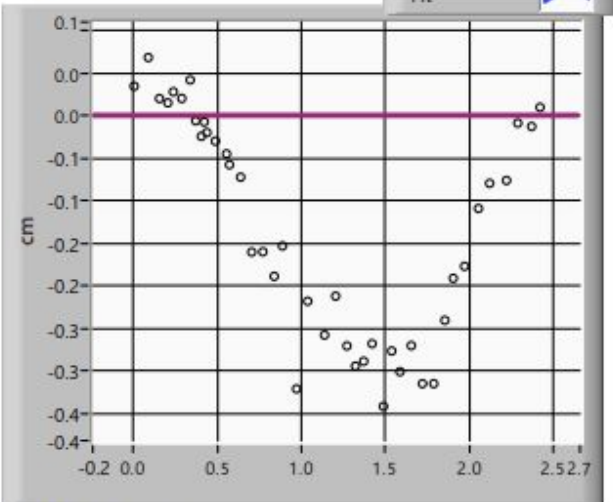
f(z) = 0 + 0 z

X Velocity vs. Time

Vx Prediction Equation

f(z) = 40.17 + 0 z

Vx Fit Equation

f(z) = 38.8 + 0 z

Y Velocity vs. Time
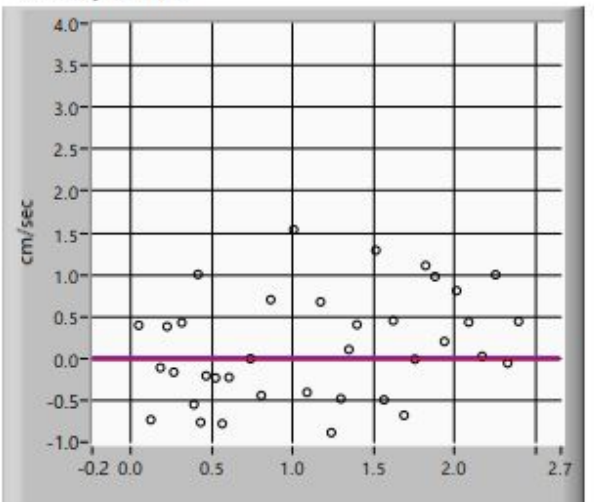
Vy Prediction Equation

f(z) = 0 + 0 z

Vy Fit Equation

f(z) = 0 + 0 z

X Position vs. Time

Y Position vs. Time

Fit

x Prediction Equation

f(z) = 0 + 34 z

y Prediction Equation

f(z) = 0 + 0 z

x Fit Equation

f(z) = 0 + 35 z

y Fit Equation

f(z) = 0 + 0 z

X Velocity vs. Time

Y Velocity vs. Time

Vx Prediction Equation

f(z) = 34 + 0 z

Vy Prediction Equation

f(z) = 0 + 0 z
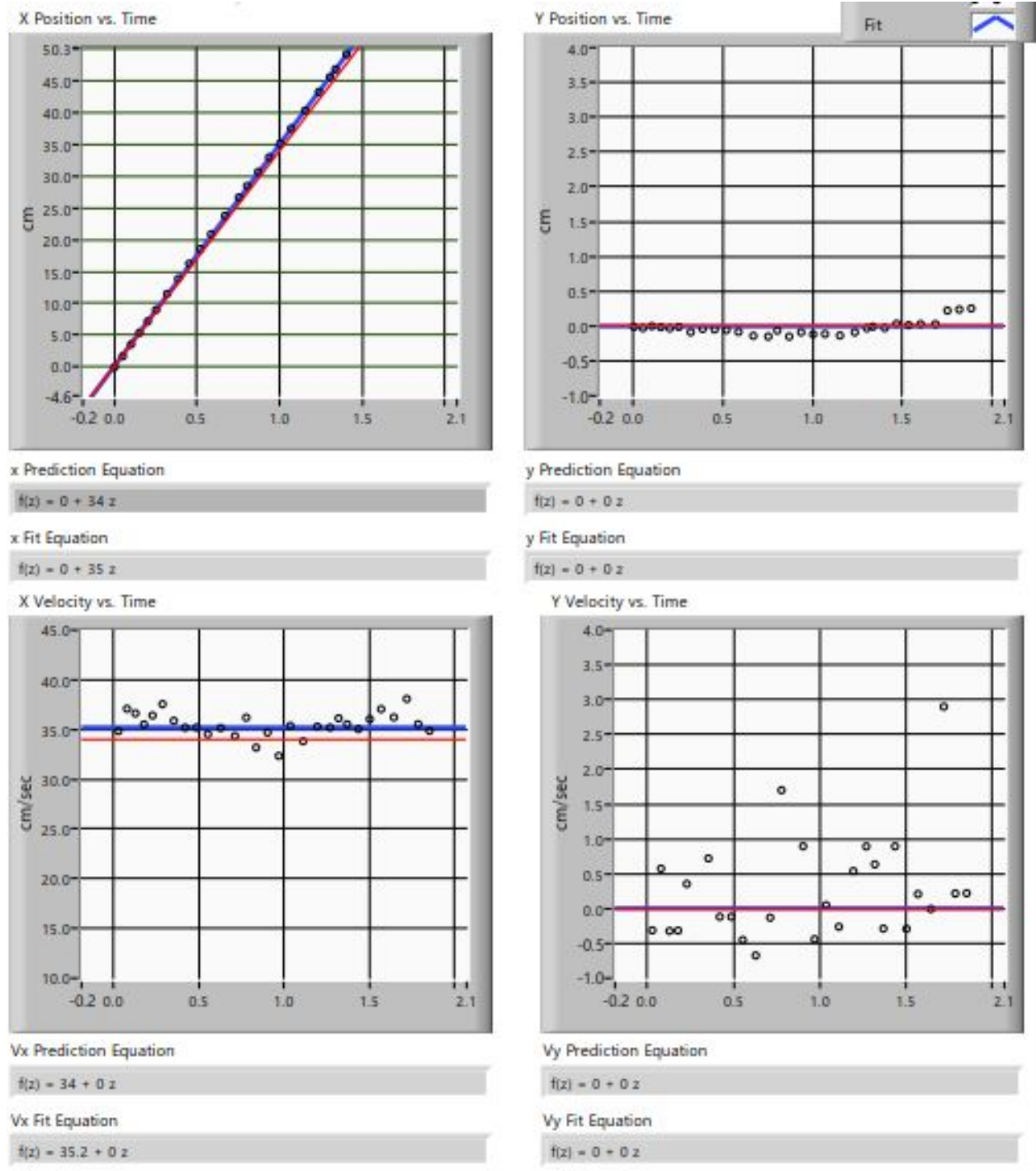
Vx Fit Equation

f(z) = 35.2 + 0 z

Vy Fit Equation

f(z) = 0 + 0 z

Analysis

We can calculate the velocity from the MotionLab fit functions. To do this, we use the formula $x = x0 + v0t + 1/2at^2$. Then v is just the coefficient of z in the position fits. This gives us

the proper calibration: v=37.9
the miscalibration: v=35

The velocity can also be calculated using the formula v=v0+at. Then v is just the constant in the velocity fits. This gives us

the proper calibration: v=38.8
the miscalibration: v=35.2

We know that the velocity found in the video analysis with the proper calibration is 37.9 cm/s, so we need to compare the velocity value we get with a miscalibration procedure to this number. Looking at the data from the fits, we can see that they are close to each other, so the error in this lab must not be significant.

We need to analyze the sources of error in the lab to interpret our result. One is human error, which can never be totally eliminated. Another error is the error in MotionLab. This is obvious because the data points don't lie right on the fit, but are spread out around it. Another error is that the meter stick could only measure the distances to +/-0.05cm. There was error in the fisheye effect of the camera lens. The buggy didn't move on a straight line exactly but we tried to fix it.

Conclusion

We predicted that %E would be 10.6%, and what we measured, 7.7%, is very close to this. The errors are therefore not significant to our result. We can say that the error due to miscalibration can be calculated as in prediction section. This experiment was definitely a success.

Physics Lab Report Rubric

Name:_____ID#:_____

Course, Lab, Problem: _____

Date Performed:_____

Lab Partners' Names:_____

	Possible	Earned
Warm-Up Questions		
Laboratory Notebook		
Participation		

Earns No Points	Earns Full Points	Possible	Earned
Argument			
- no or unclear argument - logic does not flow - gaps in content - leaves reader with questions	- complete, cogent, flowing argument - content, execution, analysis, conclusion all present - leaves reader satisfied		
Technical Style			
- vocabulary, syntax, etc. inappropriate for scientific writing - necessary nonverbal media absent or poorly constructed - subjective, fanciful, or appealing to emotions - jarringly inconsistent - no or confusing sections	- language appropriate for scientific writing - nonverbal media present where appropriate, well-constructed, well incorporated - objective, indicative, logical style - consistent - division into sections is helpful		
Use of Physics			
- predictions unjustified - experiment physically unjustified - experiment tests wrong phenomenon - theory absent from consideration of premise, predictions, and results	- predictions justified with physical theory - experiment is physically sound and tests phenomenon in question - results interpreted with theory to clear, appropriate conclusion		
Quantitativeness			
- statements are vague or arbitrary - analysis is inappropriately qualitative - uncertainty analysis not used to evaluate prediction or find result - numbers, equations, units, uncertainties missing or inappropriate	- consistently quantitative - equations, numbers with units, uncertainties throughout - prediction confirmed or denied, result found by some form of uncertainty analysis - results, conclusions based on data		
Total			