University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

WINTER 2009 – PART I

Thursday, January 15, 2009 – 9:00 am to 12:00 noon

Part 1 of this exam consists of 12 problems of equal weight. You will be graded on your 10 best efforts.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, "page 1", "page 2", etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Some physical constants and quantities in SI and natural units

Layout by Krister Henriksson, krister.henriksson@helsinki.fi (rev. April 7 2003)

Physical constants in SI units

| Constant	Symbol	Value
Pi	π	3.141592654
Universal gravitational constant	G	$6.67259 \cdot 10^{-11}$ Nm^2/kg^2
Standard acceleration of gravity	g_n	9.80665 m/s^2
Standard temperature	T_0	273.15 K
Standard pressure	p_0	1 atm = 760 Torr = 101325 Pa = 1.01325 bar
Molar volume at STP	V_m	$22.42420 \cdot 10^{-3}$ m^3/mol
Avogadro's constant	N_{A}	$6.0221367 \cdot 10^{23}$ 1/mol
Gas constant	R	8.314510 J/(mol·K)
Boltzmann's constant, R/N_{A}	k_B	$1.380658 \cdot 10^{-23}$ J/K
		$= 8.617385 \cdot 10^{-5}$ eV/K
Speed of light in vacuum	c	$2.99792458 \cdot 10^{8}$ m/s
Fundamental unit of electric charge	e	$1.60217733 \cdot 10^{-19}$ C
Mass of electron	m_e	$9.1093897 \cdot 10^{-31}$ kg
		$= 0.51099906$ MeV/c^2
Mass of proton	m_p	$1.6726231 \cdot 10^{-27}$ kg
		$= 938.27231$ MeV/c^2
Mass of neutron	m_n	$1.6749286 \cdot 10^{-27}$ kg
		$= 939.56563$ MeV/c^2
Universal atomic mass unit	u	$1.6605402 \cdot 10^{-27}$ kg
		$= 931.49435$ MeV/c^2
Permeability of vacuum	μ_0	$4\pi \cdot 10^{-7}$ N/A^2
		$= 1.256637061 \cdot 10^{-6}$ N/A^2
Permittivity of vacuum	$\varepsilon_0 = 1/(\mu_0 c^2)$	$8.85418782 \cdot 10^{-12}$ F/m
Coefficient in Coulomb's law	$1/(4\pi\varepsilon_0)$	$8.98755 \cdot 10^{9}$ Nm^2/C^2
Planck's constant	h	$6.6260755 \cdot 10^{-34}$ Js
		$= 4.1356692 \cdot 10^{-15}$ eVs
	$\hbar = h/(2\pi)$	$1.05457266 \cdot 10^{-34}$ Js
		$= 6.5821220 \cdot 10^{-16}$ eVs
Electromagnetic coupling constant	$\alpha = e^2/(4\pi\varepsilon_0\hbar c)$	$7.29735308 \cdot 10^{-3}$
(fine structure constant)		$= 1/137.0359895$
Rydberg's constant	$R_\infty = m_e c \alpha^2/(2h)$	$1.0973731534 \cdot 10^{7}$ 1/m
Stefan & Boltzmann's constant	$\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$	$5.67051 \cdot 10^{-8}$ W/(m^2·K^4)
		$= 3.53925 \cdot 10^{11}$ eV/(m^2·K^4)

1. Consider a simple model for the thermal properties of the planets in which the planets act like black bodies. If, in such a model, the surface temperature of the earth is 278 K, what is the predicted surface temperature of Venus? Venus is about 0.72 AU from the sun, where 1 AU is the distance from the earth to the sun. (Ignore effects of the atmospheres of the planets.)

2. A straight cylindrical wire of radius a carries a current I and has resistance per unit length R/L. The wire is embedded in a cylindrical medium of thermal conductivity κ. If the embedding medium is in contact with a thermal bath at temperature T_b at a distance b from the center of the wire, what is the temperature at the surface of the wire?

3. A long straight wire is bent into a circular loop of radius R near its midpoint and this loop is twisted so that its plane is perpendicular to the wire. The wire carries current I. What is the magnetic field at the *center* of the loop in the wire?
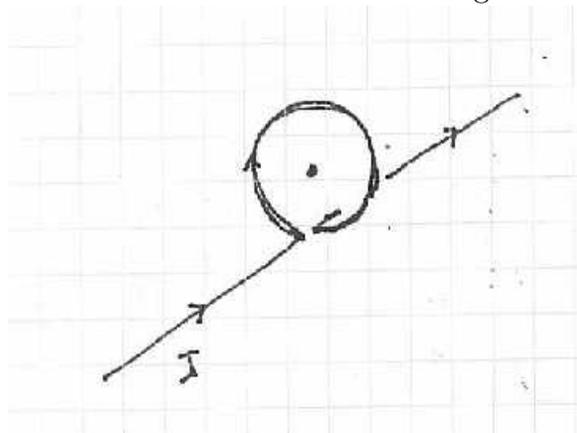
4. Consider a ball rolling down an inclined plane with grade angle θ. If the coefficient of friction between the ball and the slope is μ, find the angle at which the surface of the ball starts to slip with respect to the surface.

5. A ball bouncing vertically off a smooth horizontal surface has a coefficient of restitution e_r, meaning that immediately after each bounce its speed is a fraction e_r of its speed immediately before that bounce. Find the time which elapses between the moment when it is dropped and when it finally comes to rest(no velocity or acceleration) .

6. In a recent report in Science, a planet was observed moving around a star in an orbit of radius 115 AU with an estimated speed, relative to the star, of 5.5 km/sec. If the orbit is circular, what is the mass of the star, in units of our sun's mass? 1AU=1.48 x 10^8 km.

7. A thin uniform circular ring of radius r, carrying uniformly distributed charge q and mass m, is placed in the xy-plane. The ring rotates with angular speed ω about its symmetry axis, parallel to the z-axis.

a. Find the ratio of its magnetic dipole moment to its angular momentum. This is called the gyromagnetic ratio g.

b. What is g for a uniform spinning charged sphere of radius R, charge Q, and mass M? Hint: Use the result from part a.

8. The 'contact term' in the hyperfine interaction between the magnetic moments of the electron and the nucleus of a hydrogen atom takes the form $H' = A\vec{I}\cdot\vec{S}$ where $\vec{I}$ is the angular momentum operator of the nucleus $\vec{S}$ is the spin angular momentum operator of the electron and A is a constant. Treating this as a perturbation, find the first order corrrections to the ground state energy of the hydrogen atom which it implies. Give the energies of the perturbed states and their degeneracies and describe their angular momentum properties.

9. An observer sees a supersonic airplane passing over a building 20 kilometers away and moving perpendicular to a line between himself and the building. A moment later the aircraft, still moving in a straight line, passes over another building which the observer knows to be 10 kilometers away from the first building and, at this moment, he hears the sound of the airplane *for the first time*. Find the ratio of the speed of the airplane to the speed of sound in the air.

10. Evidence has been reported of cosmic ray particles incident on the atmosphere of the earth having energies of 10^{20} eV.

a. If such particles were protons, what would their velocities be? (Give the difference between the speed of light and the speed of the particles.)

b. If the particles originated on the other side of our galaxy (about 10^4 light years away) how much time would have elapsed, in the rest frame of the particle, during its trip to the earth, assuming again that the particles are protons?

11. Write an expression for the vibrational specific heat per molecule of a gas of diatomic molecules modeled (quantum mechanically) as harmonic oscillators. Let the harmonic force be described by a spring constant K and the masses of the atoms in the molecules be M and m. Show that your result reduces to the expected equipartition result in the appropriate high temperature limit, which should be defined.

12. A permanent magnet of cylindrical shape is dropped through a vertically oriented cylindrical coil. The inside diameter of the coil is slightly larger than the diameter of the magnet and the magnet falls smoothly through. The coil is in series with an ammeter and a resistance R. Assume that the length of the cylindrical magnet is much less than the length of the coil and neglect any effect of eddy currents induced in the magnet.

a. Make a qualitative sketch of the emf $\mathcal{E}(t)$ across the coil as a function of time as the magnet passes into, through and out the bottom end of the coil. Make sketches indicating the direction of the induced currents assuming that the magnets north pole is facing down as it is dropped, as shown below.

b. Taking account of the self inductance L of the coil, write an integral which would permit the current in the circuit to be calculated as a function of time, given $\mathcal{E}(t)$.
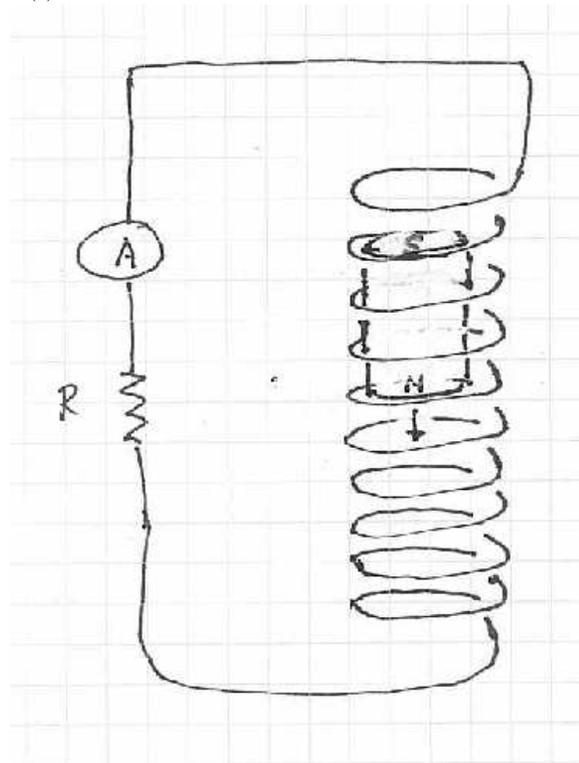
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GRADUATE WRITTEN EXAMINATION

WINTER 2009 – PART 2

Friday, January 16, 2009 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 6 problems of equal weight. You will be graded on your 5 best efforts.

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1. Consider two identical particles, having spin $1/2$, in a one dimensional box of length a. They are nonrelativistic and have mass m. In the first part of the problem they are not interacting.

a. Write down the three lowest energy eigenstates of the system which are eigenstates of total spin, including both the spatial and spin parts of the wave functions. Write the energy of each state and make an energy diagram indicating the energies and degeneracies.

b. Now introduce a delta function interaction

$$V(x_1 - x_2) = \lambda \delta(x_1 - x_2)$$

Find the resulting shifts in the energies of the states found in a) in first order perturbation theory in λ.

2. An extremely long uniform string of linear mass density μ under tension τ has a small bead of mass m attached to it at $x = 0$ (the center of the string). A transverse harmonic wave of form $ARe(exp(ikx - i\omega t))$ travels from one end of the string toward the bead where part of it is reflected and part is transmitted. ($\omega = \sqrt{\tau/\mu}k$ but you will not need to use this in this problem.) Ignore the effects of the ends of the string far to the left and right of the origin.

a. What are the boundary conditions at the bead which constrain the amplitudes of the incident, reflected and transmitted waves?

b. Find explicit functional forms for the steady state reflected and transmitted waves in terms of the parameters of the incoming wave.

3. The neutron has an average lifetime of 887s.

a. The neutron decays into a proton and two leptons. What are the leptons?

b. If neutrons from a cosmic ray interaction 2 light years from earth reach the earth with probability of e^{-1}, what must their energy be (in a coordinate system at rest with respect to the earth)?

c. If the leptons are emitted in directions perpendicular to direction of the neutron flight path in the center of mass frame, what is the angle which the daughter anti-neutrino makes with the flight path of the neutron in a frame of reference fixed with respect to the earth? (You may assume that the proton is at rest in the center of mass frame.)

d. What is the energy (in the earth's frame) of the anti-neutrinos emitted at this angle?

4. A spherical particle of uniform mass density ρ and radius a falls starting with zero velocity in an isothermal atmosphere from height L above the surface of a planet. L is much less than the curvature of the planet surface. The acceleration due to gravity at the surface is g. The atmospheric mass density at the planet surface is ρ_0 and the average molecular mass in the atmosphere is m. Taking bouyancy forces but not effects of atmospheric friction into account, what is the vertical velocity of the particle just before it hits the surface? Simplify the expression in the limits $L >> k_B T/mg$ and $L << k_B T/mg$. Draw a qualitative graph of the final velocity as a function of L including the behavior in both limits.

5. Two spherical cavities, of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed, call these charges q_a and q_b and let the distance between them be r_{ab}.

a. Find the surface charge densities σ_a, σ_b and and σ_R on the surfaces of the two cavities and the outer surface of the conducting sphere.

b. What is the field outside the conductor?

c. What is the force on q_a and q_b?
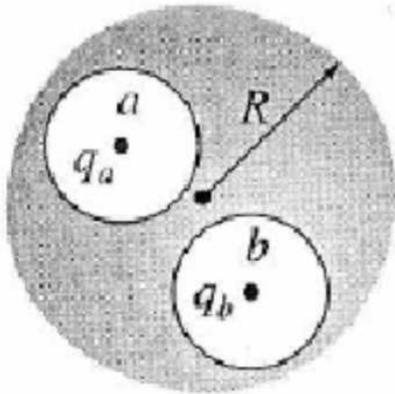
6. Suppose, as was thought at one time, that neutrinos, which are spin $1/2$ fermions, are massless, with energy momentum relation $\epsilon(\vec{p}) = |\vec{p}|c$ where c is the speed of light. (Though neutrinos have spin $1/2$, only one of the two possible spin values is observed in nature.) Neutrinos have extremely weak interactions with each other and everything else so we will neglect interactions in this model, in which neutrinos are confined in a box of volume V at absolute temperature T.

a. Find the number density of neutrinos as a function of the parameters given T, V and $\hbar, c$, and k_B. (There will be a dimensionless integral which you don't have to evaluate.)

b. Find the energy, the entropy and the Helmholtz free energy of the neutrinos in terms of the same variables. Use the Helmholtz free energy to calculate the pressure of the neutrinos.

c. Estimate the order of magnitude of your results for the density and pressure in SI units, assuming that the temperature is about 3K, as in the universe.