Optical design of COrE+

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November 23, 2015

The optical designs for COrE+ were made by Darragh McCarthy and Neil Trappe at Maynooth University and Karl Young and Shaul Hanany at University of Minnesota. The details given here are what was done at Minesota. Two optical designs were produced; a 1.5m Gregorian-Dragone for COrE+ Light and a 1.5m Open (also called front-fed) Dragone for COrE+ Extended.

The optical perfomance of the two systems is summarized in Table 1.

Gregorian, COrE+Light	
Primary mirror projected diameter	1.5 m
Secondary mirror projected size	1.3 m × 1.08 m
Focal ratio ($f/\#$)	2.05
FOV, Focal plane diameter at Strehl = 0.8	
@ 60 GHz	10.9 deg × 11.4 deg, 61.6 cm × 64.2 cm
@ 160 GHz	7.15 deg × 6.44 deg, 39.3 cm × 36.2 cm
@ 600 GHz	3.00 deg × 1.33 deg, 16.1 cm × 7.9 cm
Open-Dragone, COrE+Extended	
Primary mirror projected diameter	1.5 m
Secondary mirror projected axes	1.74 m × 2.1 m
Tertiary mirror projected axes	1.5 m × 1.26 m
Focal ratio ($f/\#$)	2.05
Focal plane at Strehl = 0.8	
@ 60 GHz	13.5 deg × 13.1 deg, 69.7 cm × 67.5 cm
@ 160 GHz	8.42 deg × 7.54 deg, 43.4 cm × 38.8 cm
@ 600 GHz	3.06 deg × 3.03 deg, 15.8 cm × 15.6 cm

Table 1: Optical performance of both COrE+ designs. Focal plane size is given in field angle on the sky and in cm on the focal plane. Focal plane size is defined at each frequency by where Strehl = 0.8 for Gaussian illumination of the primary with 20 dB edge taper. The Strehl = 0.8 contours are not ellipses easily defined by x and y diameters, so the dimensions above are approximate. At the end of this memo are plots showing all Strehl = 0.8 contours.

1 COrE+ Light

The ray trace for COrE+ Light is shown in Figure 1 along with a diagram showing the geometry of a Gregorian-Dragone. A Gregorian-Dragone consists of an off-axis parabolic primary and an elliptical secondary. When the design conditions are met there is zero cross polarization at the center of the focal plane. The parameters given below follow the conventions shown in this diagram. The fundamental design of the light version was produced by Darragh. He found

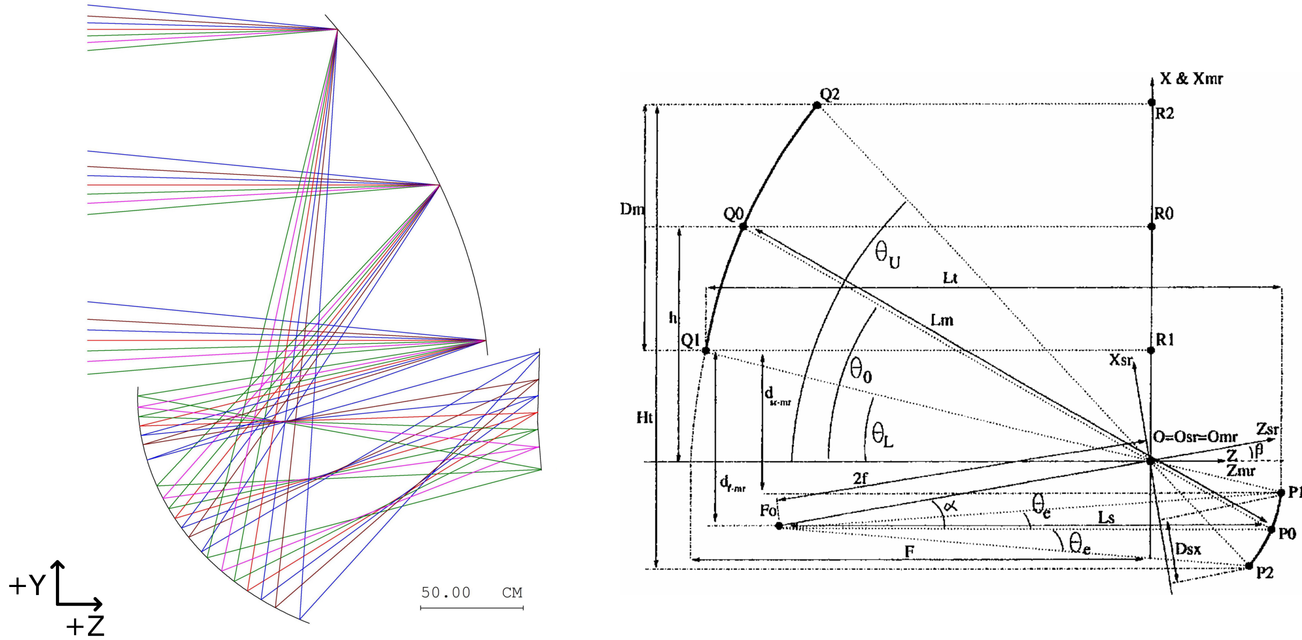

Figure 1: Left: Ray trace of the COrE+ Light design. Going from top to bottom rays are at field angles +5.5(blue), +3.0 (brown), +1.5 (blue), 0.0 (red), -1.5 (green), -3.0 (magenta), -5.0 (green) degrees in elevation. Right: Diagram showing angles and lengths that define a Gregorian-Dragone. Positive angles are in the counterclockwise direction. From Granet 2002 [1].

a design with good performance following the Gregorian-Dragone design criteria described in Granet 2002 [1]. Using Zemax, Darragh optimized the focal plane (FP) for this base version; allowing the FP angle, α , the FP distance from the secondary, and the FP shape to vary. The FP distance from the secondary is the distance along the Z_{sr} axis from the mirror surface to the focal plane. It is, $D_{FP-SR} = 2f + \frac{f}{e}(1-e)$. The FP shape is defined by radius of curvature, R_{FP}, and conic constant, k_{FP}. The parameters of Darragh's final design are below.

Karl took this design and optimized it in CodeV to increase the usable FOV. CodeV's 'Automatic Design' tool was used with the standard error function, transverse ray aberration. The optimizer used 25 field angles with maximum of ± 4 degrees in Az and El spaced evenly across the FP. Four frequencies; 130, 160, 220, 600 GHz were optimized simultaneously.

The optimizer was allowed to vary a total of 21 parameters. Both mirrors were changed to 12th order aspherics meaning they are described by 7 parameters; radius of curvature, R, conic constant, k, and 5 aspheric terms A thru E. The aspheric surface equation used by CodeV is,

$$z(r) = \frac{r^2/R}{1+\sqrt{1(1+k)r^2/R^2}} + Ar^4 + Br^6 + Cr^8 + Dr^{10} + Er^{12},$$

where r is the radial coordinate and z is the height relative to the center of the surface at $r = 0$. Setting both mirrors to aspherics creates 14 (7 per mirror) free parameters. The other 7 parameters varied are k_{FP}, R_{FP}, Y_{FP} (vertical offset of the focal plane). As well as β, α, $2f$, and F as defined in the diagram above, Figure 1.

These 21 parameters are not typically all varied simultaneously as the the optimizer tends to diverge in that case. The focal plane parameters are always allowed to vary. Then the mirrors are optimized as aspherics. Then the angles and inter-mirror distances are added. This optimization

Gregorian parameters	
β	5.6 degrees
D_M	1500 mm
F	890.15 mm
$2f$	1300 mm
e of secondary	0.554686
θ_e	14.2571 degrees
θ_0	-56.3558 degrees
Modified parameters	
α	-23.623 degrees
D_{FP-SR}	1830.729 mm
R_{FP}	3.508E-020 mm
k_{FP}	-30.141

Table 2: Initial design parameters for COrE+ Light from Darragh. There are further modified by optimization in CodeV.

tends to prefer a 'flatter' telescope, α and β going toward 0, but this puts the secondary in the ray path. To avoid this and limit optimization failures I added three constraints; $\beta > 3$ degrees, effective focal length = 311cm, and secondary to FP distance $D_{FP-SR} < 195$cm. These last two constraints limit the overall size of the telescope which tends to explode under CodeV optimization.

Physical apertures for the primary and secondary are found by tracing edge rays, and enlarging the mirrors until all rays are passed. We kept the edge of the secondary 8cm from the most extreme incoming ray and the edge of the primary 5cm from the nearest ray to limit diffraction. The projected optical size, perpendicular to the chief ray, of the primary is still 1.5m; its physical aperture has been increased to avoid diffraction at the mirror edges and is given in Table 3. The apertures given in Table 3 are defined in the coordinate system of each mirror. For the primary this means that X, Y, and Z (RH coordinates) are as shown in the lower left of the ray trace, Figure 1. For the secondary there has been a coordinate rotation, β, so $Z \rightarrow Z_{sr}$ where Z_{sr} is defined in the diagram. Y is rotated in the same manner so Y, as I give it, corresponds to X_{sr} in the diagram, Figure 1. Also the origin for the secondary aperture is on the Z_{sr} axis, so there is a ΔY shift of the entire aperture. The origin of the primary aperture is taken to be the impact point of the chief ray, equivalent to Q_0 in Figure 1.

The final parameters of the 1.5m Gregorian are in Table 3. If a parameter is not given then it is unchanged from its value in the true Gregorian system that Darragh designed with parameters in Table 2.

Mirrors							
	R	k	A	B	C	D	E
Primary	-210.41	-0.98127	-1.2550e-009	1.2013e-013	-7.0258e-018	1.8743e-022	-1.9303e-027
Secondary	98.353	-0.31929	5.4255e-010	2.3141e-013	2.5270e-016	-3.6456e-020	2.0265e-024
Other design parameters							
F	β	D_{FP-SR}	α	R_{FP}	k_{FB}	ΔY_{FP}	
-118.84	3.00	195.00	-11.702	3.508e-020	-30.141	-6.3795	

Apertures				
	X radius (cm)	Y radius (cm)	Y offset (cm)	
Primary	82.0	82.0	0.0	
Secondary	65.0	54.0	-54.5	

Table 3: Design parameters of COrE+ Light. All lengths in cm and angles in degrees. Negative signs reflect the coordinate definitions in CodeV. In other ray tracing programs the signs of various lengths may be different. Angles are positive in the counterclockwise direction. The apertures coordinates are described in the text. Important to note: these parameters are no longer related by the Gregorian conditions. You can't use them to solve for other values in the diagram. For that you have to use the initial design values in Table 2.

2 COrE+ Extended

COrE+ Extended is a 1.5m Open Dragone with a tertiary fold mirror added. The Open Dragone consists of an off-axis parabolic primary and a hyperbolic secondary. It has a larger FOV than the Gregorian-Dragone and also has zero cross polarization for the center feed. A ray trace of the final design is shown in Figure 2 along with a diagram of a typical Open Dragone.

The design process for COrE+ Extended was simpler than for COrE+ Light. A compact Open Dragone with large FOV was developed according to the Dragone conditions as outlined in Chang & Prata 1999 [2]. An equivalent method is in Granet 2001 [3]. No additional optimization in Code V using aspheric mirrors was done. The design was arrived at by adjusting the 5 defining parameters for an Open Dragone, producing a ray trace of each trial, and converging on a final design that was as compact as possible while maintaining a large FOV. The final parameters are given in Table 4.

Open Dragone parameters	
D	150 cm
θ_e	14.475 degrees
θ_0	-100.0 degrees
θ_p	-180.0 degrees
l	217.5 cm
Tertiary mirror parameters	
D_{SR_TR}	97.5 cm
θ_X	35 degrees
θ_Y	10 degrees

Table 4: Defining parameters for the COrE+ Extended Open Dragone. Definitions of lengths and angles are in Figure 2.

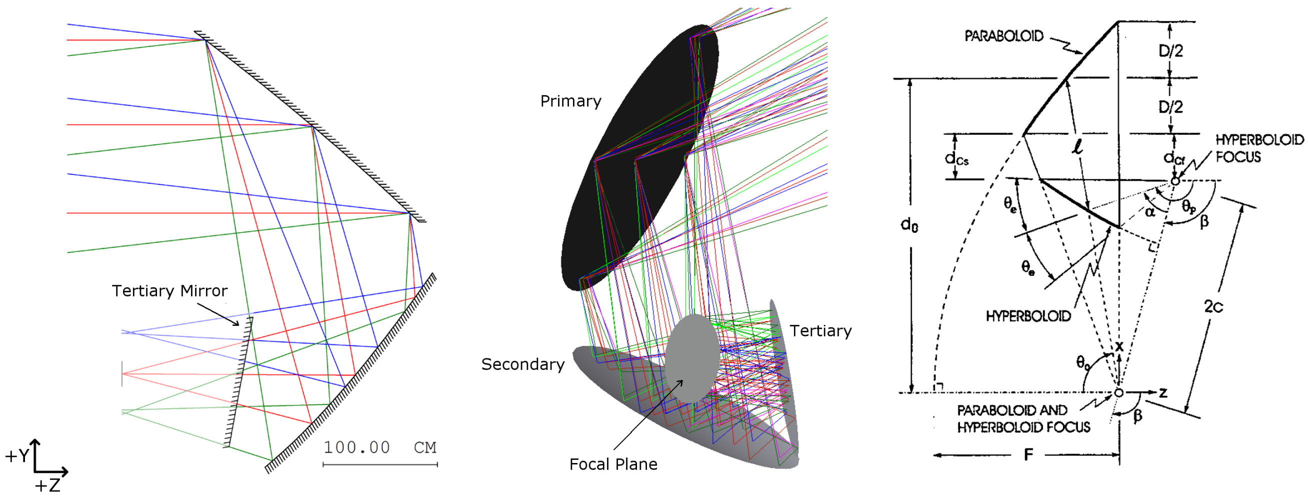

Figure 2: Left: Ray trace of the COrE+ Extended design. The rays are at 0 and ±6.5 degrees. The faded rays passing the tertiary mirror are actually being folded into the page, but are shown here to illustrate the Gregorian nature of the system. Center: A 3D view of the entire system showing how the tertiary folds the ray bundle. Right: Diagram showing angles and lengths that define an Open Dragone. Positive angles are in the clockwise direction. From Chang & Pratta [2]

A tertiary fold mirror was added to move the focal plane towards the base of the spacecraft and make the design more compact. Having the focal plane near the main body of the spacecraft reduces necessary support structure for the focal plane. A more compact overall design is needed to fit the telescope within the rocket faring. The tertiary can be a plain flat mirror, or a reflective polarization modulator. The position and angles defining this mirror are in Table 4. D_{SR-TR} is the distance from the secondary to the tertiary along the path of the chief ray. The angle θ_X refers to a rotation in the X-Z plane with the Z axis corresponding to 0. This is a rotation into the page; see left panel of Figure 2. θ_Y is a rotation in the Y-Z plane, also referenced from the Z axis with positive angles in the $+Y$ direction. The order of these rotation is θ_Y, θ_X.

To summarize, the tertiary mirror is created perpendicular to the chief ray going from the secondary to the focal plane at a distance $D_{SR_TR} = 97.5$cm. Then it is rotated upward by 10 degrees in the plane of the page. Then it is rotated perpendicular to the page by 35 degrees moving the focal plane into the page. A 3D view of the system is shown in the center of Figure 2.

Apertures that do not clip any rays are then found in CodeV and listed in Table 5. The values are for each mirror's projected aperture in the coordinates of that surface.

For the primary this coordinate system is shown in the lower left of Figure 2 with the origin located at the focal point of the parabola that defines the mirror. This means that the Y offset of the aperture is equivalent to d_0 in the diagram in the right panel of Figure 2 and the X and Y projected radii equal $D/2$.

The coordinates of the secondary have been rotated by β. In these new coordinates the Z axis runs from the paraboloid focus to the hyperboloid focus. The Y axis is in the plane of the page and negative towards the primary, and the X axis is perpendicular to the page.

The coordinates of the tertiary have been rotated three times. First by α, then by θ_Y, and finally by θ_X. This makes the new coordinates difficult to visualize, but the Z-axis is perpendicular to the tertiary mirror and positive away from the secondary. Referring to the left panel of Figure 2 it is angled into the page. The Y axis is in the plane of the page and negative toward the primary. The X axis is angled out of the page and positive toward the focal plane. Using the center panel of Figure 2 for a different description; the Z-axis is positive to the right, the X-axis comes out of the page, and the Y-axis points downward. The origin of this coordinate system is the impact point of the chief ray.

Apertures					
	X radius	Y radius	X offset	Y offset	Rotation angle
Primary	82.5 cm	82.5 cm	0.0 cm	703.8 cm	0.0 degrees
Secondary	87.0 cm	105.0 cm	0.0 cm	-172.5 cm	0.0 degrees
Tertiary	75.0 cm	63.0 cm	-10.5 cm	7.5 cm	-30.0 degrees

Table 5: Aperture specifications for COrE+ Extended. X and Y coordinates are defined in the text for each surface. Rotation angle is the angle the elliptical aperture is rotated after it is drawn in the XY plane. A positive rotation angle rotates the X axis of the ellipse towards the Y axis of the coordinate system.

It's worth noting that the design for COrE+ Extended is simply an Open Dragone. It has not been optimized in any way in CodeV; with aspheric mirrors or by adjusting tilt angles. This suggests that some increases in the focal plane size and image quality could be found by an optimization process similar to that used for COrE+ Light. Also the focal plane for COrE+ Extended is currently flat and nearly telecentric, within 3 degrees, simplifying the detector design.

The COrE+ Extended design is physically larger than the Light version and is on the edge of fitting into the 3.8 m Soyuz fairing that was being considered in 2014. The optics themselves fit easily, but it was never fully determined if the V-grooves used for radiative cooling would all fit. There were conflicts at the 25 cm level so this design should easily fit in the 4.5 m fairing being considered in 2015.

References

[1] Granet, Chistopher. "Designing Classical Offset Cassegrain or Gregorian Dual-Reflector Anetnnas from Combinations of Prescribed Geometric Parameters." *IEEE Antennas and Propagation Magazine*, Vol. 44 No. 3, June 2002.

[2] Chang, S. and Prata, A., Jr. "A design procedure for classical offset Dragonian antennas with circular apertures." *Antennas and Propagation Society International Symposium, 1999. IEEE*, Vol. 2, pp.1140-1143 vol.2, July 1999.

[3] Granet, Chistopher. "Designing Classical Dragonian Offset Dual-Reflector Antennas from Combinations of Prescribed Geometric Parameters." *IEEE Antennas and Propagation Magazine*, Vol. 43 No. 6, Dec. 2001.

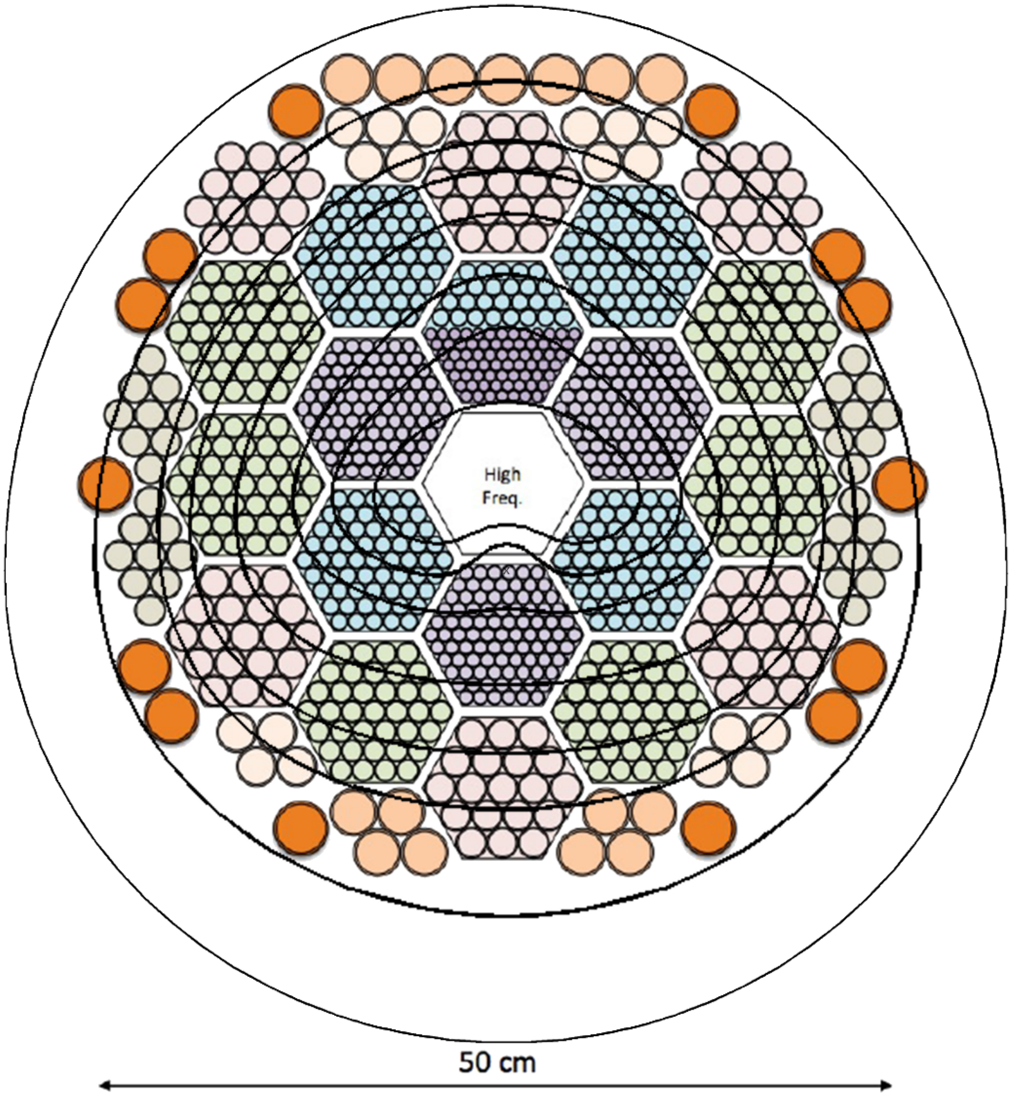

Figure 3: Sketch of the focal plane of COrE+ Light. Contours are Strehl= 0.8 for 60, 90, 130, 160, 220, 340, 450, and 600 GHz. The small colored circles correspond to feedhorns at these frequencies.

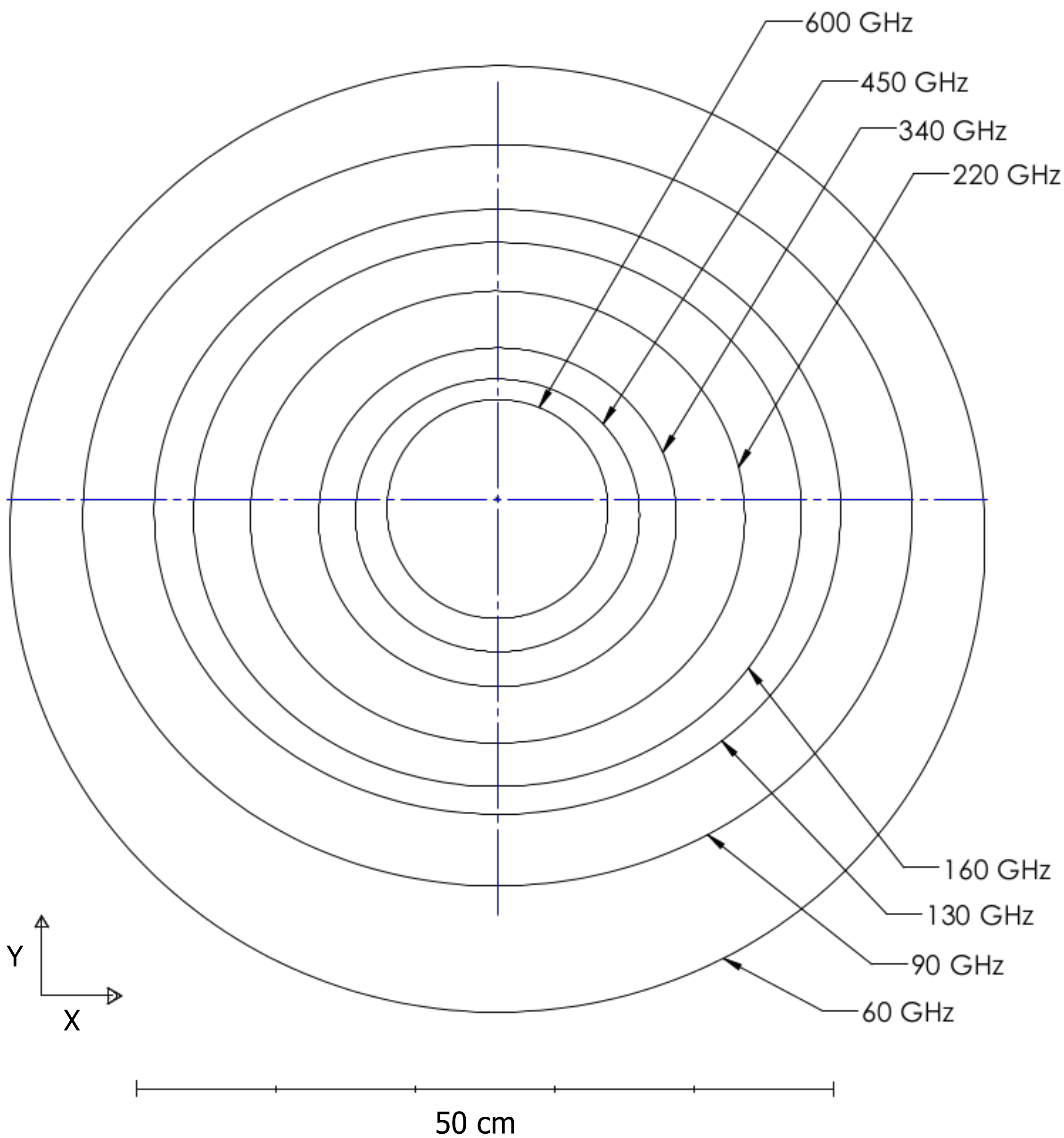

Figure 4: Sketch of the focal plane of COrE+ Extended. Contours are Strehl= 0.8 at each frequency.