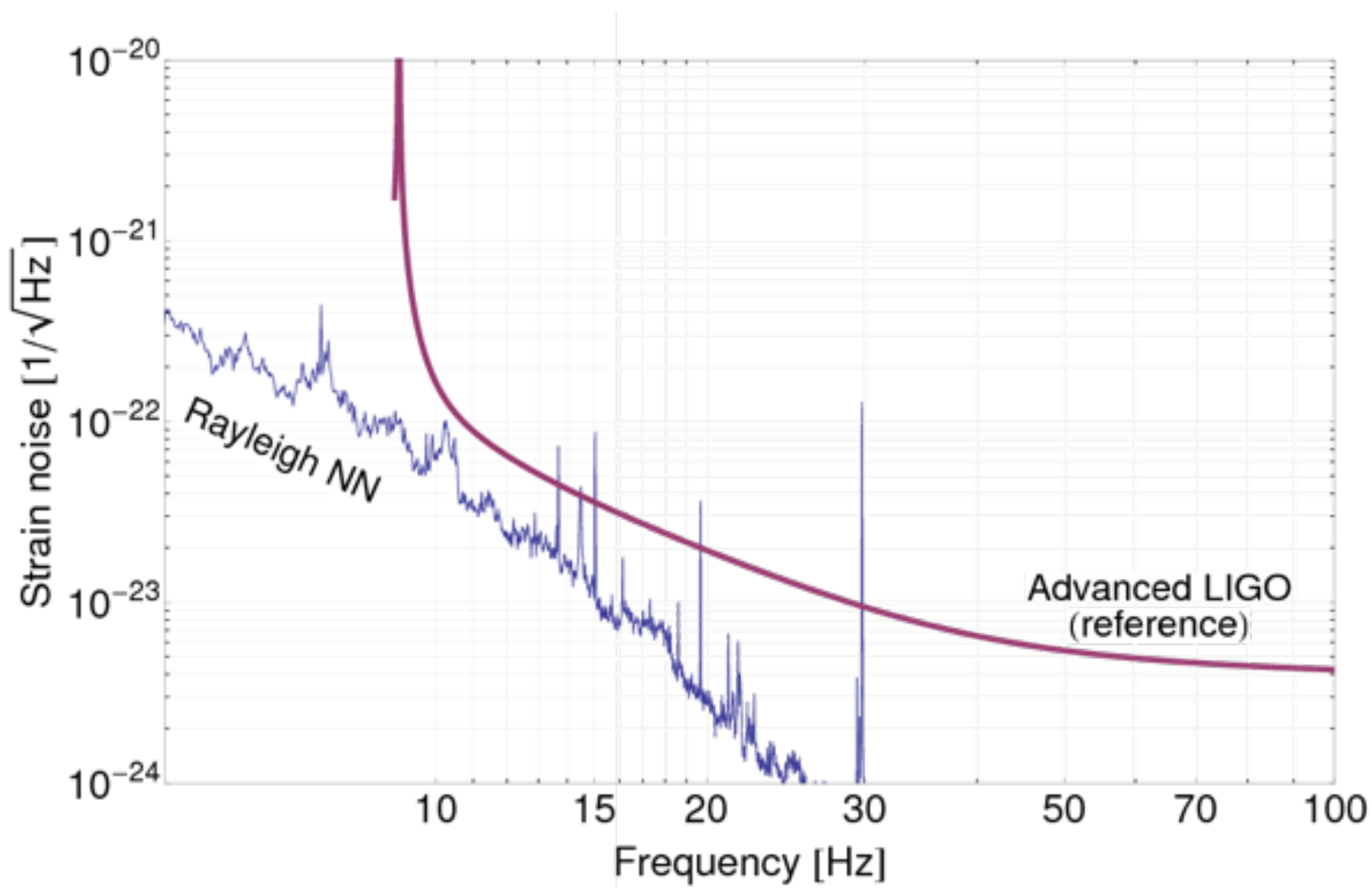
ANDREW MATAS, PAT MEYERS, VUK MANDIC

NEWTONIAN NOISE ESTIMATE

MOTIVATION

Newtonian noise (fluctuations in local gravitational field due to fluctuations in atmosphere, seismic,…) is a fundamental noise source for interferometric GW detectors at low frequencies



Cannot screen a gravitational field!

Major problem for stochastic searches as the signal ~f^(-3)

Coughlin, Michael *et al.* Class.Quant.Grav. 33 (2016) no.24, 244001

NEWTONIAN NOISE FROM SEISMIC FIELDS

Density fluctuations source Newtonian potential

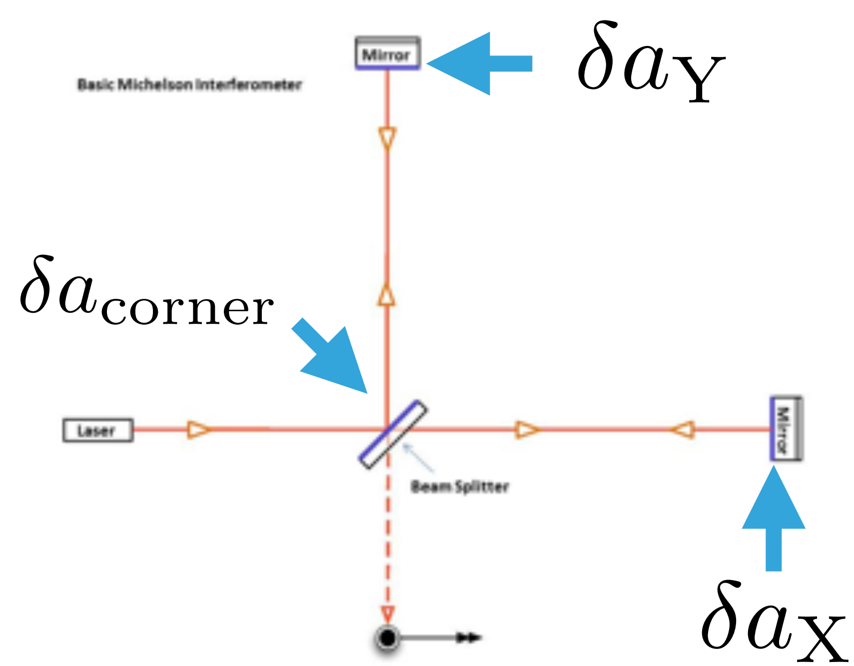
$$\nabla^2 \phi = 4\pi G \rho$$

Analytic solutions for potential exist assuming a homogenous half space (bi-exponential model)

Harms, Living Reviews in Relativity 18 (2015) no.1, 3

Gravitational acceleration computed from Newtonian potential

$$\delta \vec{a} = \nabla \phi$$



Differential acceleration leads to strain noise

$$h \sim \frac{\delta x}{L} \sim \frac{\delta a}{\omega^2 L}$$

METHOD

Take bi-exponential model coefficients as a function of frequency from Pat's PE results
f=[0.2,0.3,…,1,1.1] Hz

Convert coefficients to conventions used in Jan's review

Take measured seismic spectra for 3 stations:
a surface station (DEAD)
a mid-depth station (800),
and a deep station (A4850)

Assume seismic field is pure Rayleigh and isotropic

Assume density is constant (2.5 g/cm^3)

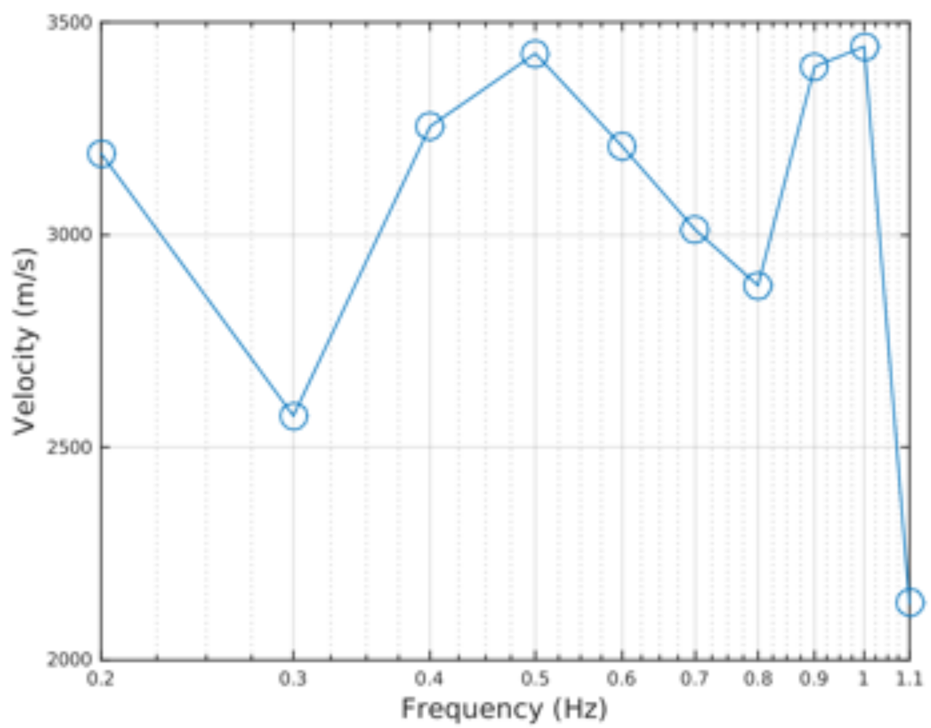
Compute total acceleration spectrum by the summing acceleration spectra from multiple propagation directions incoherently. For each direction and frequency, we can use the seismic spectrum and eigenfunctions to estimate the amplitude of NN acceleration. Then we sum over directions incoherently.

R-WAVE EIGENFUNCTION MEASUREMENTS

Bi-exponential model parameters measured at 10 different frequencies from 0.2-1.1 Hz using method described in [Pat's slides from July telecon]

$$r_z(z) = c_1 e^{-a_1 kz} + (1 - c_1)e^{-a_2 kz}$$
$$r_h(z) = c_3 e^{-a_3 kz} + (N_h - c_3)e^{-a_4 kz}$$

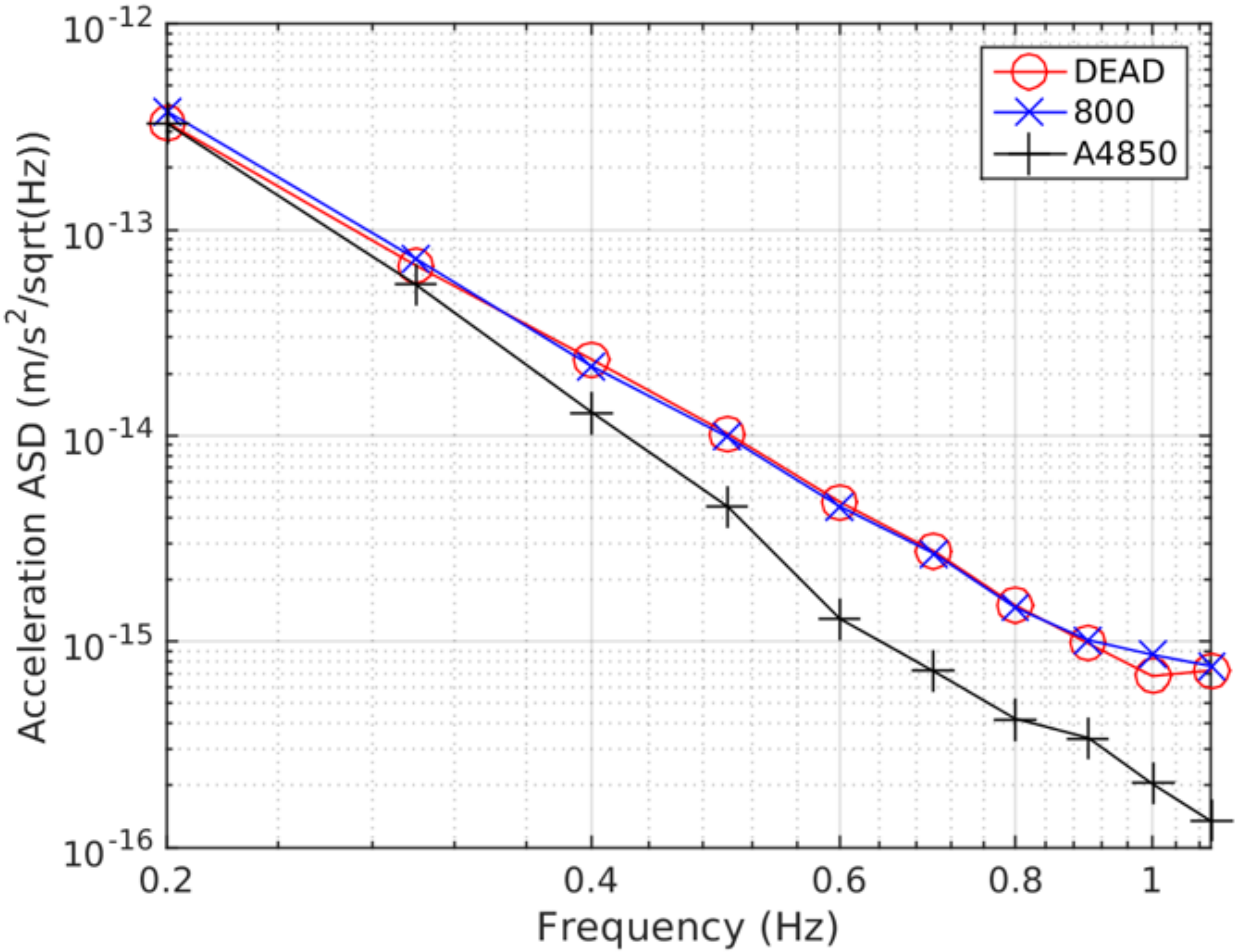
$$k = 2\pi f / v$$



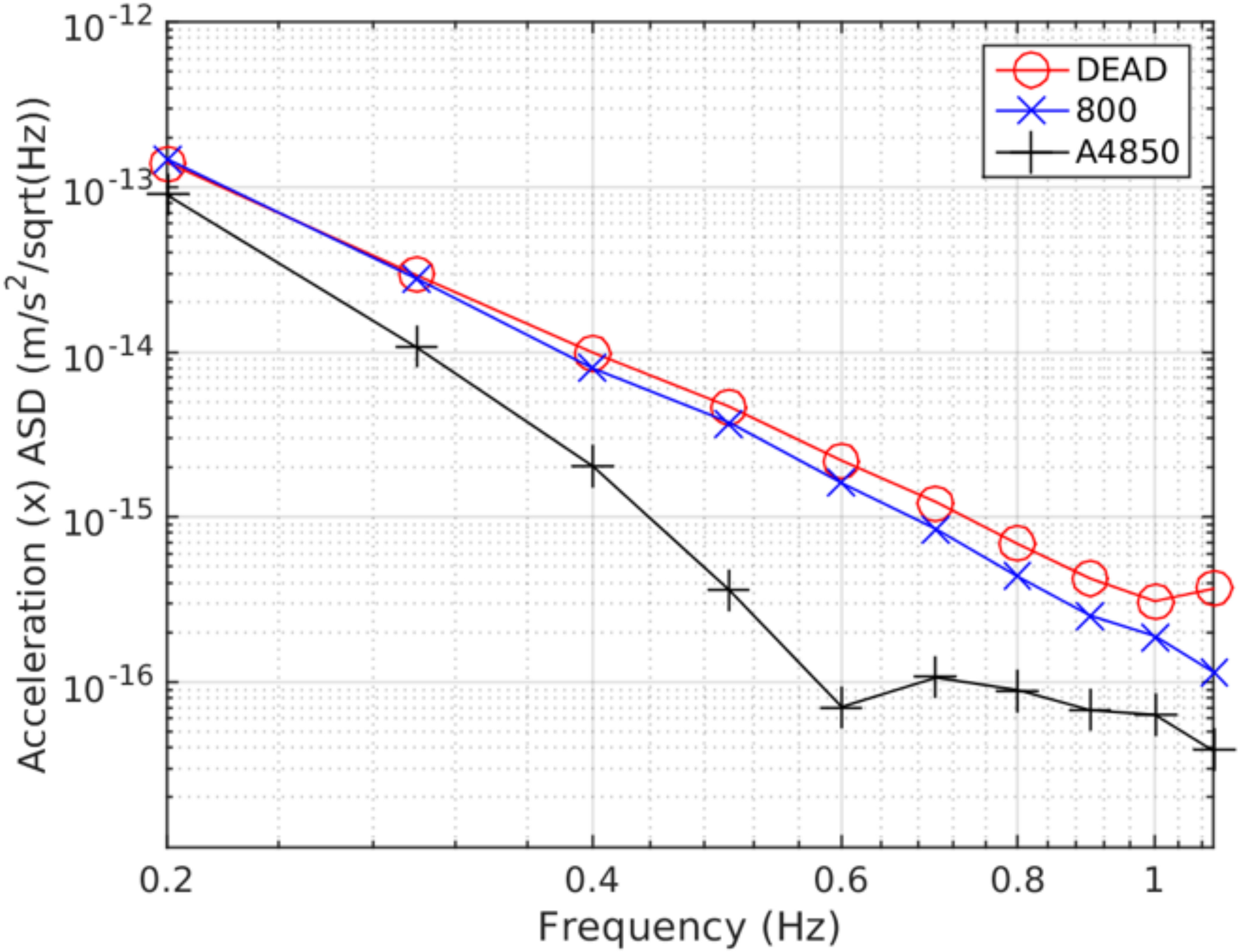
Best fit velocity as a function of frequency

Ranges from 2000-3500m/s

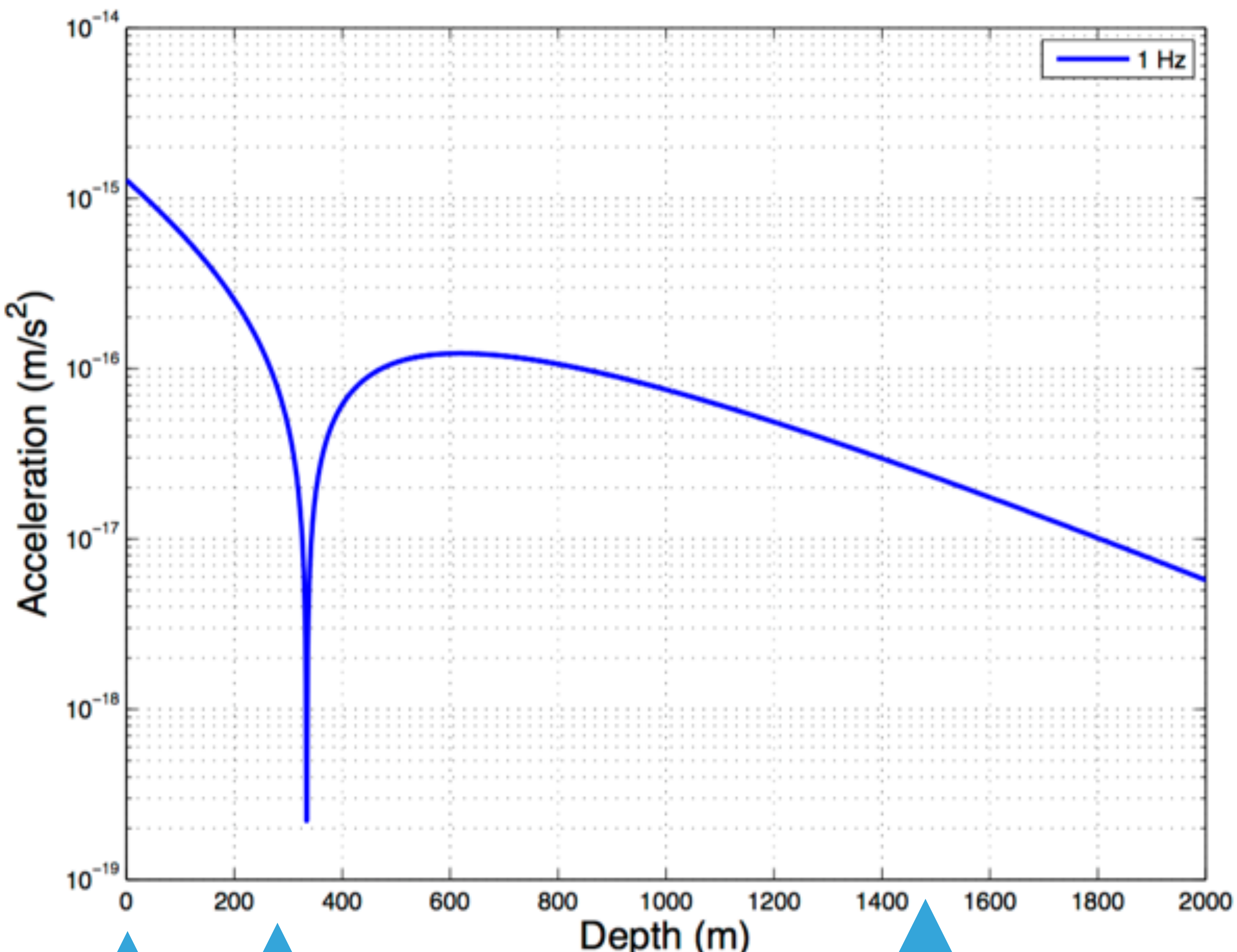
RMS ACCELERATION SPECTRUM



X-COMPONENT OF ACCELERATION SPECTRUM



COMPARE WITH DEPTH DEPENDENCE OF R WAVE



DEAD
depth=0

800
depth=244m

A4850
depth=1478m

This is an R-wave using fiducial parameters from Vuk's note

Acceleration is in range 1e-17-1e-15 at 1Hz, consistent with plot on previous page

CONVERTING TO STRAIN

Assuming acceleration is incoherent at corner and end stations

Relate strain to acceleration

$$h = \frac{\delta x - \delta y}{L} = \frac{\delta a_x - \delta a_y}{\omega^2 L}$$

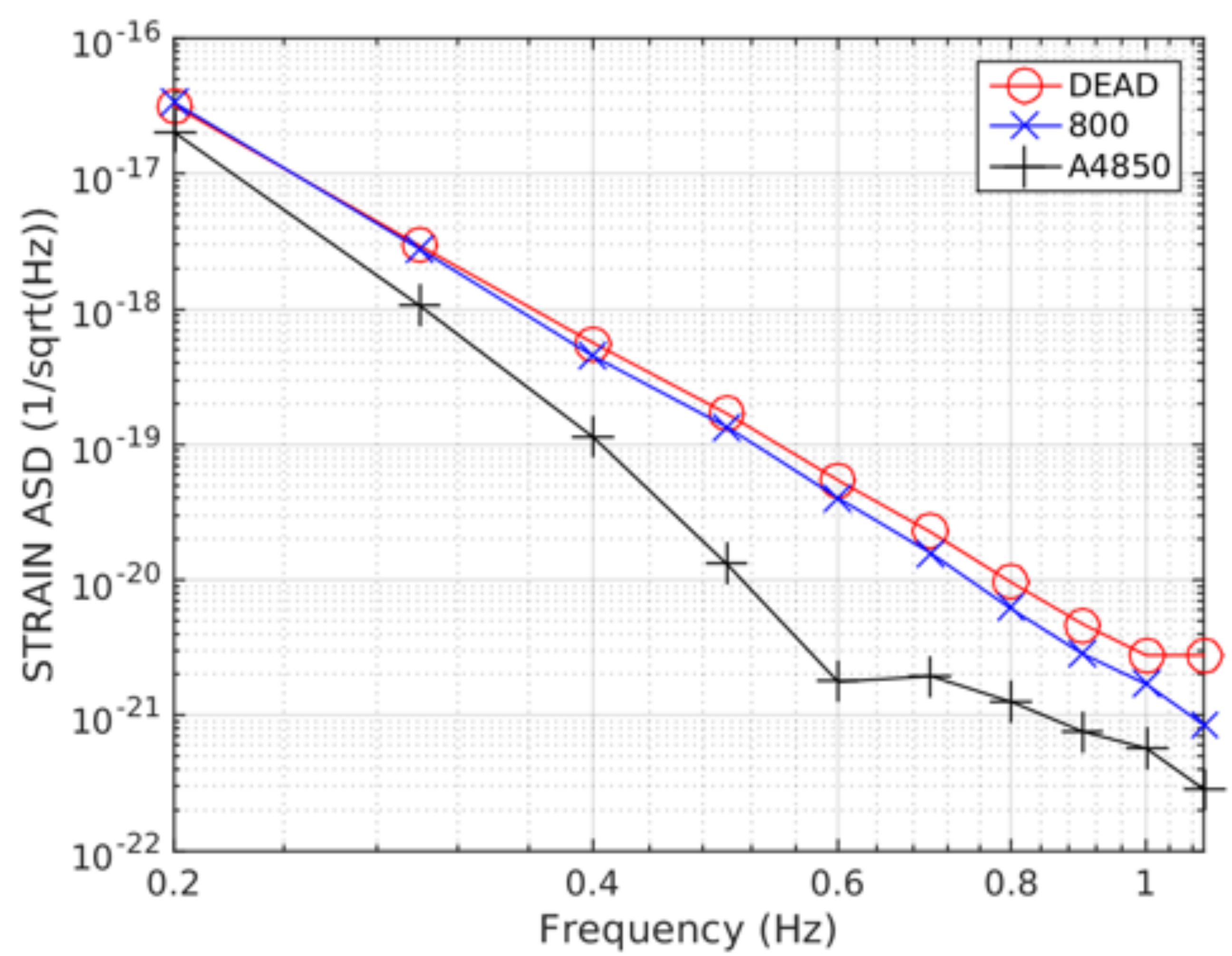
assume mass is free

Strain power is an incoherent sum

$$h_{rms} = \sqrt{2}\frac{\delta a_{x,rms}}{\omega^2 L}$$

Plugging in numbers....

$$L = 4\text{km}$$
$$\omega = 2\pi f$$

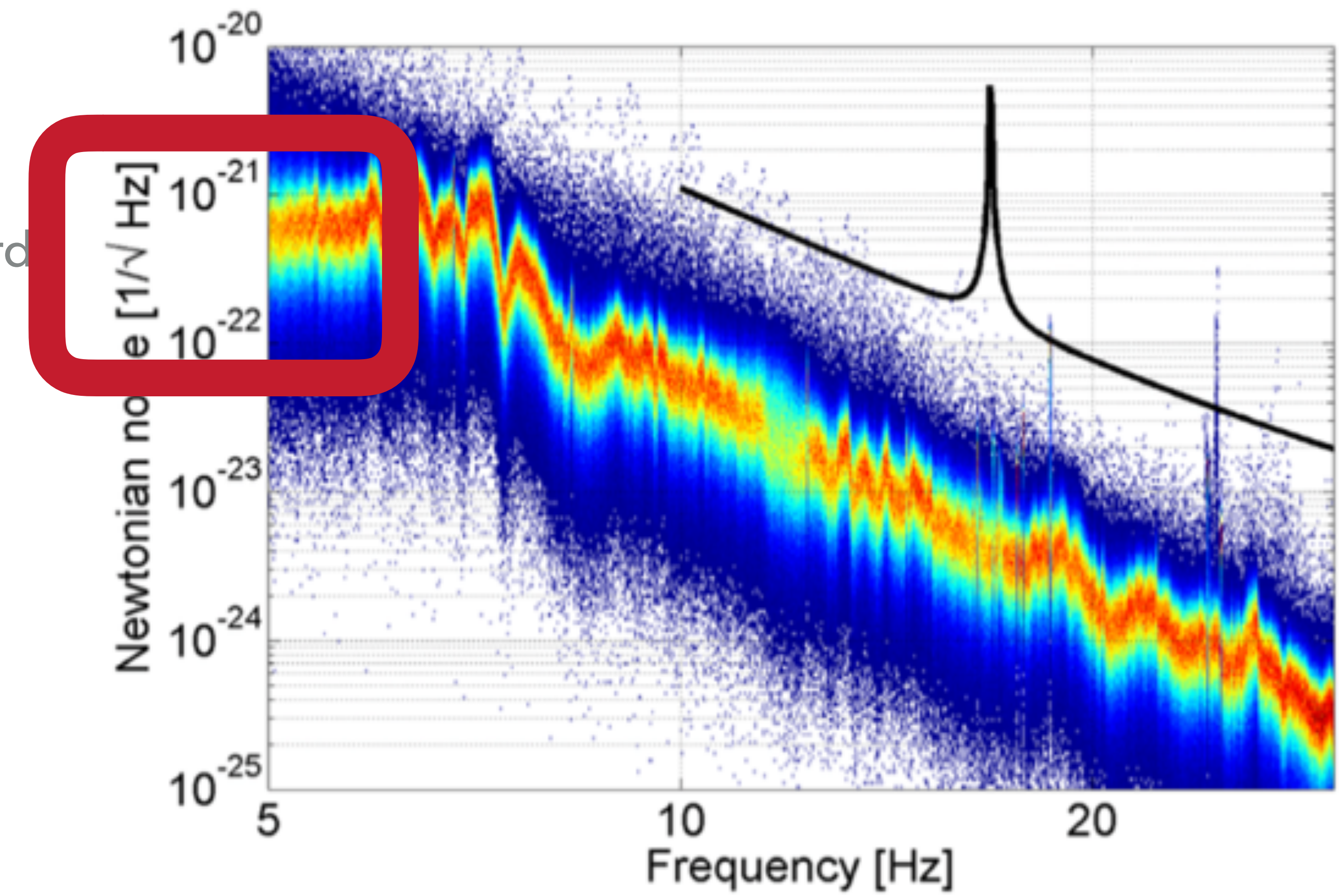


COMPARE WITH SPECTRUM AT HANFORD

Newtonian noise spectrum for Hanford

Note the min frequency on the plot is 5Hz

NN at 5Hz is slightly smaller (but in ballpark of) NN from previous result



WORK IN PROGRESS

- Study spectrum from an anisotropic background
- Account for detectors at multiple locations (ie, corner and end stations of an interferometer)
  - Allows us to compute correlation functions like: $$S(\delta a(L\hat{e}_x) - \delta a(0))$$

    which is really what we should use to estimate NN
  - Challenges so far: getting accurate PSD (seem to get a lot of spectral leakage, trying different windows hasn't helped so far)
- Go to higher frequencies
- Long term goal: use radiometer to characterize direction and polarization of seismic field, estimate Newtonian Noise

EXTRA SLIDES

MORE DETAILS…

To compute the spectrum from multiple directions…

1. Seismic spectrum at frequency f is $\xi_i(f) \qquad i = \{x, y, z\}$

2. Split circle up into N directions (uniformly). We suppose there are waves coming from each direction with random phases (so power from each direction adds incoherently)

3. The spectral density from each direction is $\xi_i(f)/\sqrt{N}$

4. Compute newtonian noise amplitude from each direction independently using GGN.m (GGN returns acceleration amplitude given seismic amplitude and direction of propagation)

5. Sum up the newtonian noise powers in quadrature for each direction

$$\delta a_i(f) = \sqrt{\sum_{directions} GGN\left(\frac{\xi_i(f)}{\sqrt{N}}, direction\right)^2}$$

ANALYTIC EXPRESSIONS FOR POTENTIAL UNDERGROUND

$$
\begin{aligned}
\vec{\xi}(\vec{r},t) &= \xi_k(\vec{r},t)\hat{k} + \xi_z(\vec{r},t)\hat{z} \\
\xi_k(\vec{r},t) &= i\left(H_1 e^{h_1 r_z} + H_2 e^{h_2 r_z}\right) e^{i(\vec{k}_\rho\cdot\vec{\rho}-\omega t)} \\
\xi_z(\vec{r},t) &= \left(V_1 e^{v_1 r_z} + V_2 e^{v_2 r_z}\right) e^{i(\vec{k}_\rho\cdot\vec{\rho}-\omega t)}
\end{aligned}
$$

$$
\begin{aligned}
\delta a_x &= -2\pi i G\rho_0 e^{i(\vec{k}_\rho\cdot\vec{\rho}-\omega t)} k_{\rho,x}(A_2 + A_1) \\
\delta a_y &= -2\pi i G\rho_0 e^{i(\vec{k}_\rho\cdot\vec{\rho}-\omega t)} k_{\rho,y}(A_2 + A_1) \\
\delta a_z &= -2\pi G\rho_0 e^{i(\vec{k}_\rho\cdot\vec{\rho}-\omega t)} (A_3 + k_\rho A_1) \\
A_1 &= e^{-hk_\rho}\left(\frac{H_1}{k_\rho - h_1} + \frac{H_2}{k_\rho - h_2} - \frac{V_1}{k_\rho - v_1} - \frac{V_2}{k_\rho - v_2}\right) \\
A_2 &= \frac{2H_1 k_\rho e^{-hh_1}}{h_1^2 - k_\rho^2} + \frac{2H_2 k_\rho e^{-hh_2}}{h_2^2 - k_\rho^2} + \frac{2V_1 v_1 e^{-hv_1}}{k_\rho^2 - v_1^2} + \frac{2V_2 v_2 e^{-hv_2}}{k_\rho^2 - v_2^2} \\
A_3 &= \frac{2H_1 h_1 k_\rho e^{-hh_1}}{h_1^2 - k_\rho^2} + \frac{2H_2 h_2 k_\rho e^{-hh_2}}{h_2^2 - k_\rho^2} + \frac{2V_1 v_1^2 e^{-hv_1}}{k_\rho^2 - v_1^2} + \frac{2V_2 v_2^2 e^{-hv_2}}{k_\rho^2 - v_2^2}
\end{aligned}
$$