r-wave eigenfunction estimation

Pat Meyers
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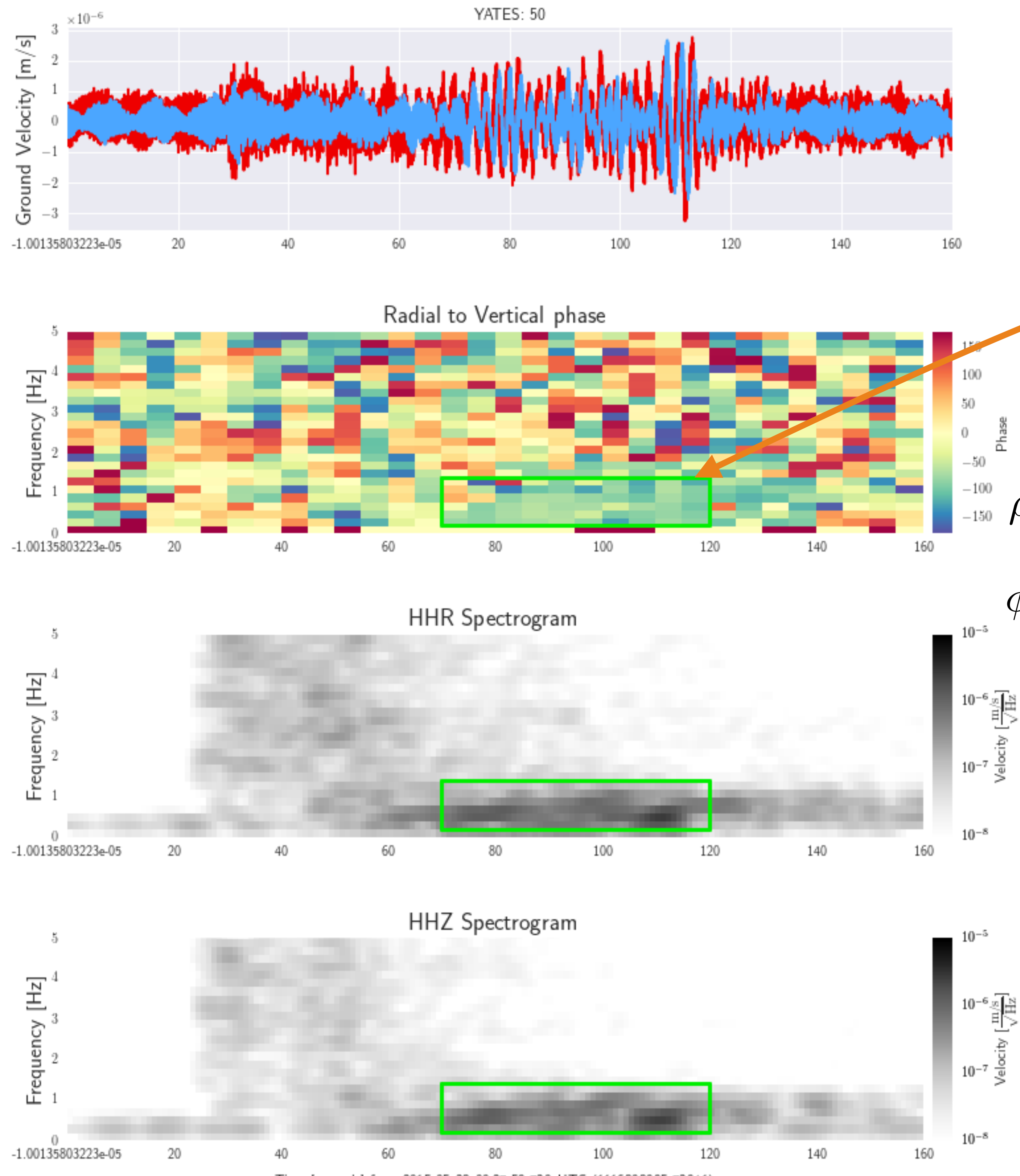
Homestake call

Overview

- Goal: measure r-wave eigenfunction
- Outline:
  - Event selection
  - Data treatment
  - Parameter Estimation
  - Results for 1 Hz
  - Discussion/questions
  - GGN estimates

Method: finding events

- Identify mine-blast events from Gary’s database that have distinct r-wave phase
- Identify the frequencies and times of r-wave phase for each event
- 7 events were used



- Phase:

$$\rho(f) = R(f)^* \times Z(f)$$

$$\phi(f) = \arctan\left(\frac{\mathrm{Imag}\,\rho(f)}{\mathrm{Re}\,\rho(f)}\right)$$

Data treatment: 1

- After identification we extract phase information like on previous slide

- amplitude information is extracted by:

   1. Filter with a gaussian filter in frequency domain then transform back to time domain

   2. Take peak of filtered waveform for R and Z components

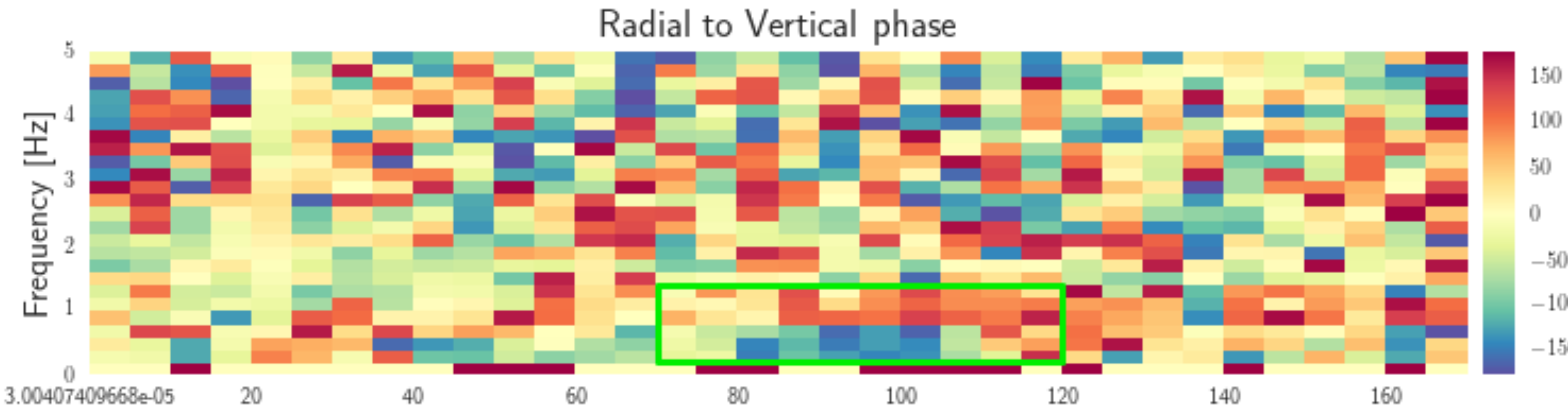
   3. Normalize peak by average of vertical component for all surface stations for that event.

- Note: I'm considering either moving everything to the frequency domain or everything to the time-domain since right now I'm using one for phase and the other for amplitude.

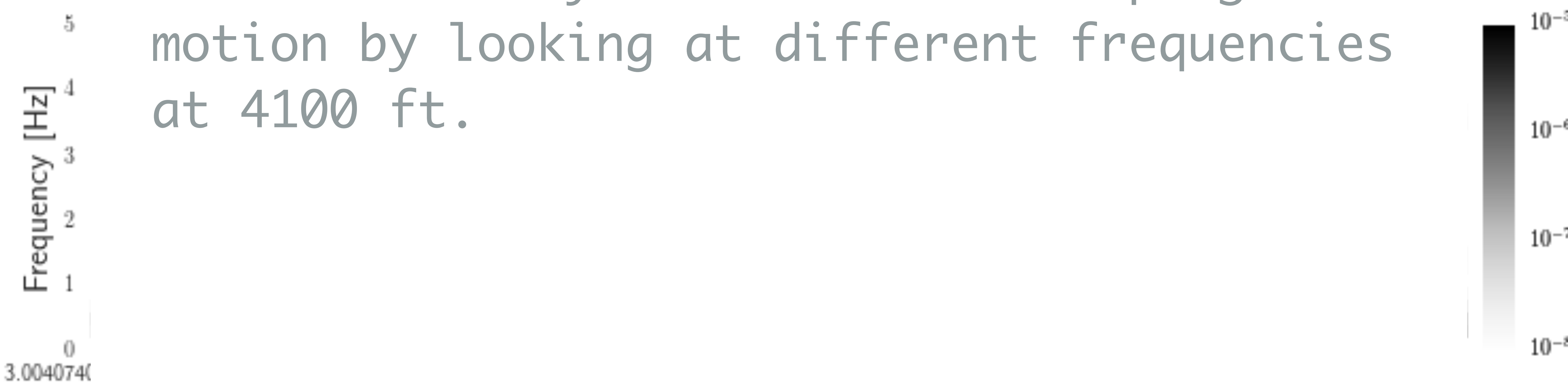
Data treatment: 2

- We then further change the Horizontal data by insisting that we take only the part that is 90 degrees in phase or out of phase: $\mathcal{R}(f) = \mathrm{Re}\left\{R(f) \times e^{(i\pi/2+\phi)}\right\}$
- Allows for treatment of prograde motion as well
    - Positive: retrograde
    - negative: prograde

Prograde example



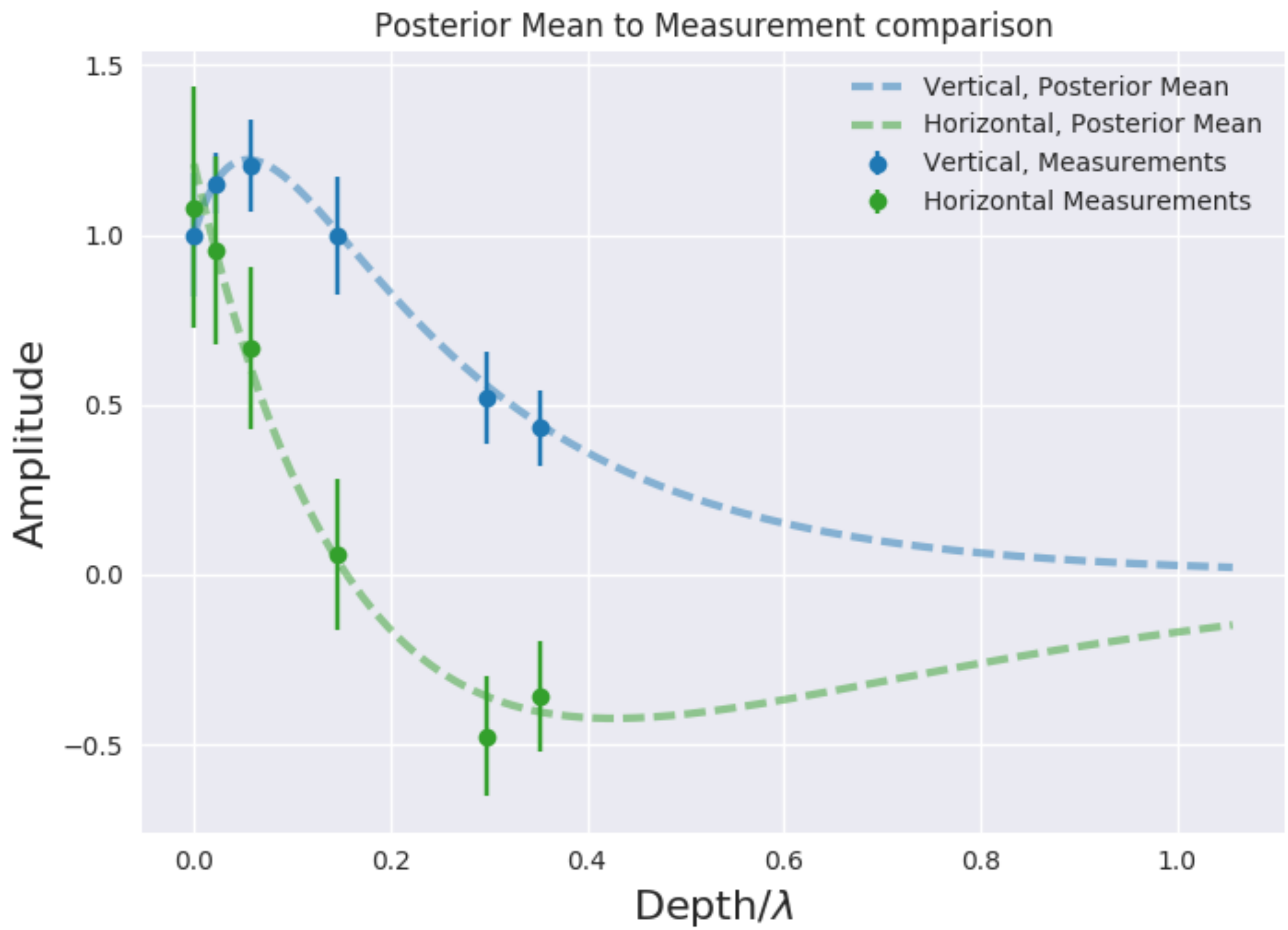
- We can actually see the shift to prograde motion by looking at different frequencies at 4100 ft.



Final eigenfunction data

- After averaging over 7 events, and all stations at each depth we find the data at right

- (lines will be explained in a moment)



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Biexponential parameter estimation

- Model has 8 parameters:

$$r_z(z) = c_1 e^{-a_1 kz} + (1 - c_1)e^{-a_2 kz}$$

$$r_h(z) = c_3 e^{-a_3 kz} + (N_h - c_3)e^{-a_4 kz}$$

$$k = 2\pi f / v$$

- Moving towards a free scaling parameter and an overall normalization for each function has reduced degeneracy greatly!

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Biexponential parameter estimation

parameters

Data

$$\log \mathcal{L} \propto -\frac{1}{2}\left(\sum_z \frac{(r_z(z,\vec{\theta}_z) - R_z(z))^2}{\sigma_z^2} + \sum_z \frac{(r_h(z,\vec{\theta}_h) - R_h(z))^2}{\sigma_h^2}\right)$$

models

measurement uncertainty

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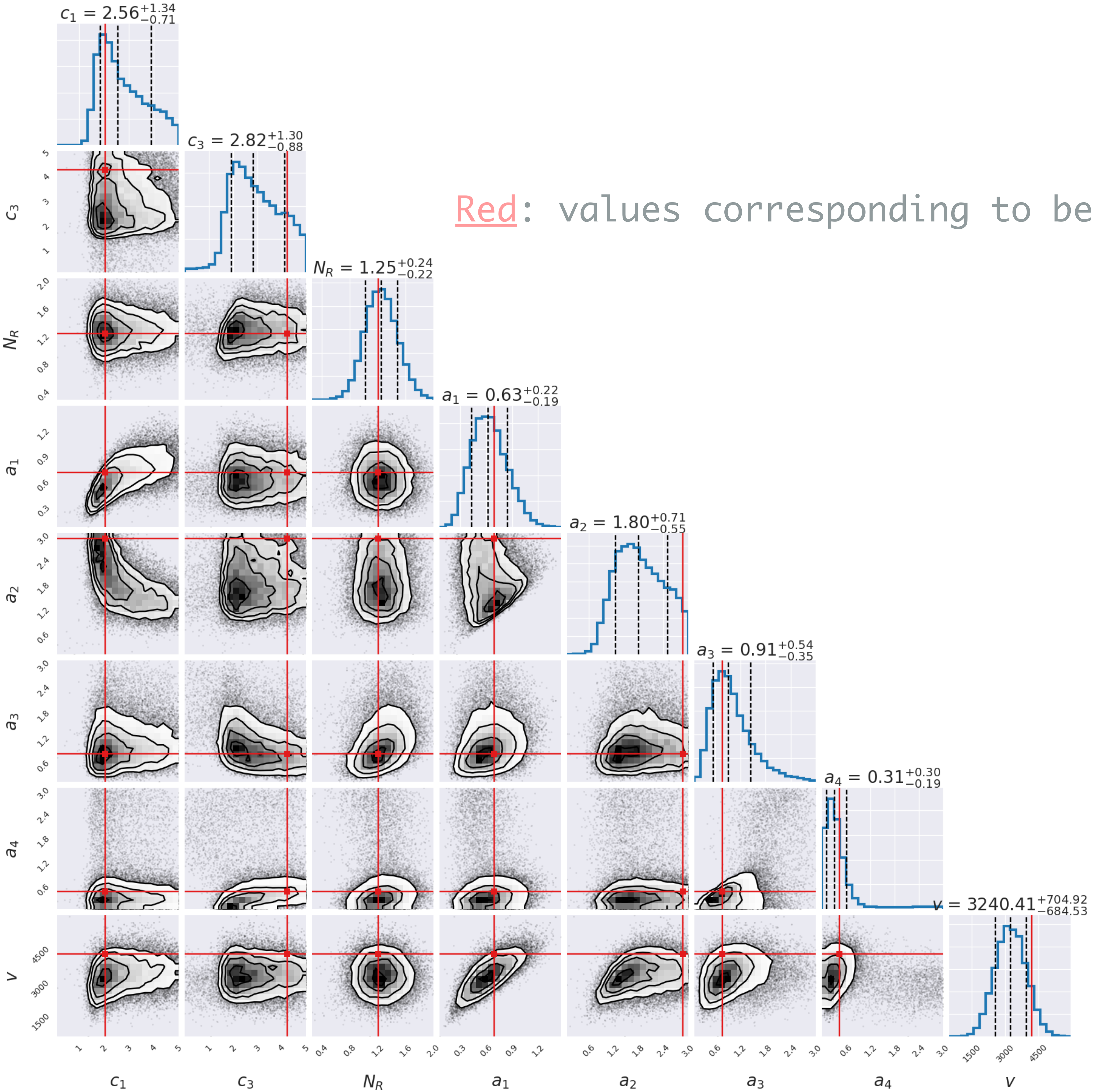
Biexponential parameter estimation

- We use nested sampling to efficiently probe 8-D parameter space and attempt to estimate values for parameters [1]

1. Feroz, F., Hobson, M. P., & Bridges, M. (2009). MultiNest: an efficient and robust Bayesian inference tool for cosmology and particle physics. Monthly Notices of the Royal Astronomical Society, 398(4), 1601–1614. http://doi.org/10.1111/j.1365-2966.2009.14548.x



Red: values corresponding to best fit

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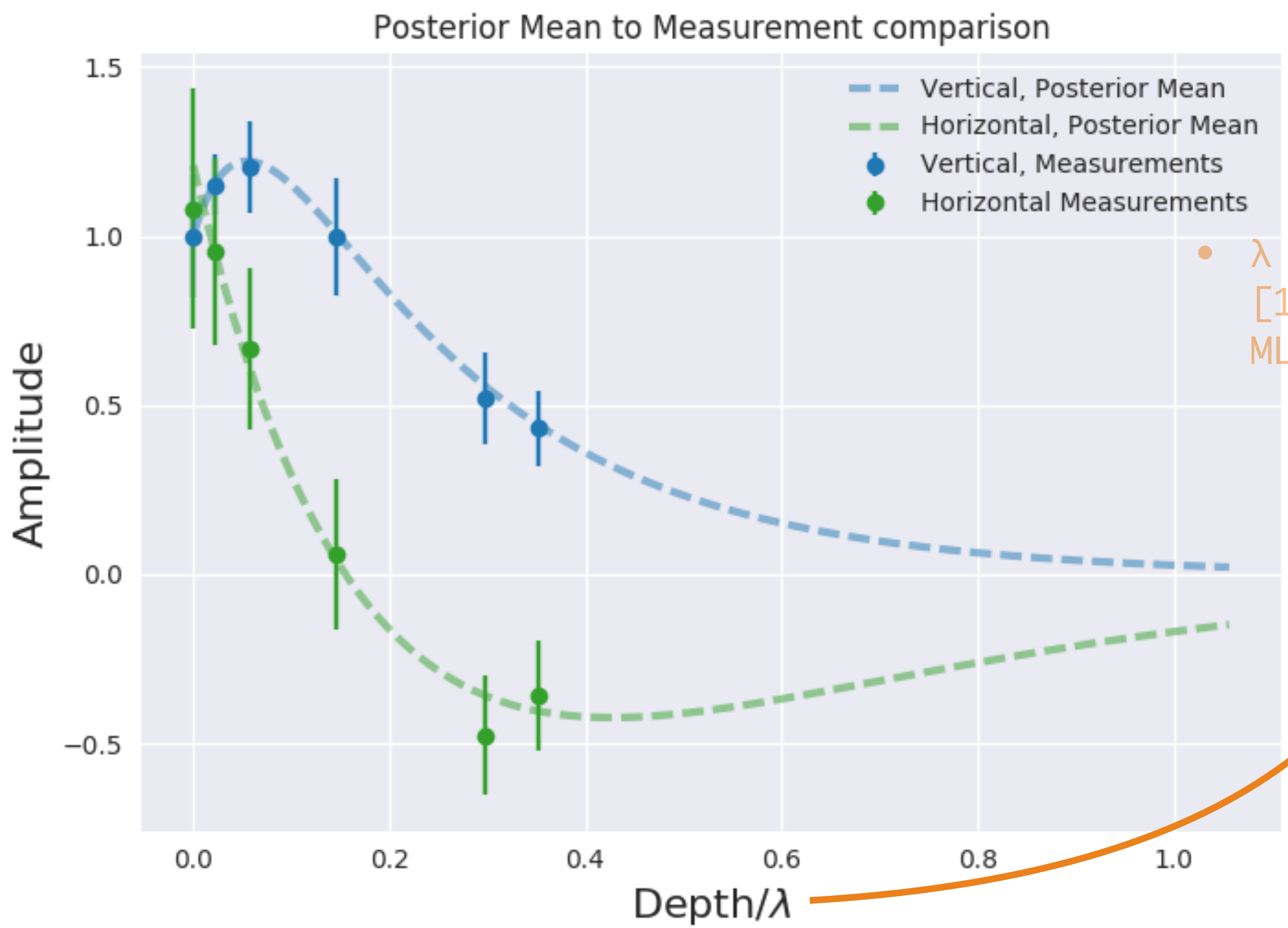
Best fit illustration



- λ is from frequency [1 Hz] and velocity ML fit parameter

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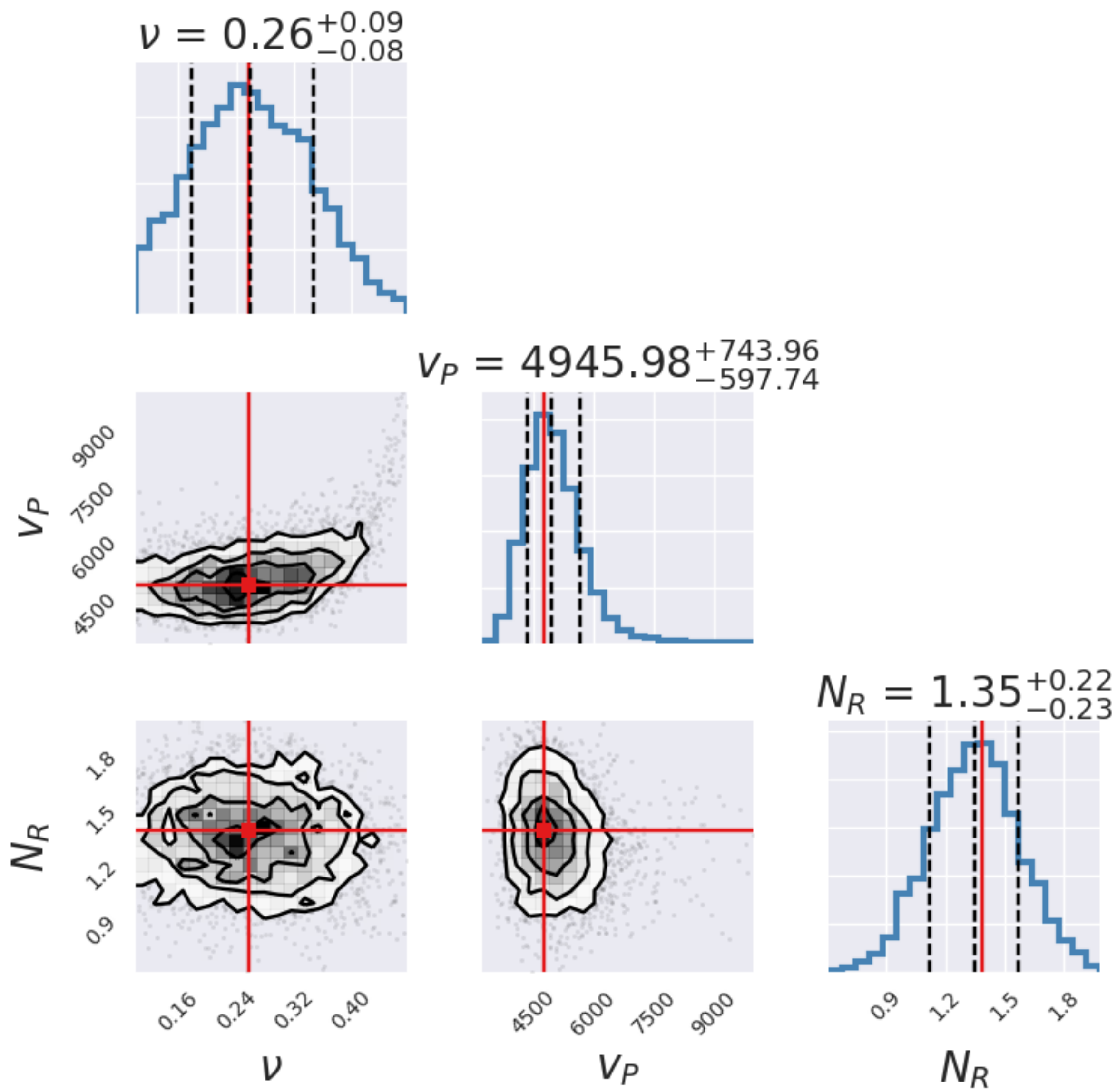
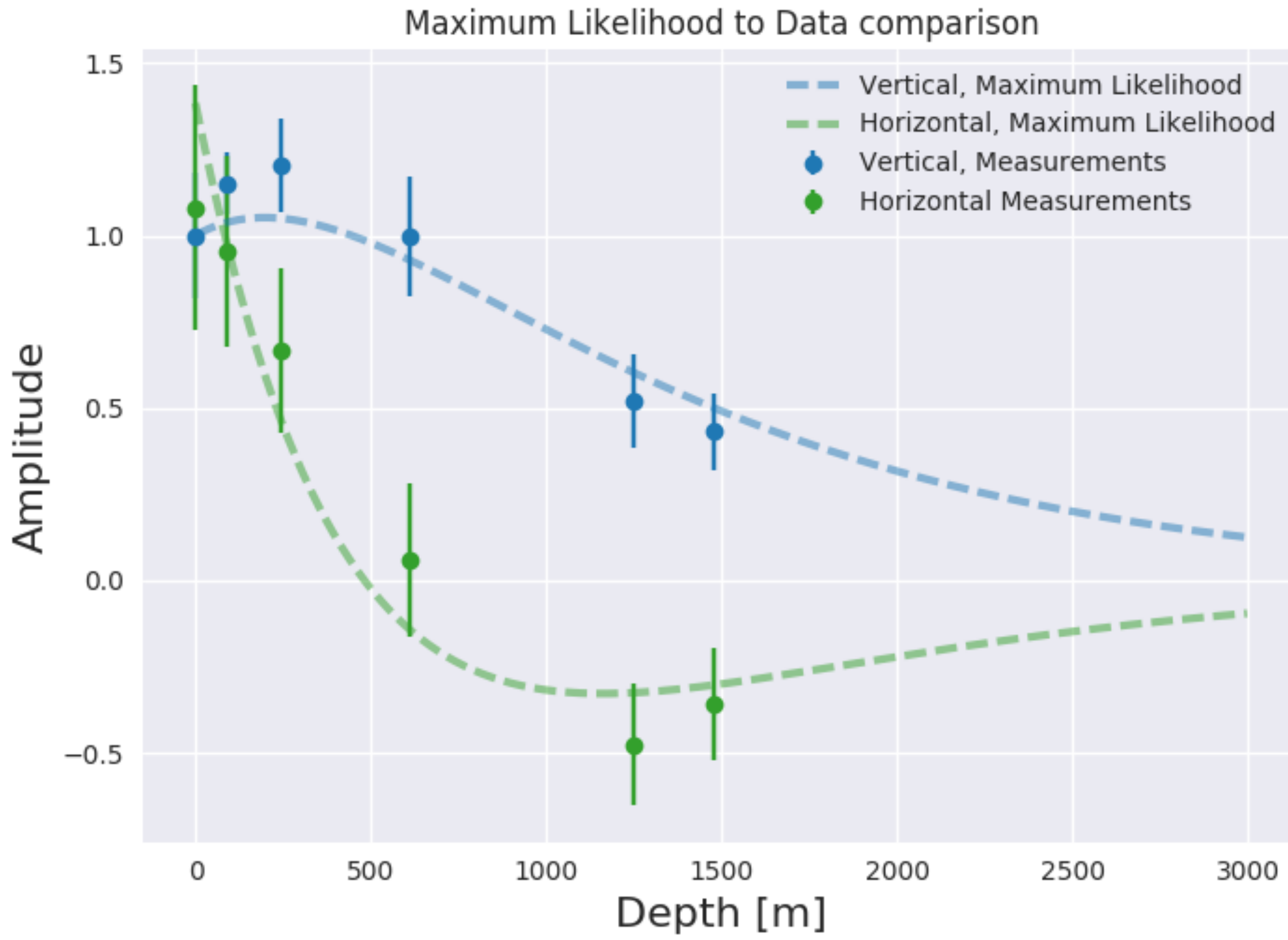
Best fit using homogeneous half-space





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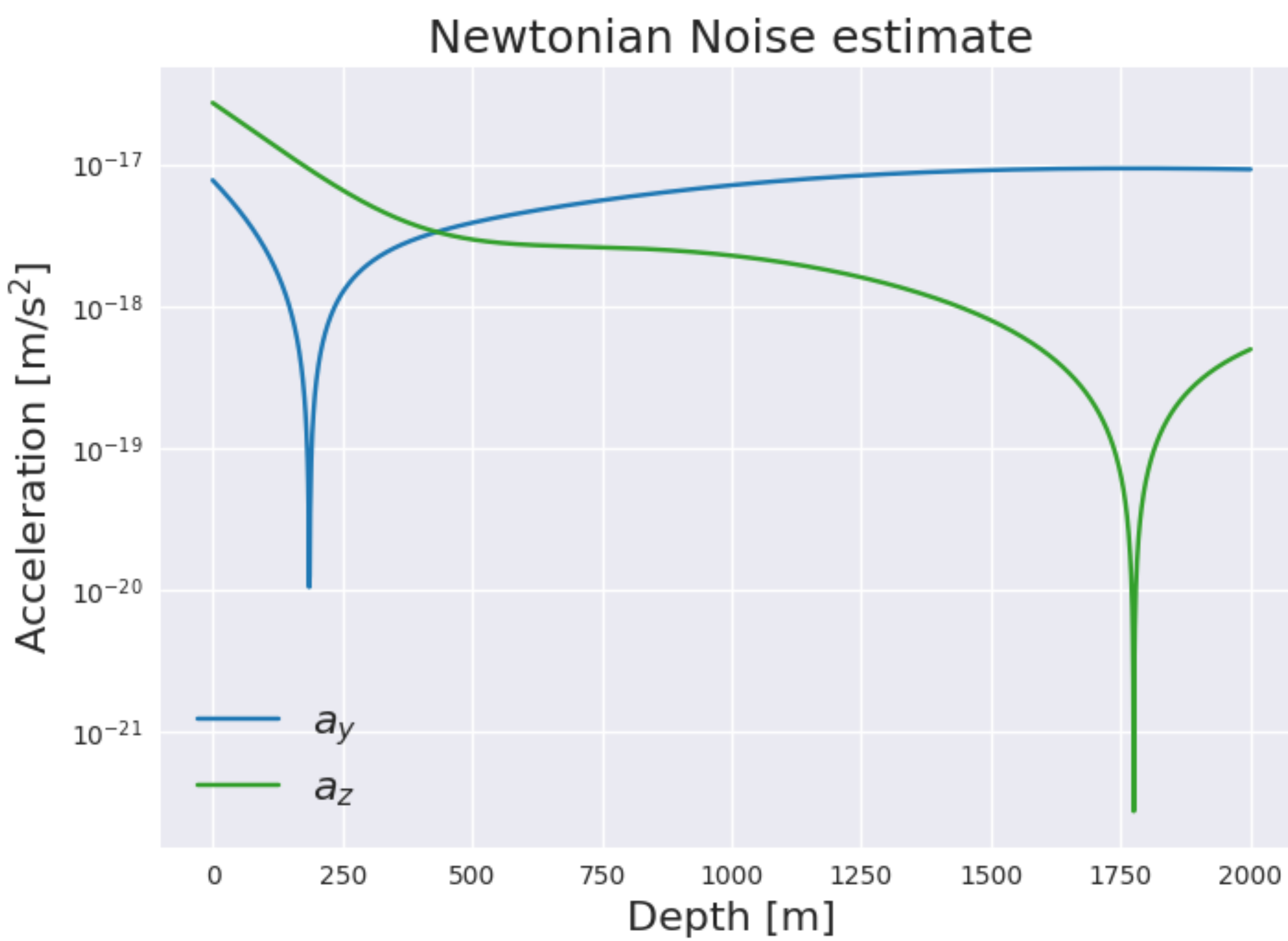
Questions/uncertainties

- Is it best to keep vertical amplitude as max, but change horizontal based on phase?
  - Should both change?
- H/V < 1, at surface from what I understand…
  - That doesn't seem to be the case. Is there some sort of free-surface reflection term I'm missing (like a factor of 2 for stations that are surface or near-surface)?
  - It would have to apply only to one of the two components if I am missing it, obviously…

Extras

- We can use the eigenfunction to estimate acceleration noise from GGN at a hypothetical test mass buried at some depth [1]
- Assumptions:
  - r-wave travels only in “x” direction
  - Has amplitude at surface equal to roughly median 1 Hz amplitude over our surface stations throughout run
  - Some estimate of rock density is assumed



1. Harms, J. (2015). Terrestrial Gravity Fluctuations. Living Reviews in Relativity, 18(1), 074001. http://doi.org/10.1007/lrr-2015-3