

Radiometer results

Pat Meyers
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Note from the start

- This represents “the” characterization of the radiometer analysis as far as I’m concerned
- Not sure how much farther we can go characterizing our ability to this method for our the array
- There are a few more things on the that aren’t presented/are in process that I’ll mention

Systematic characterization

- On the right is process we do for solving radiometer

- M = “model resolution matrix”

- Ideally would be identity

- Depends upon how we create pseudo-inverse matrix

$$Y = \gamma S_{true}$$

$$S_{model} = (\gamma^\dagger \gamma)^{-1} \gamma^\dagger Y$$

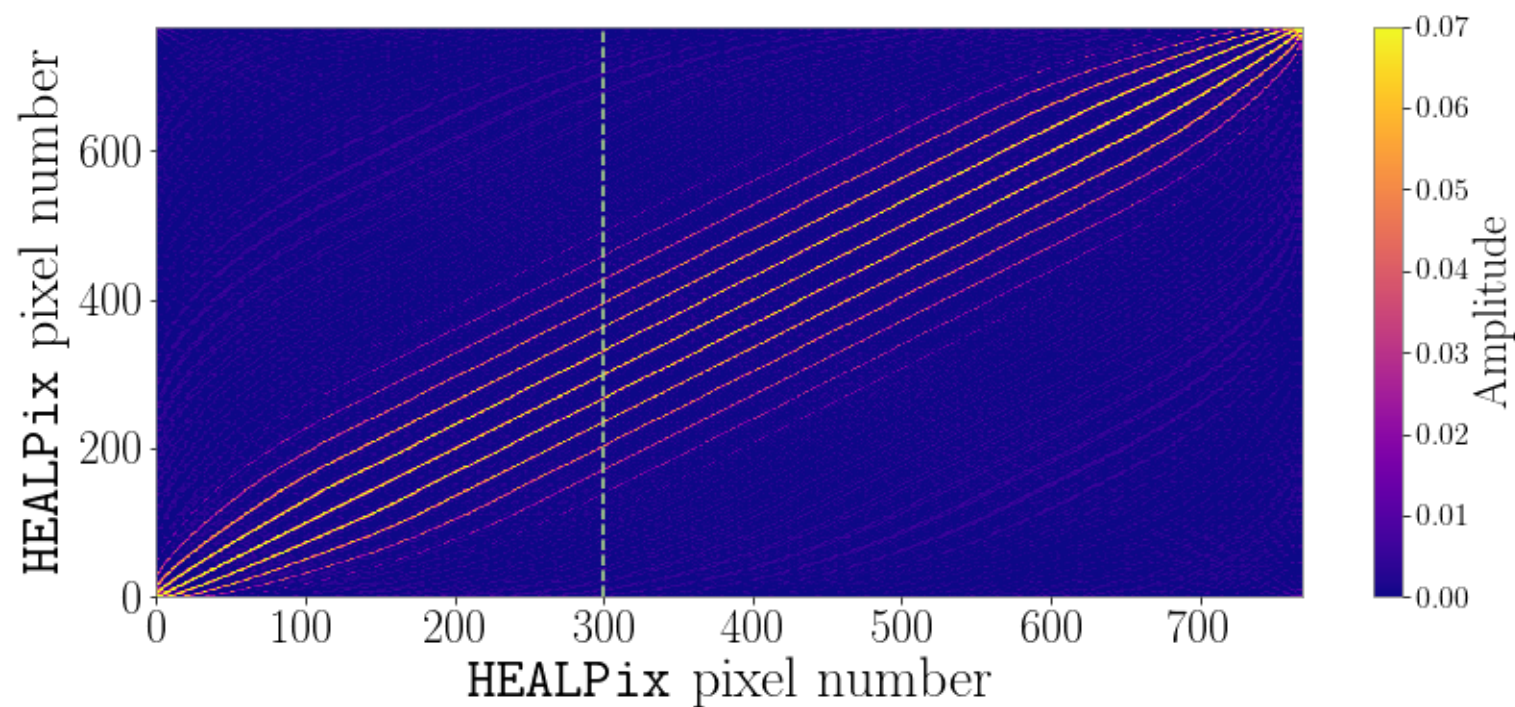
$$S_{model} = \gamma^+ Y$$

$$S_{model} = \gamma^+ \gamma S_{true}$$

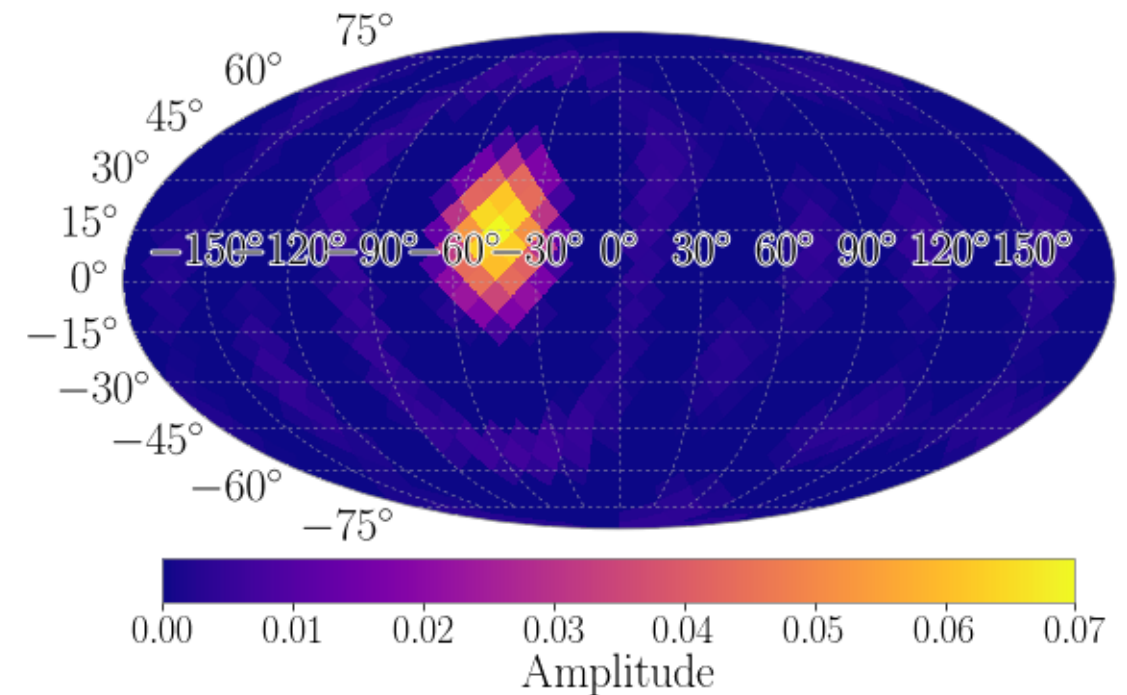
$$\mathcal{M} = \gamma^+ \gamma$$

P-wave model resolution matrix

Model resolution matrix: $\bar{\gamma}^+ \bar{\gamma}$



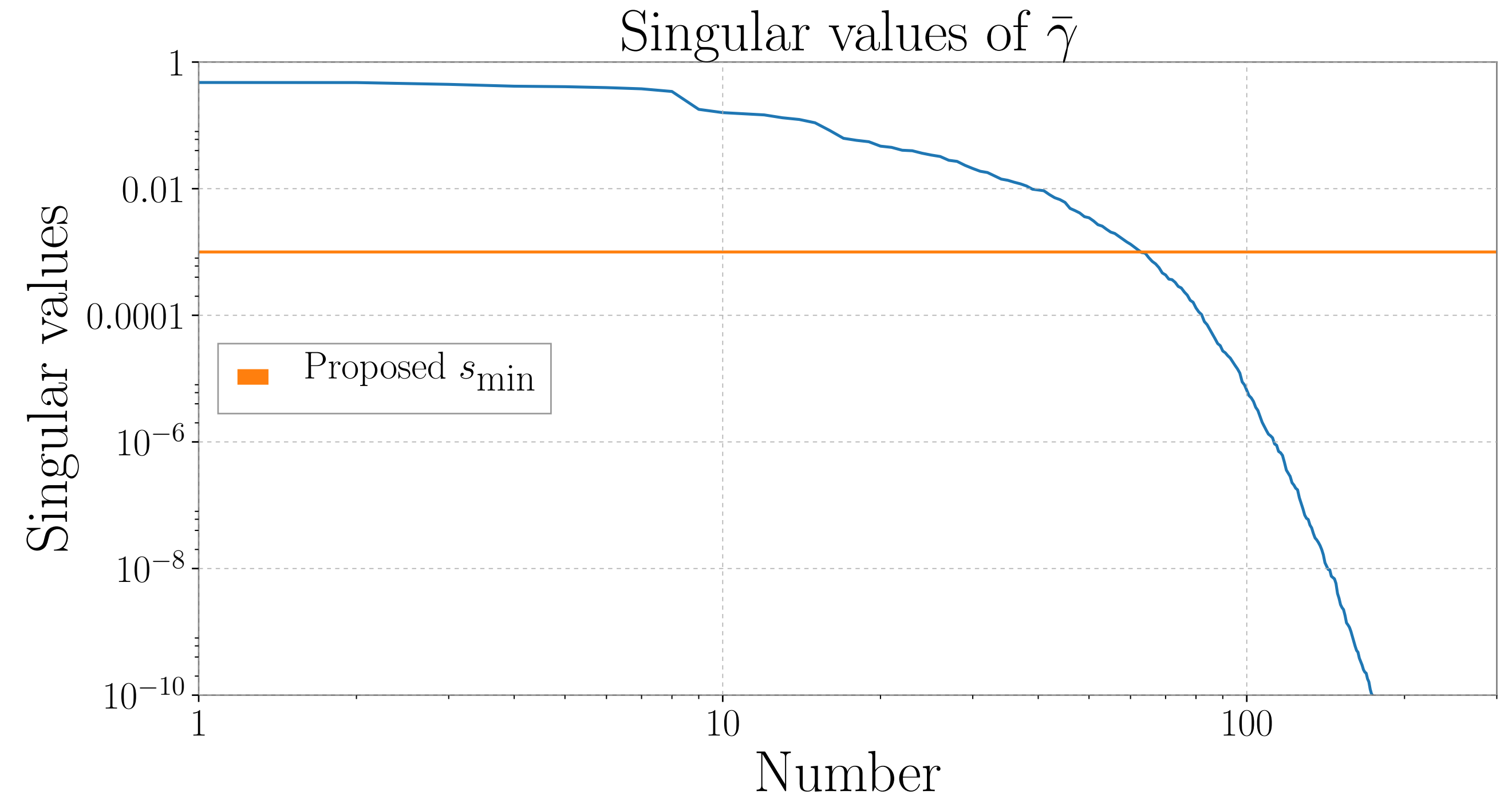
Column 300 of Model Resolution Matrix

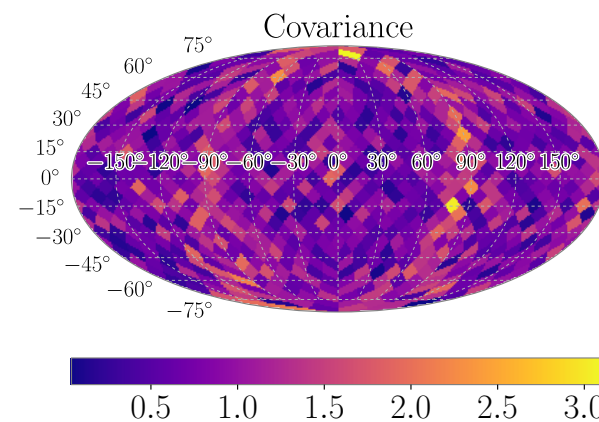
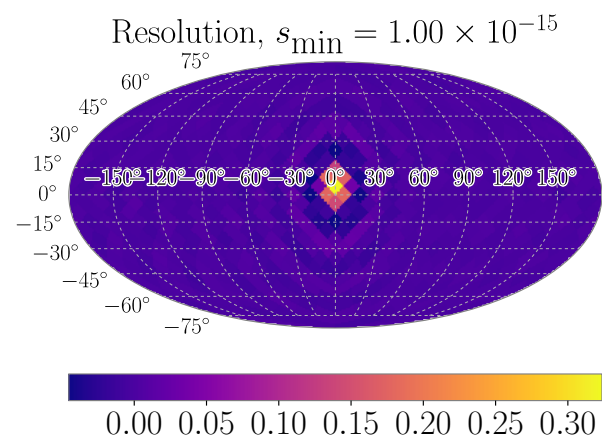


Creating pseudo-inverse

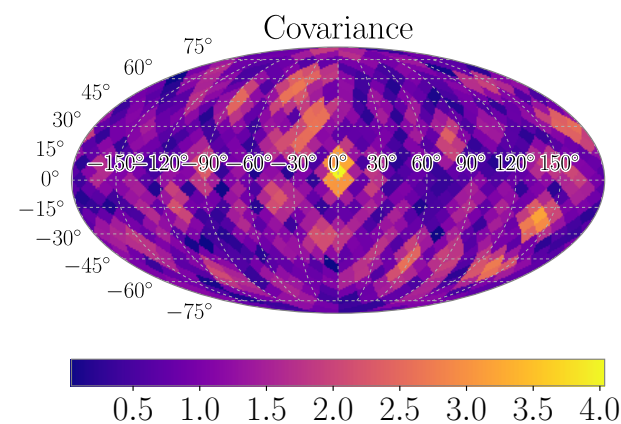
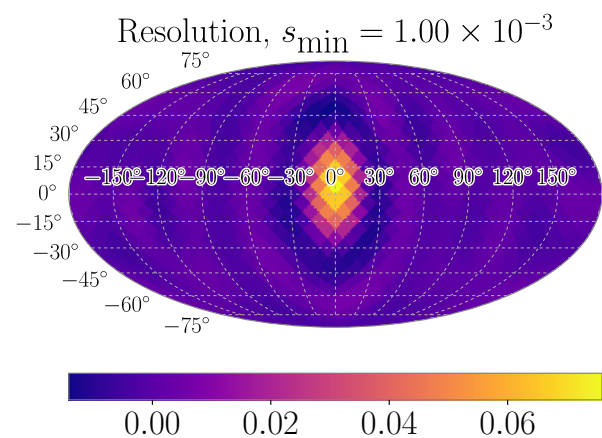
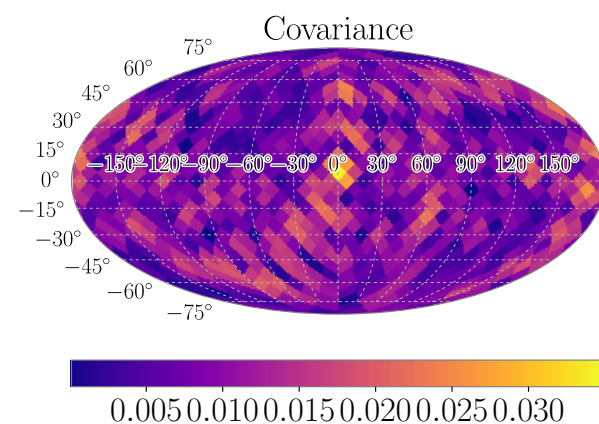
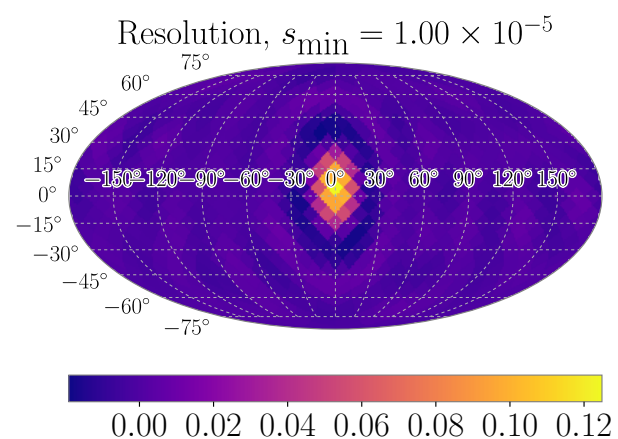
- Take singular value decomposition of γ
 - cut off at some minimum value
 - create pseudo-inverse
- Get model parameters
- Small minimum $\Rightarrow$ better resolution, higher covariance
- large minimum $\Rightarrow$ worse resolution, lower covariance

Singular values: P-wave gamma matrix

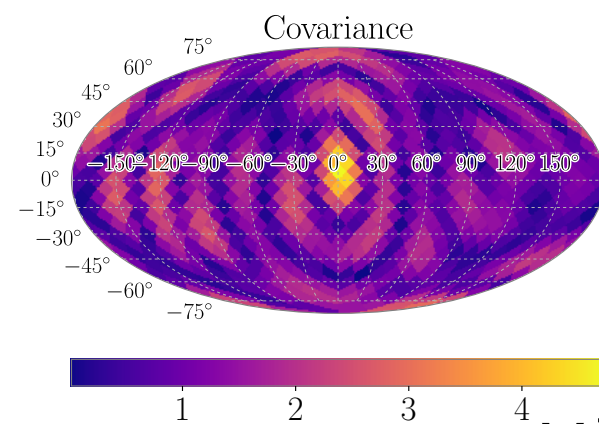
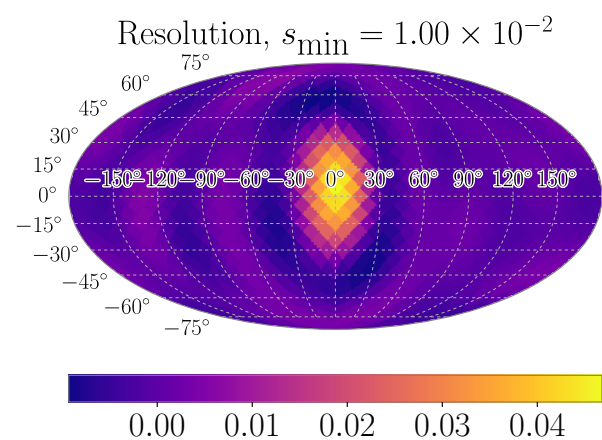




$\times 10^{18}$



$\times 10^{-6}$



$\times 10^{-8}$

Choosing best $s_{\min}$

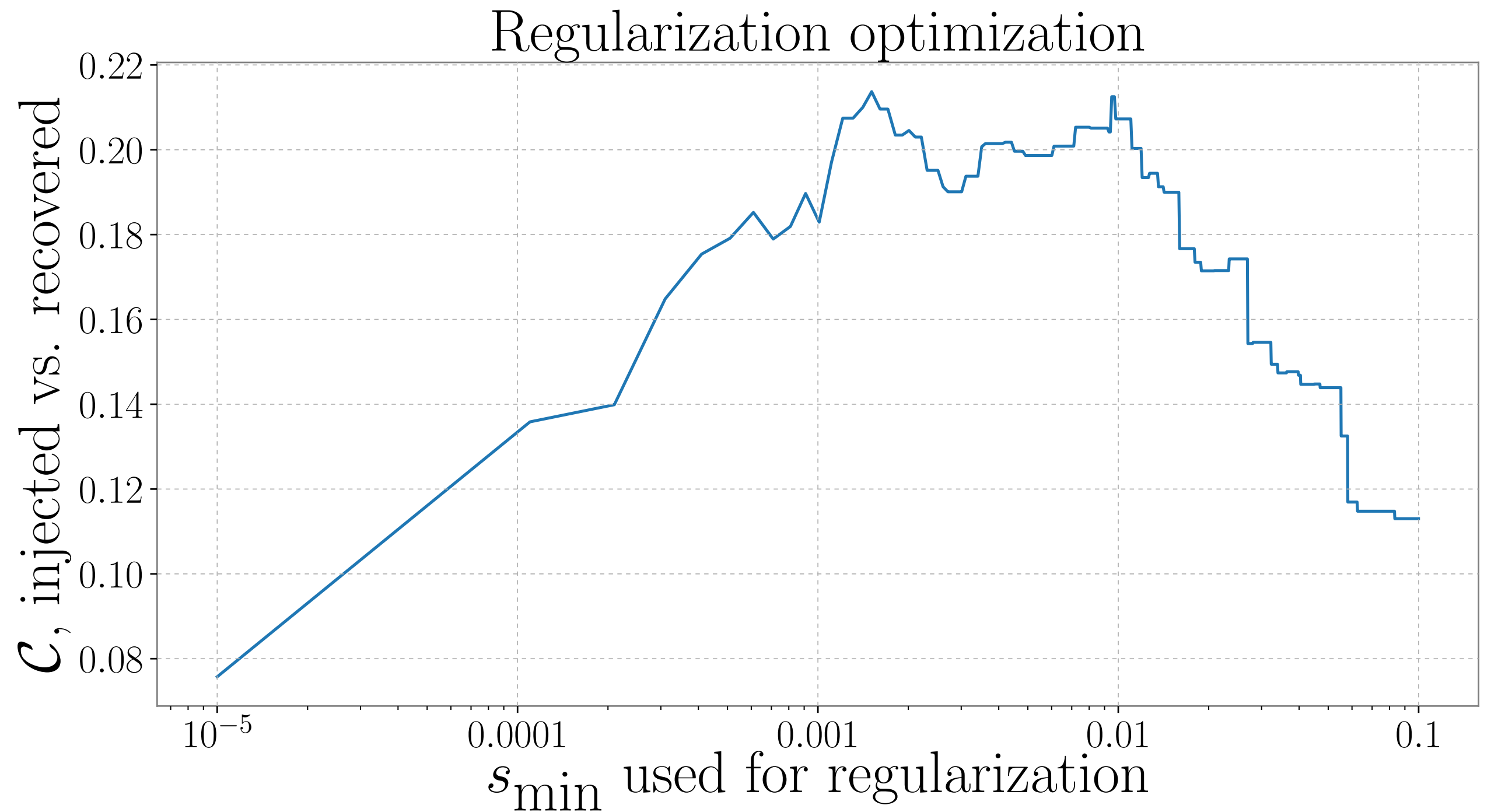
- Create injected map: single pixel with amplitude = 1, all others 0
- Get recovered map using $s_{\min}$
- take map coherence
- maximize map coherence over $s_{\min}$

$$\mathcal{C} = \frac{\frac{1}{2}(\mathbf{M}_1 \mathbf{M}_2^\dagger + \mathbf{M}_2 \mathbf{M}_1^\dagger)}{\sqrt{(\mathbf{M}_1 \mathbf{M}_1^\dagger)(\mathbf{M}_2 \mathbf{M}_2^\dagger)}}$$

Maximization subtleties

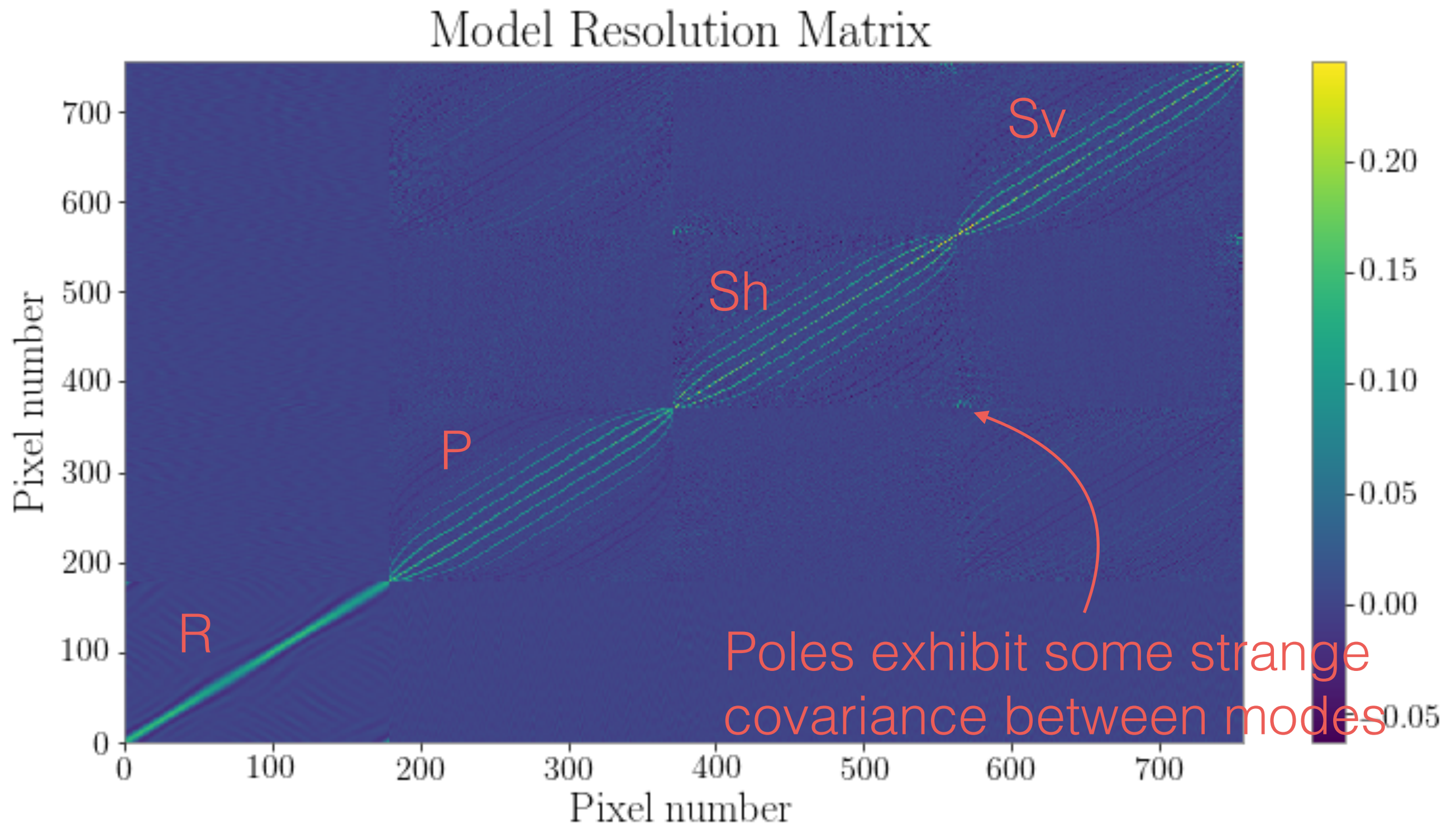
- In theory, maximization needs to be re-run for different injection types and different recovery types (i.e. if we want to recovery P, S, or R-waves

P-wave, coherence maximization

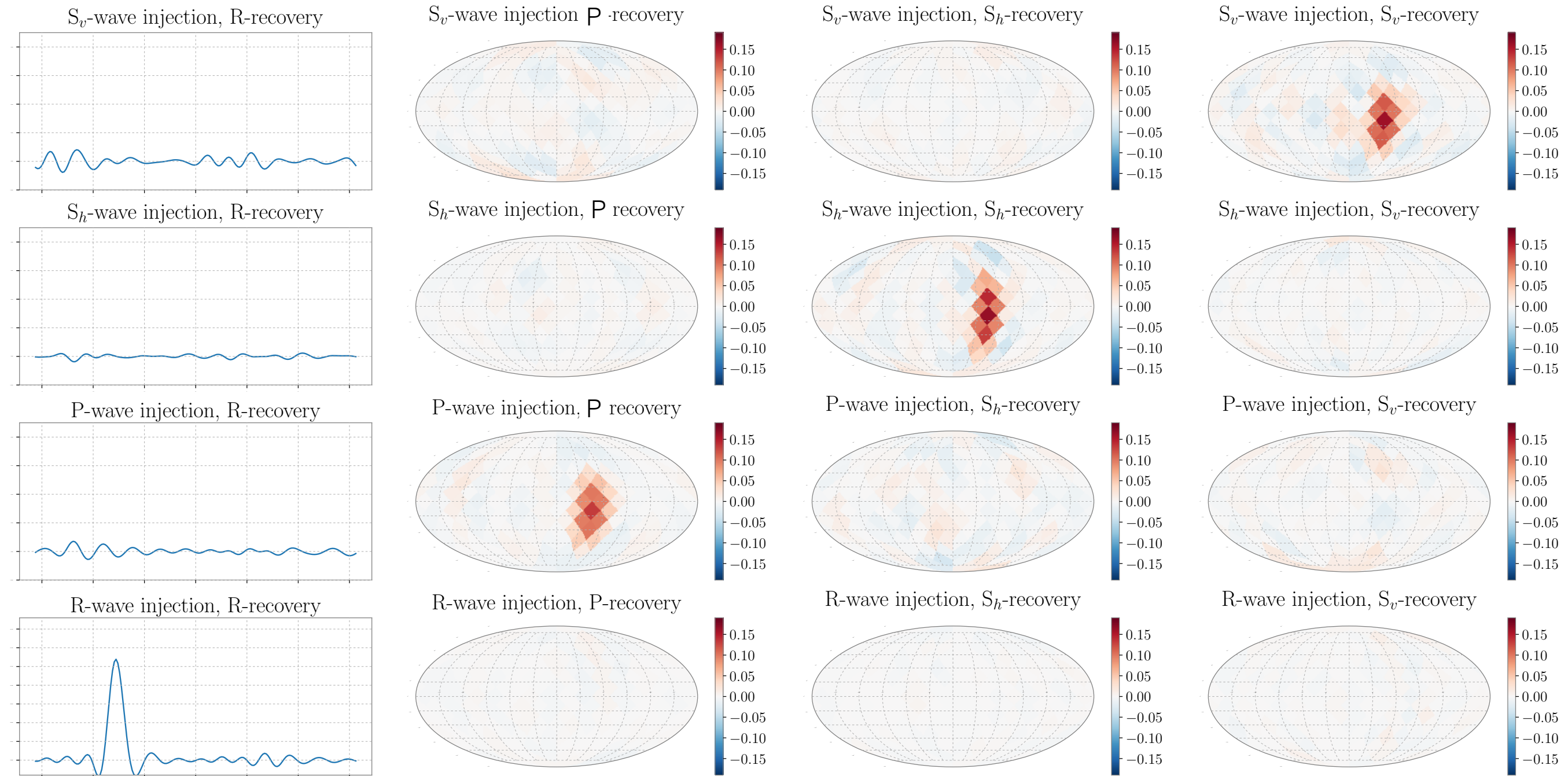


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MRM for all recovery types



MRM for all recovery types (one column from each block)



MRM for all types (conclusions)

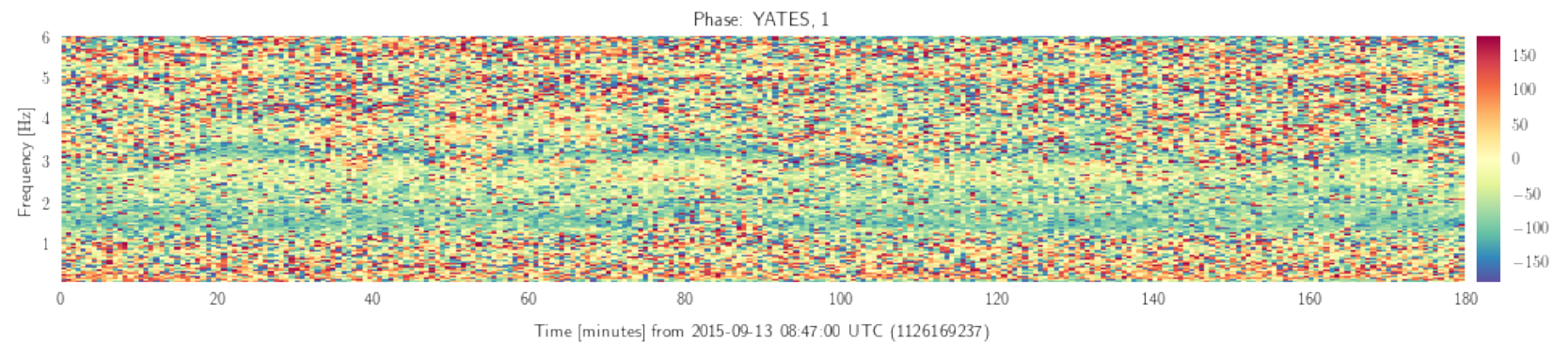
- Things tend to be fairly independent using our array.
- In the end, I ran into some issues with covariance being large when trying to recover injections of three different types at once, though

Last characterization study in progress

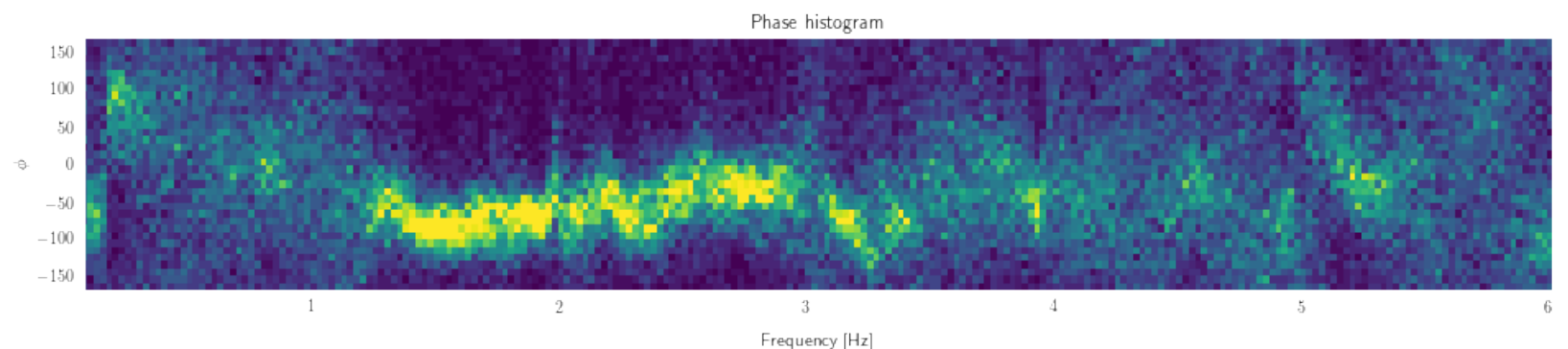
- Angular resolution vs. wavelength for each mode
 - For each mode and for each wavelength, maximize map coherence for East-pointing injection over $s_{\min}$
 - Take angular spot size from MRM column plot
 - Plot spot size vs. λ
 - Plot column rank of pseudo-inverse vs. λ as well (number of singular values larger than $s_{\min}$)

Persistent source example

- Top: Phase difference between vertical and east channel
- Bottom: Histogram over time segments
- NOTE: This source isn't really seen in the North channel at all

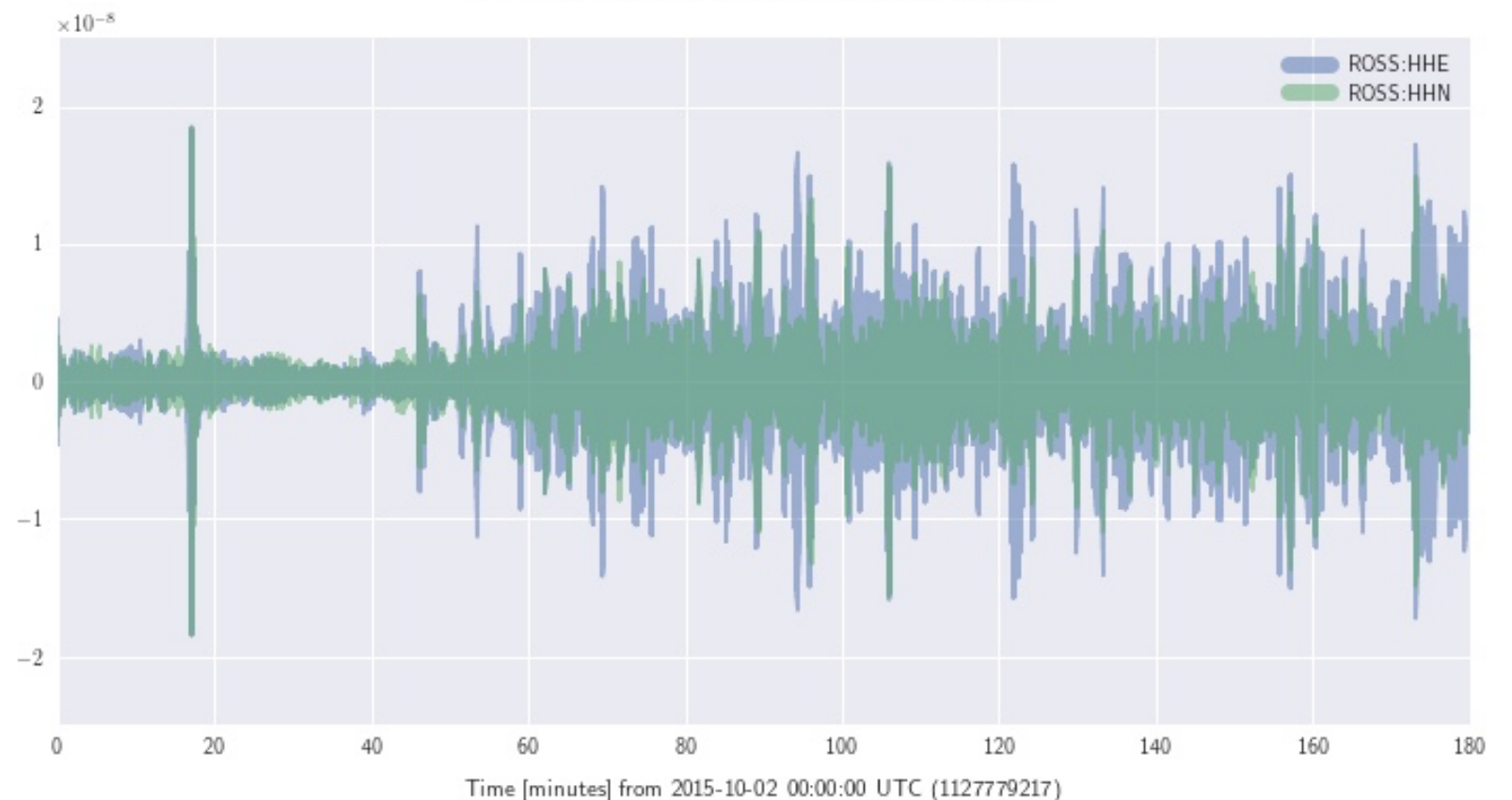
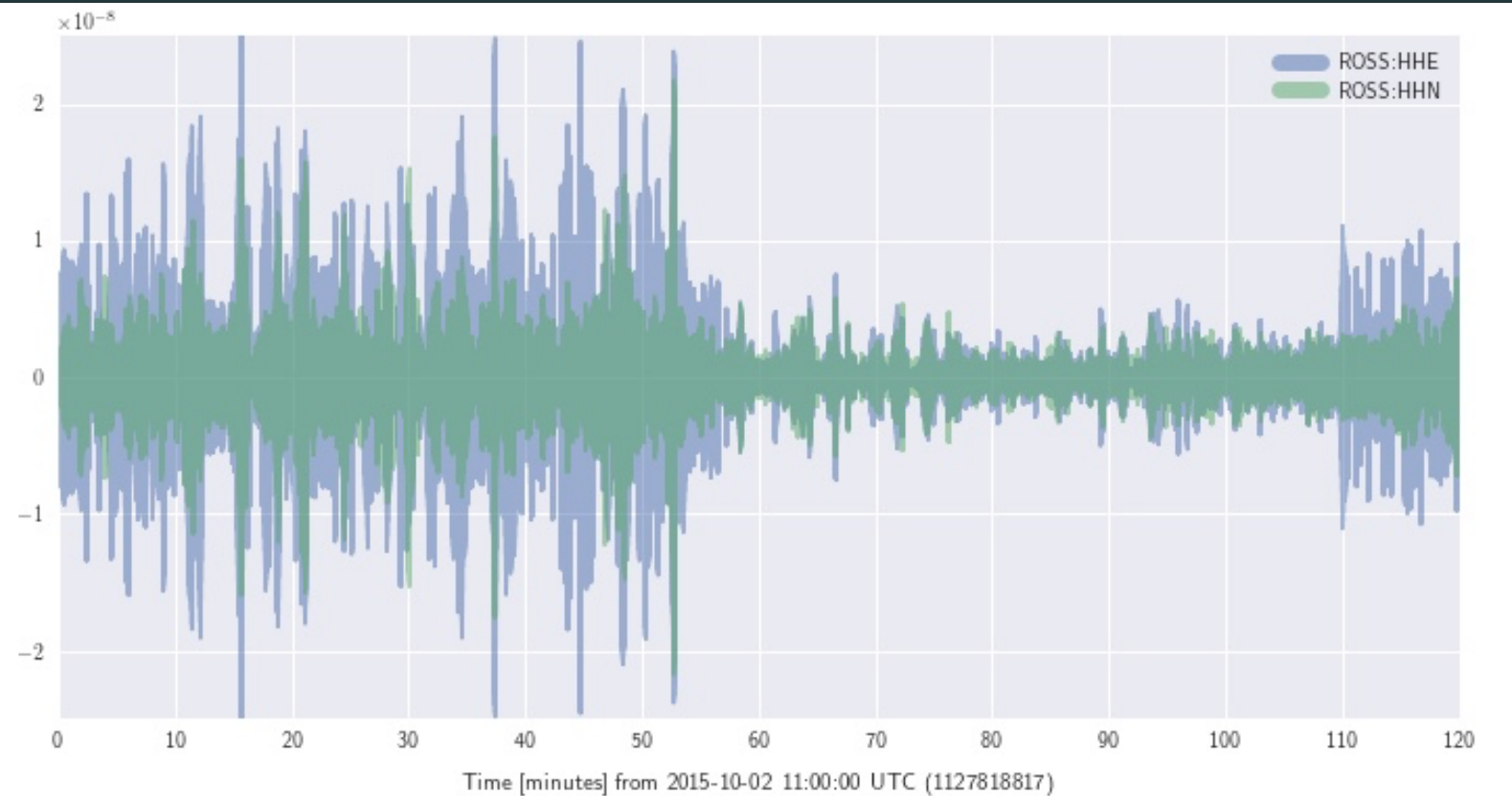


confusing information so I've covered up these plots



This source turns on and off?

- Band-pass filter from 1.4 - 1.6 Hz, look at time series
- See it turn off at 00:00 - 00:30 and 12:00 - 12:30 UTC *every day* for some reason



Quick timing analysis

- Use only phase delay between stations to estimate direction and velocity
- Assume coming in along surface
- Basically this is a “surface plane-wave” hypothesis that uses no polarization information

Likelihood

- Use a student's t-distribution over inter-site phases
- i = pairs of channels at different seismometers, m_i = number of phase measurements for that pair

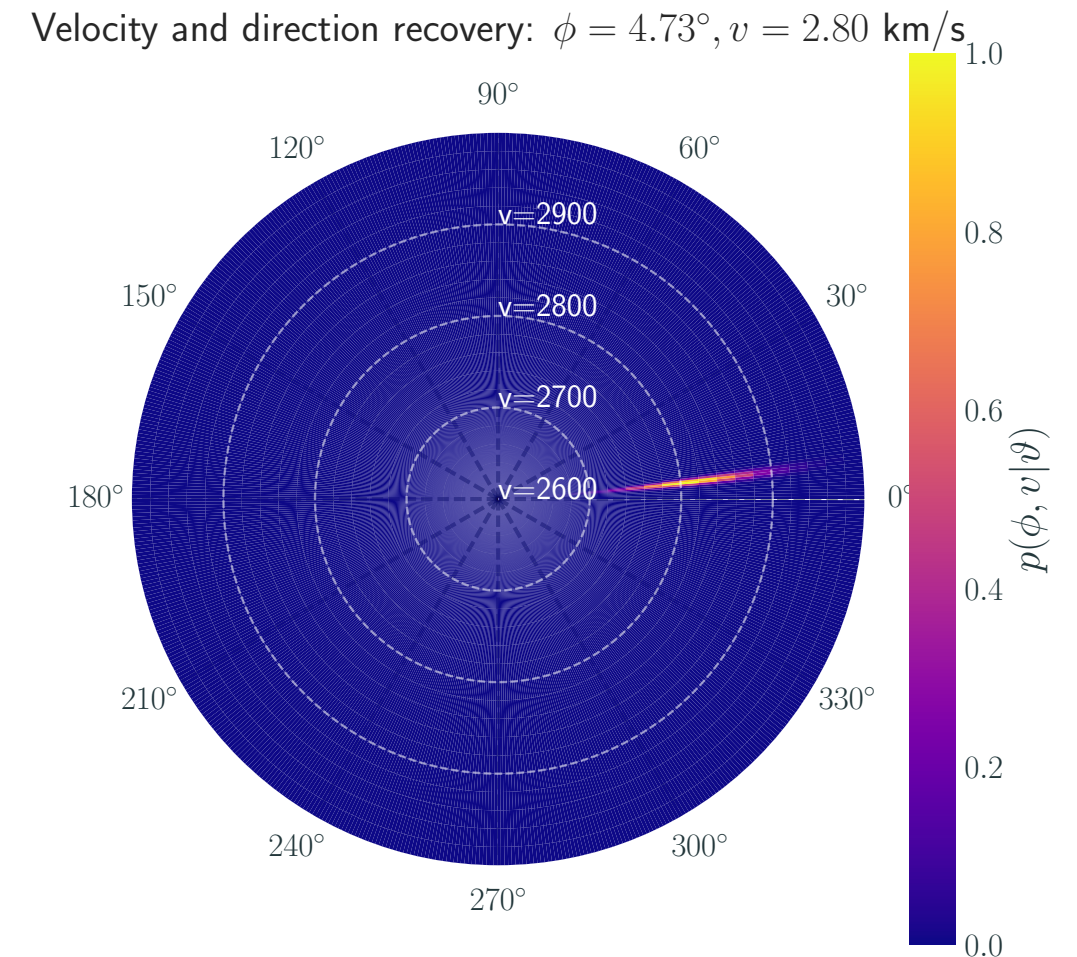
$$p(\{\vartheta\} | \hat{\Omega}, v) = \prod_{i=1}^{N_{pairs}} \frac{\Gamma(m_i/2 - 1)}{2\pi^{m_i/2-1}} \left(\sum_{k=1}^{m_i} \left| \vartheta_{i,k} - \frac{2\pi f \hat{\Omega} \cdot \Delta \vec{x}_i}{v} \right|^2 \right)^{-m_i/2},$$

Phase measurement

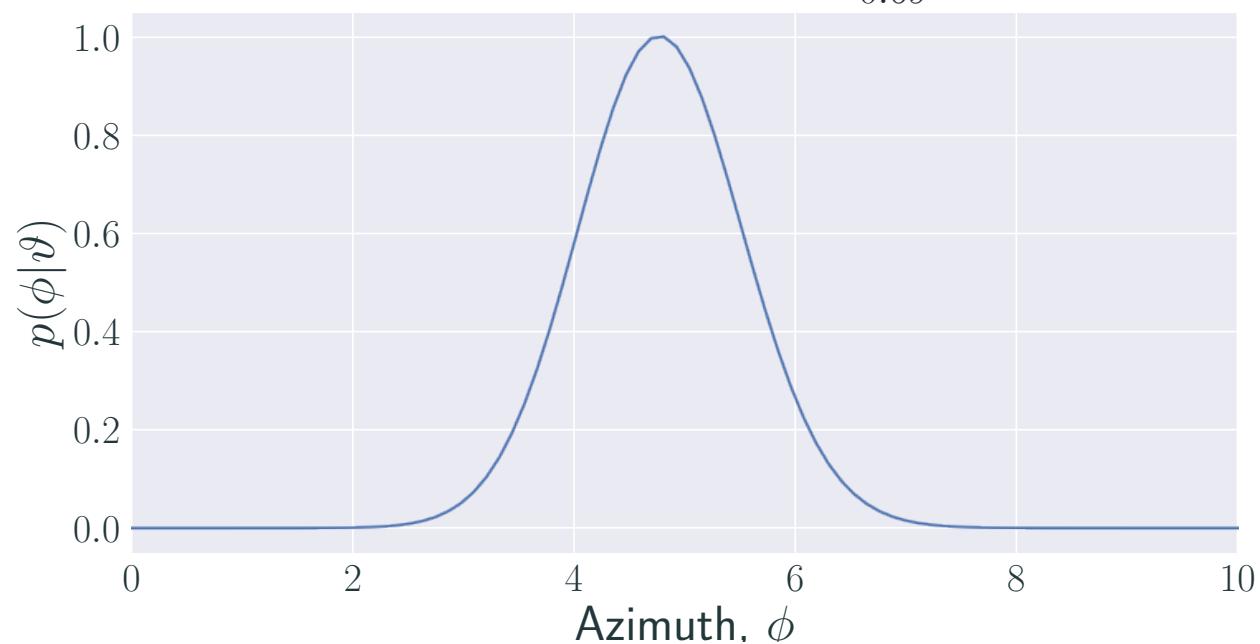
theoretical phase between stations

Results

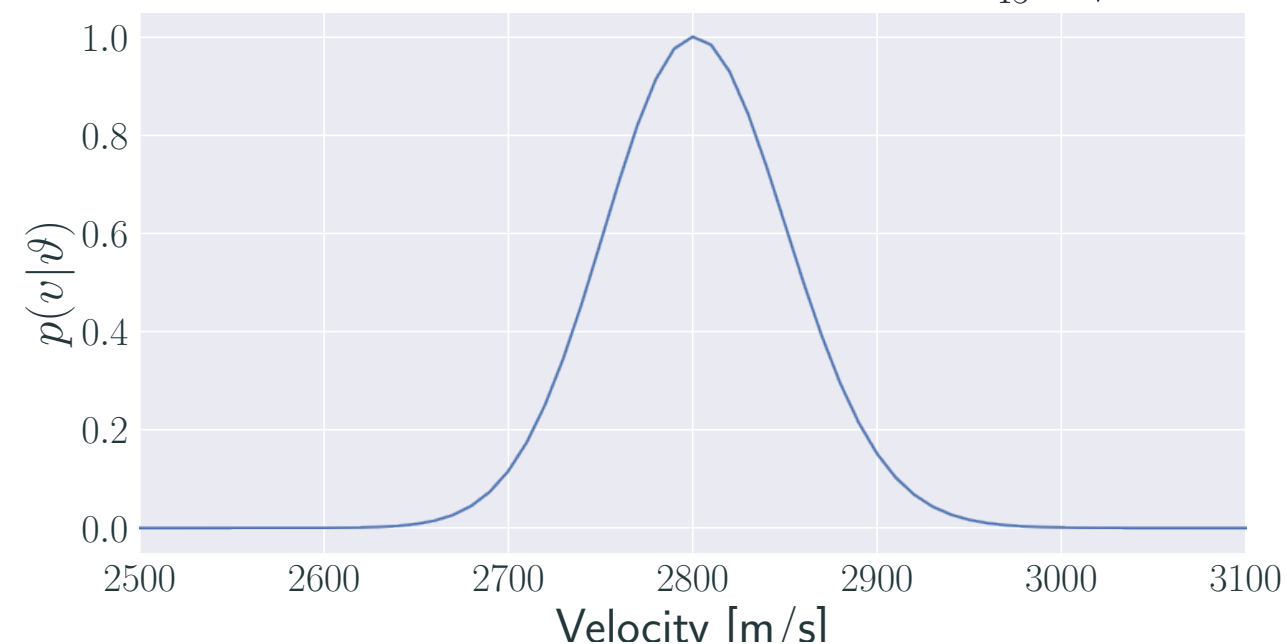
- Bayes factor between random phases and surface plane-wave assumption is ~ 250



Posterior on ϕ , $\phi = 4.81^{+0.69}_{-0.69}$ degrees



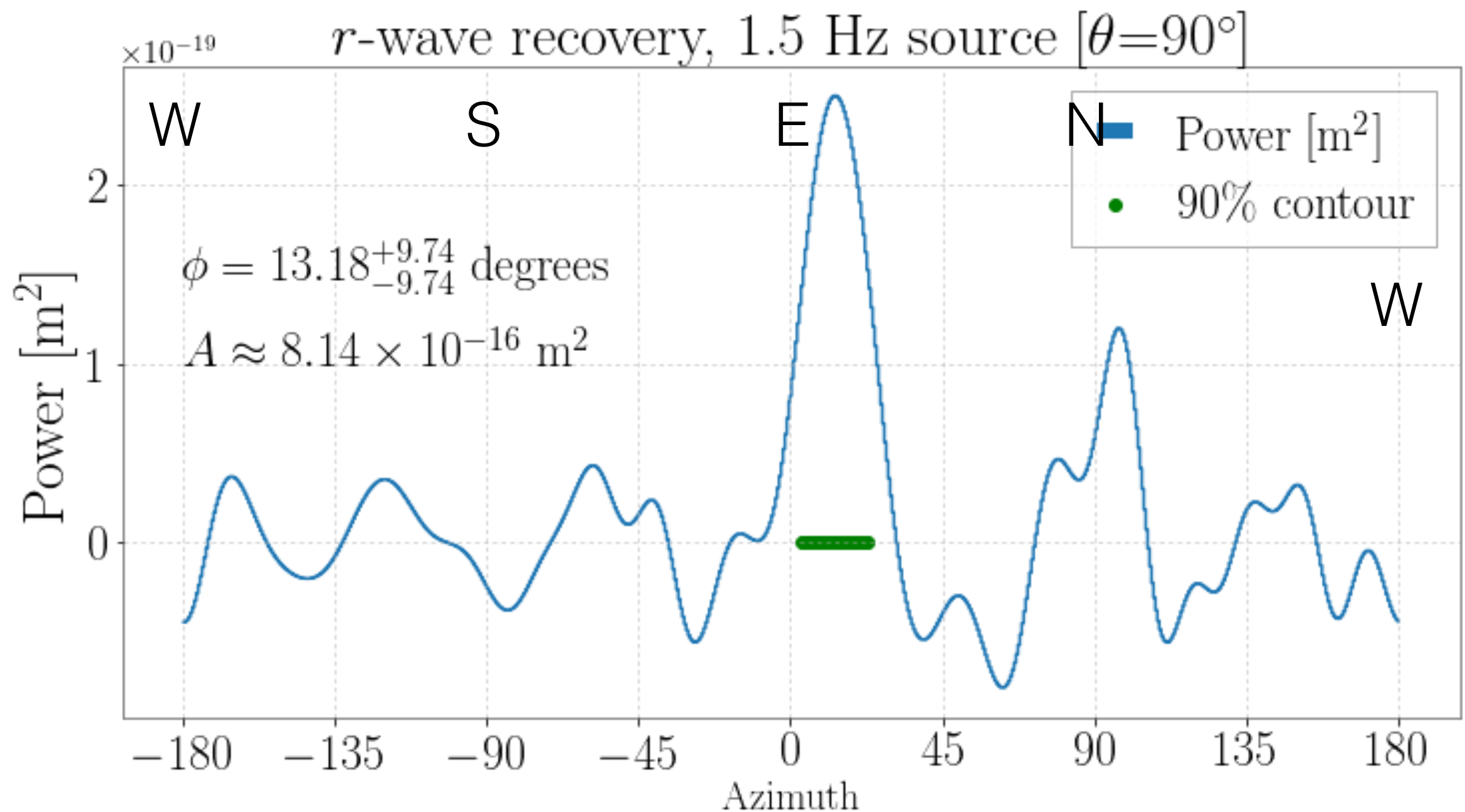
Posterior on velocity, $v = 2805^{+45}_{-45} \text{ m/s}$



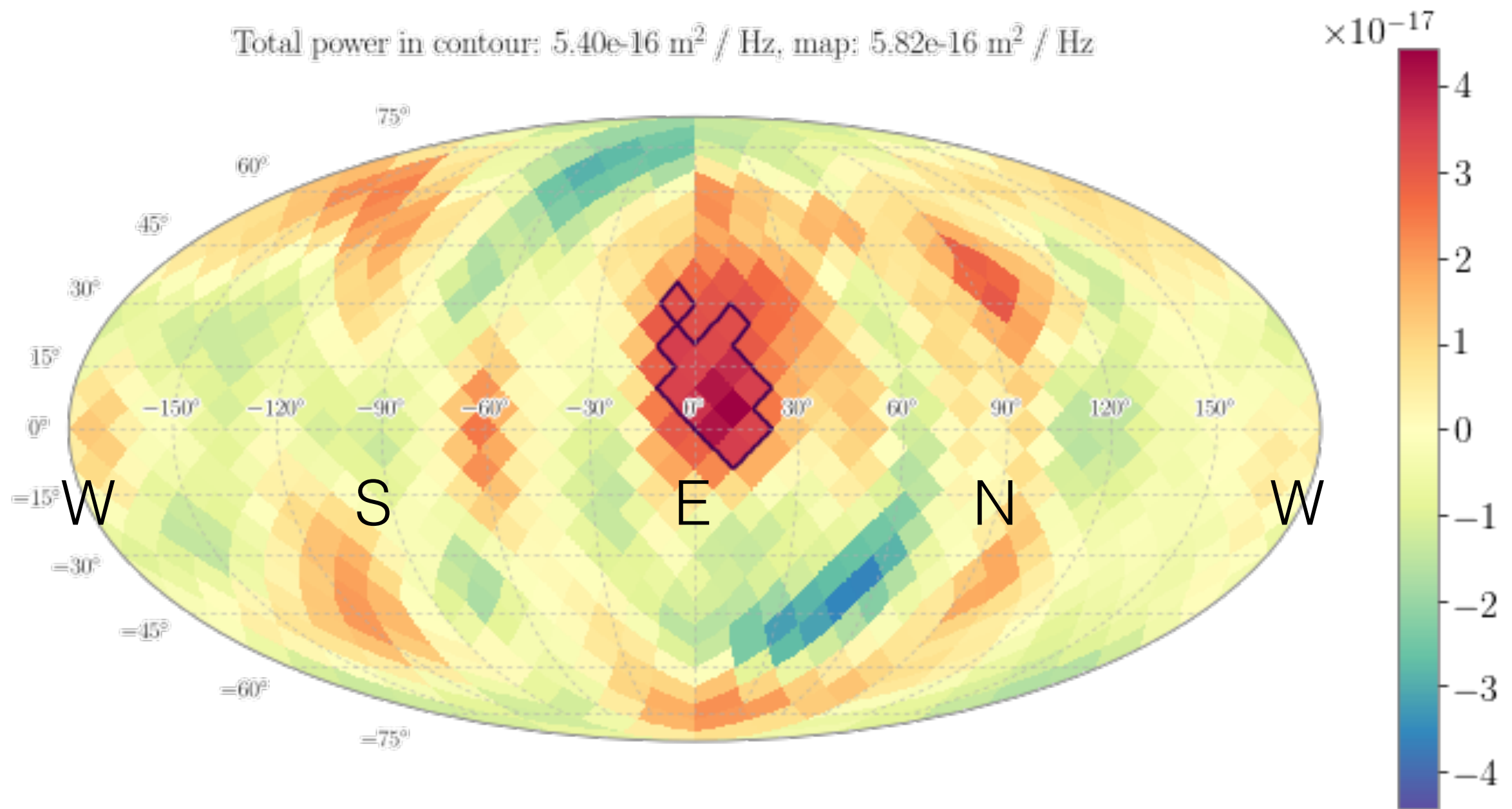
Radiometer results

- Run radiometer on same time-frame
- Only use surface stations (don't have a completely reliable eigenfunction yet to go to depth, plus it's not obviously seen at depth)
- Recover with many velocities within range of possible values, take average

R-wave map

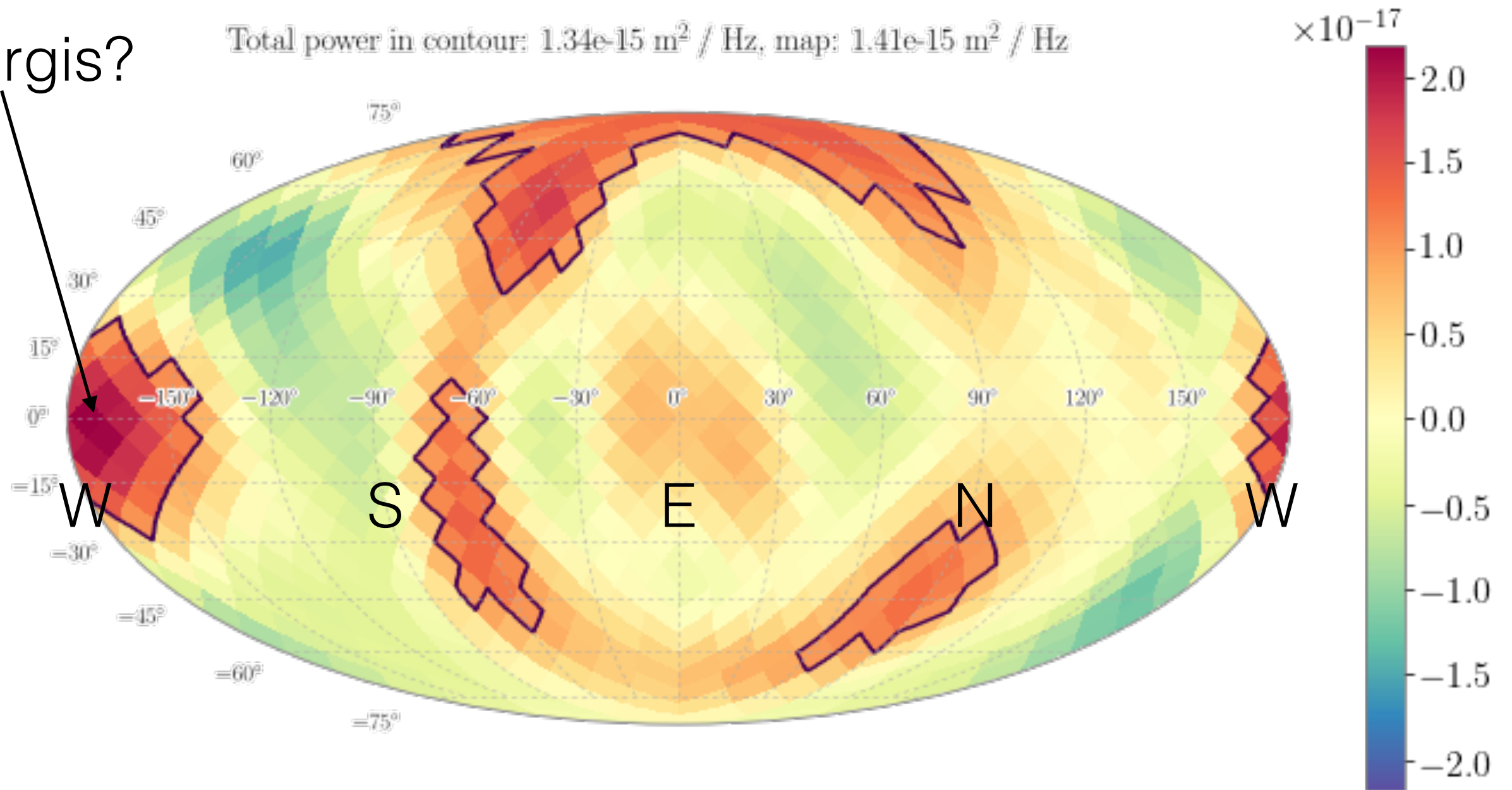


R-wave map with polar angle



P-wave map with polar angle

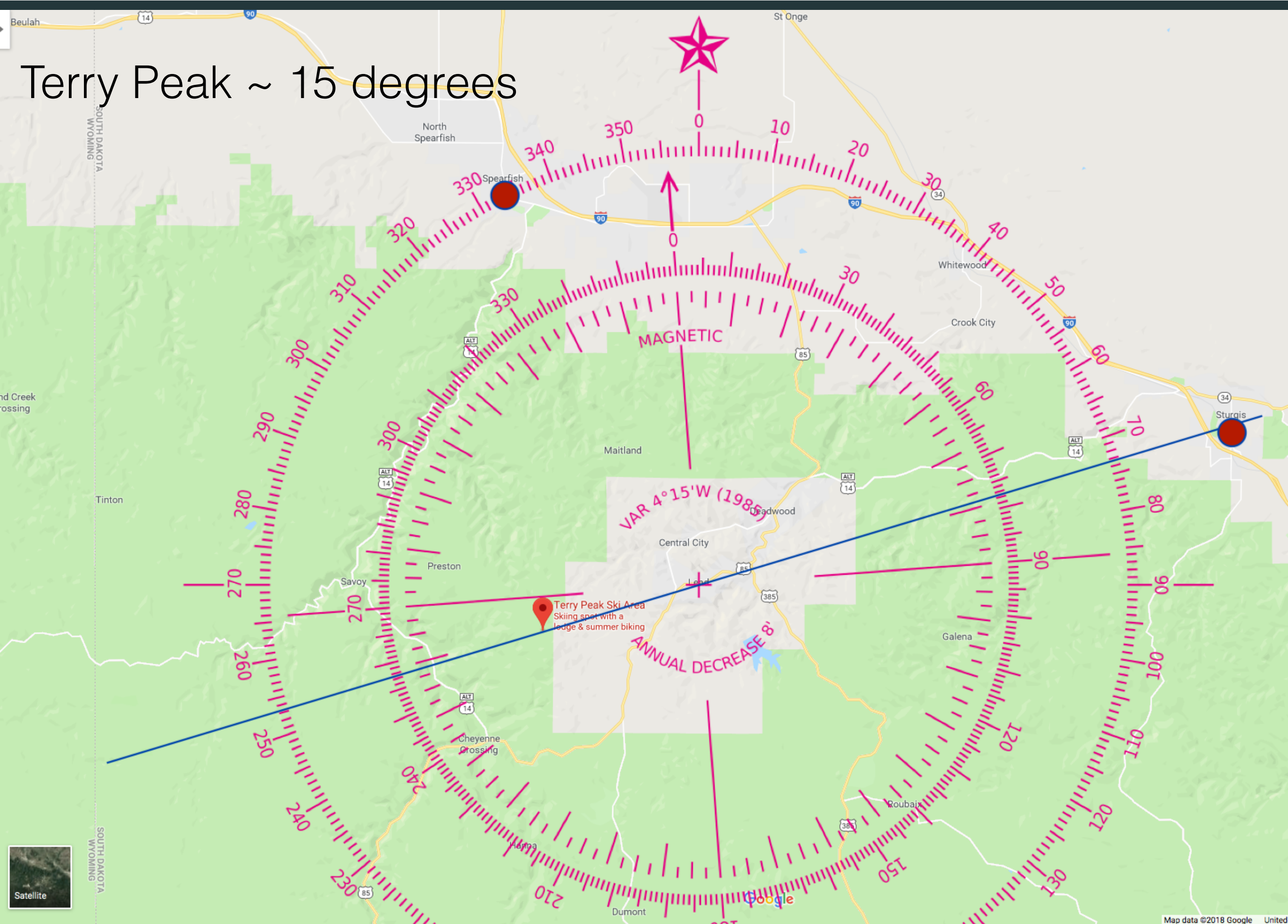
Sturgis?

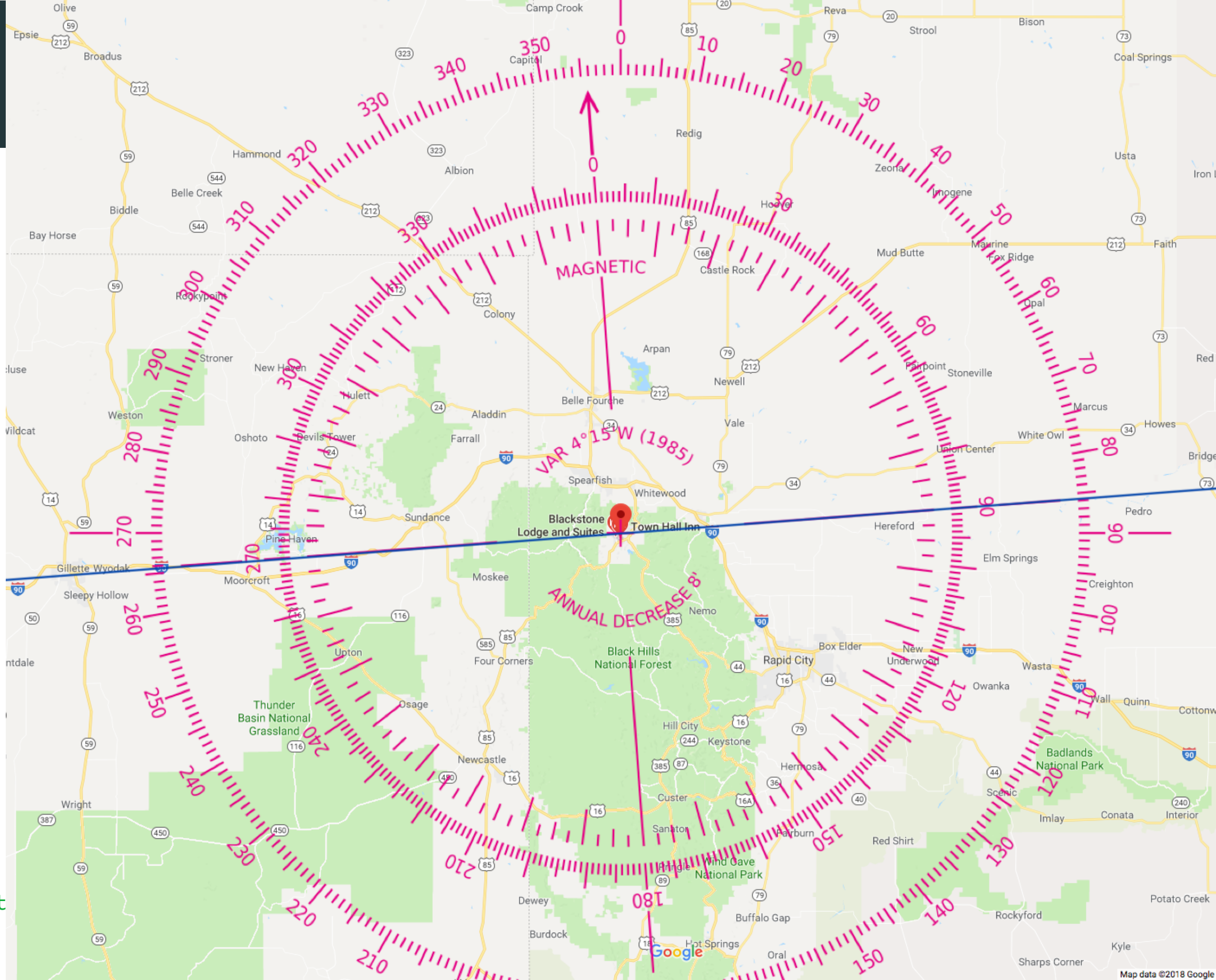


So...what is it?

- No clue! It's propagating in the East direction (that's not source direction...)
- => It's coming almost directly from the west.
- Terry Peak is roughly in the correct direction!
 - Might violate plane-wave assumption, though
- Gillette, Wyoming is in roughly the correct direction as well.

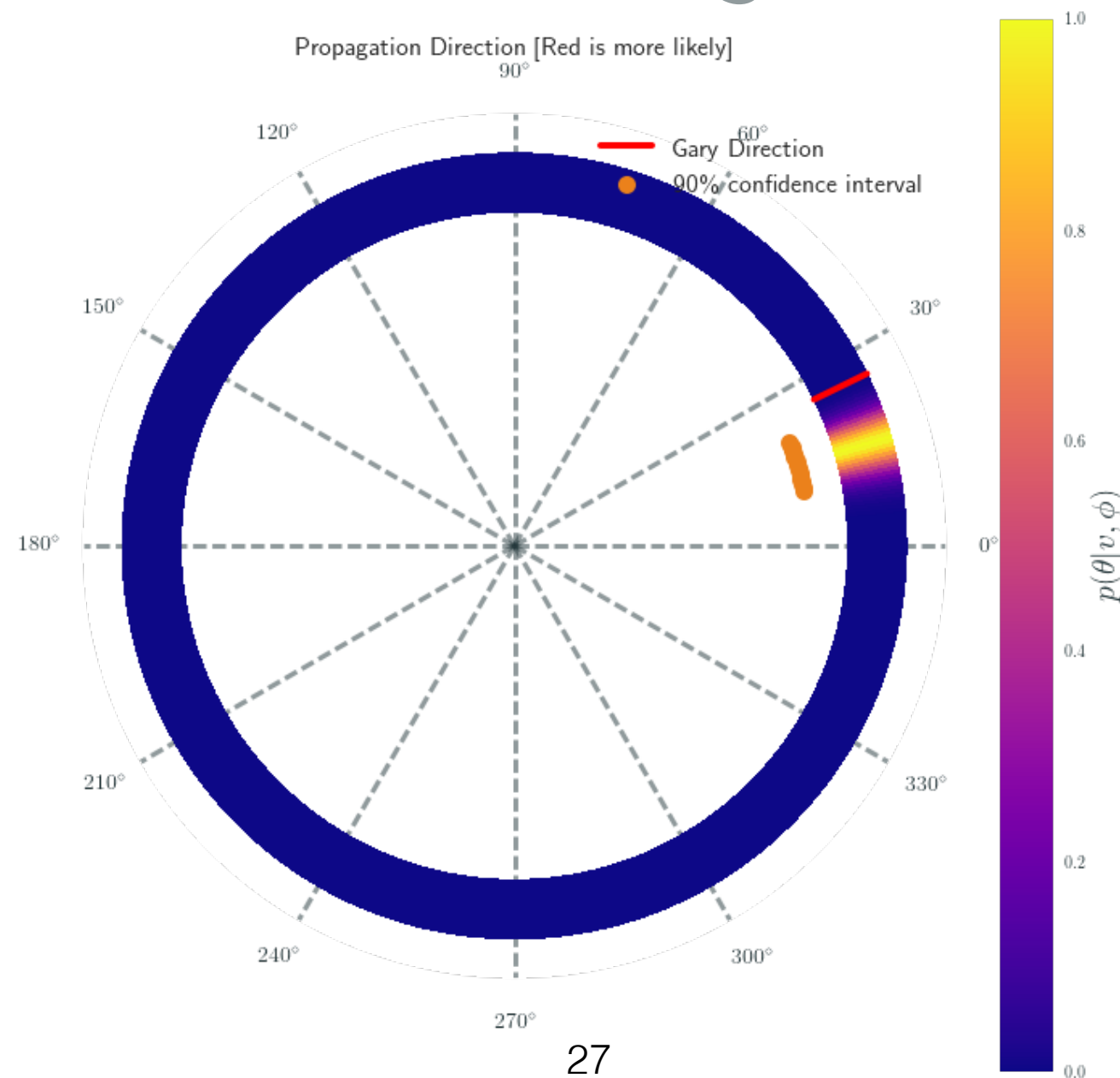
Terry Peak ~ 15 degrees





(Direction check on Gary's event)

- Timing analysis check of whether we're getting propagation direction or source direction using one of Gary's events



Next steps

- Calculate Newtonian noise from these maps
- Try to use R-wave eigenfunction so that we can include stations at several depths...although we won't have a measurement at this frequency
- Maybe try to measure R-wave eigenfunction using this as well?