University of Minnesota
School of Physics and Astronomy

GRADUATE WRITTEN EXAMINATION

Spring 2016 – PART I

Thursday, January 14th, 2016 – 9:00 am to 1:00 pm

Part 1 of this exam consists of 10 problems of equal weight. You will be graded on all 10 problems.

This is a closed-book examination. You may use a calculator. A list of some physical constants and properties that you may require is included. Please take a moment to review its contents before starting the examination.

Please put your assigned **CODE NUMBER (not your name or student ID)** in the **UPPER RIGHT-HAND CORNER** of each piece of paper that you submit, along with the relevant problem number in the **UPPER LEFT-HAND CORNER.**

BEGIN EACH PROBLEM ON A FRESH SHEET OF PAPER, so that no sheet contains work for more than one problem.

USE ONLY ONE SIDE of the paper; if you require more than one sheet, be sure to indicate, “page 1”, “page 2”, etc., under the problem number already entered on the sheet.

Once completed, all your work should be put in the manila envelope provided, **IN ORDER** of the problem numbers.

Constants	Symbols	values
Speed of light in vacuum	c	3.00×10^{8} m/s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Electron rest mass energy	$m_e c^2$	0.511 MeV
Permeability constant	μ_o	1.26×10^{-6} H/m
Permeability constant/4π	$\mu_o/4\pi$	10^{-7} H/m
Proton rest mass	m_p	1.67×10^{-27} kg
Proton rest mass energy	$m_p c^2$	938 MeV
Neutron rest mass	m_n	1.68×10^{-27} kg
Neutron rest mass energy	$m_n c^2$	940 MeV
Planck constant	h	6.63×10^{-34} J–s
Gravitational constant	G	6.67×10^{-11} m^3/s^2–kg
Molar gas constant	R	8.31 J/mol–K
Avogadro constant	N_A	6.02×10^{23} /mol
Boltzmann constant	k_B	1.38×10^{-23} J/K
Molar volume of ideal gas at STP	V_m	2.24×10^{-2} m^3/mol
Earth radius	R_E	6.38×10^{6} m
Earth's mass	M_E	5.98×10^{24} kg
Earth-Sun distance	1 AU	1.50×10^{11} m
Stirling's Approximation:	ln(N!) = Nln(N) – N + (small corrections)	

Problem 1
Two blocks of equal mass m are connected by a flexible cord of length L. One block is placed on a smooth horizontal table; the other block hangs over the edge. The cord is heavy, having a mass m'.
Find the acceleration of the block as a function of its position. (Assume that the cord is not stretchable).

Problem 2
Consider an ideal monatomic gas in thermodynamic equilibrium. Suppose it goes through a four step cycle: (1) Isothermal compression from volume $2V_0$ to volume V_0, where its pressure is P_0; (2) Adiabatic compression with $TV^{\gamma-1} = constant$ to volume $V_0/2$; (3) Isothermal expansion back to volume V_0; (4) Then adiabatic expansion to the initial volume $2V_0$ to complete the cycle.
How much work is done during the cycle by the gas on its environment? Give your answer as a function of P_0, V_0 and γ.

Problem 3
At low temperatures, an insulating ferromagnet is described as a gas of non-interacting bosons called magnons. These magnons are related to fluctuations of the magnetic moments, which propagate along the ferromagnet. The energy of a magnon is given by $E(k) = ck^2$, where c is a constant and k is the wave-vector of the magnon. The number of magnons inside the ferromagnet is not conserved.

a. What is the value of the chemical potential at equilibrium?
b. At low temperatures, the specific heat of the ferromagnet is proportional to a power of temperature, $C(T) \propto T^{\alpha}$. Calculate α for a three-dimensional ferromagnet.

Problem 4
For a large water Cherenkov detector, you need to remove all contaminants present at concentration C from the water. To estimate the amount of energy that this will take, you calculate the least energy necessary to decontaminate 99.9% of the water, leaving all the contamination in the remaining 0.1%, which can be thrown away. You know the amount of contamination and that the process takes place at room temperature.

a. Calculate the entropy as function of the number of contaminates first (considering a unit cell of volume z and no exclusion of the volume).
b. Determine the change in the entropy after the decontamination took place and the required energy.

Problem 5
A pinhole camera has no lens, just only a small hole to form an image on the screen. If the hole is too large, the image is fuzzy because of the finite hole size. If it is too small, the image is fuzzy because of diffraction. Determine the size of the hole for the sharpest image (where both effect are equally strong) of a very distant object for green light with wavelength 560 nm and the distance from the hole to the screen of 20 cm.

Problem 6

A steady charge-current flowing along a wire is known to create a magnetic field. Analogously, a steady spin-current I_s flowing along a wire creates an electric field. The infinitesimal field created by an infinitesimal segment $d\vec{l}$ of the wire at a point located at the position $\vec{r}$ (relative to the segment) is given by:

$$d\vec{E} = \frac{\mu_0 I_s}{4\pi r^3} d\vec{l} \times \left[\hat{n} - \frac{3\vec{r}(\vec{r}\cdot\hat{n})}{r^2}\right]$$

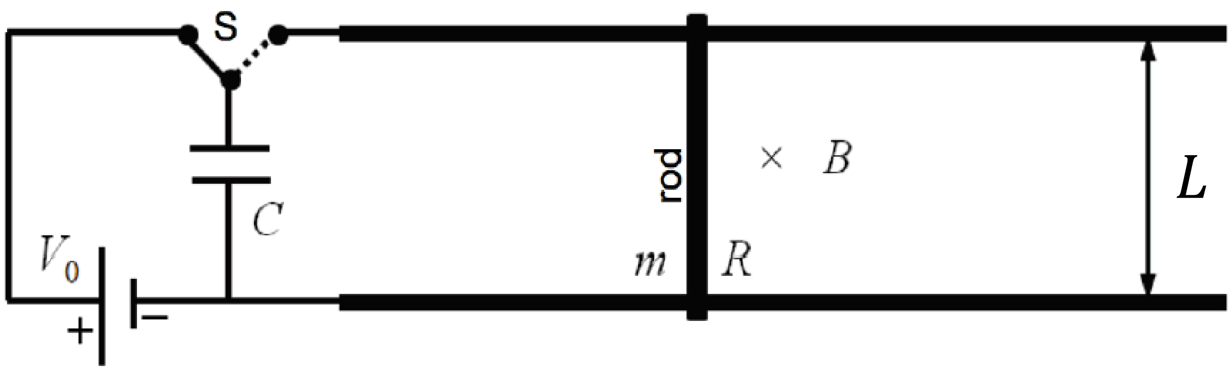
Here, $\hat{n}$ is the unit vector parallel to the spin direction. Consider a very long thin wire carrying a spin-current propagating parallel to the positive x-axis. The spins are aligned parallel to the positive z-axis, and the wire crosses the origin. The spin-current is given by $I_s = 1\frac{\mu_B}{s} = 9.27\ 10^{-24}\ Am^2/s$, where μ_B, the Bohr magneton, is the magnetic moment of an electron.

a. Calculate the direction and the magnitude of the electric field at a point distant $y = 1$ mm from the wire along the positive y-axis.
b. Compare the magnitude of this electric field to the electrostatic field generated at the same point by a single electron located at the origin and at rest.

Problem 7

One end of a horizontal track of width *L* and negligible resistance is connected to a capacitor of capacitance *C* charged to voltage V_0 of polarity shown in the figure. Since the inductance is small, the current can go up to V_0/R quickly. The system is placed in a homogeneous vertical magnetic field *B* pointing into the page. A frictionless conducting rod of mass *m* and resistance *R* is placed perpendicular onto the track. After the capacitor is fully charged the position of the switch S is changed from the position indicated by the full line to the position indicated by the dotted line, and the rod starts moving.

a. In which direction does the rod move, and why?
b. What is the maximum velocity that the rod acquires?



Problem 8

A particle of mass m is moving in a one-dimensional potential *V(x)* such that:

$$V(x) = \begin{cases} \frac{m\omega^2}{2}x^2 & \text{if } x > 0 \\ +\infty & \text{if } x \le 0 \end{cases}$$

a. Consider the motion classically. What is the period of motion in such potential and the corresponding cyclic frequency?

b. Consider the motion in quantum mechanics and express the eigenstates |m> of the problem in terms of the eigenstates |n> of the standard quantum harmonic oscillator. You don't need to solve the differential equation or write down the wavefunction corresponding to |m>.
c. Find the spectrum of the levels in the potential *V(x)*. How is the spacing between consecutive energy levels related to the frequency of the classical motion?

Problem 9

The B meson factory at the Stanford Linear Accelerator Center was designed to produce pairs of B mesons in the reaction $e^+e^- \rightarrow B\bar{B}$ by colliding electron and positron beams and to observe the decay of the B mesons in flight. At the collision point the positrons had a total energy of 9.0 GeV (per particle). The mass of the B meson is 5.28 GeV/c^2.

a. At what minimal energy of the electrons in the electron beam the pairs of B mesons started being produced?
b. If the energy in the electron beam was just above this threshold, what was the mean path length of the B meson, given that its mean lifetime (in its rest frame) is 1.6 ps?

Hint: assume that the electrons are relativistic ($E_\pm = p_\pm c$).

Problem 10

A foil of ^{57}Fe contains some nuclei in an excited state. They decay by emitting photons of energy 14.4 keV. Ignore the recoil of the nuclei. The foil is at temperature 300 K, at which temperature its molar heat capacity is *3R*. Calculate the approximate root-mean-square velocity of the iron nuclei and calculate how the center of the observed frequency range of the emitted photons varies as the temperature *T* increases, $(d\nu/dT)$ (in Hz/K). (This quantity is also sometimes called the relativistic temperature coefficient of the frequency.)

Hint: the shift of the central frequency occurs due to the time dilation.

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GRADUATE WRITTEN EXAMINATION

Spring 2016 – PART II

Friday, January 15th, 2016 – 9:00 am to 1:00 pm

Part 2 of this exam consists of 5 problems of equal weight. You will be graded on all 5 problems.

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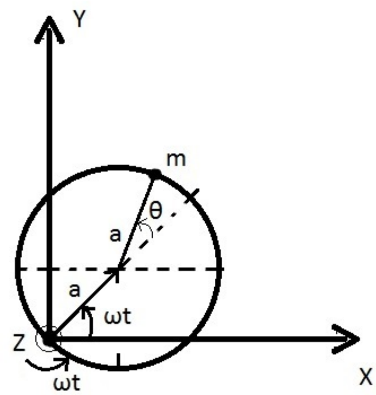
Problem 1

A bead of mass *m* is free to slide without friction on a wire in the shape of a circle (i.e. a hoop) of radius a. There is no friction and no gravity. The hoop rotates with constant angular velocity ω about an axis perpendicular to the hoop and passing through the edge of the hoop (imagine a hula-hoop).

The angle θ measures the displacement of the bead from the diameter of the hoop that passes through the origin. In this figure, the hoop is in the *xy* plane, and the axis of rotation is *z*, orthogonal to the page. The origin of this coordinate system is on the edge of the hoop.

a. Find the Hamiltonian of this system
b. Derive the equation of motion for θ (there is NO need to solve it).



Problem 2

A long molecule is composed of *N* non-interacting chemical 'monomers' each of which can be in one of two states of different lengths *a* and *b*, were $b > a$. The whole molecule therefore can be between *Na* and *Nb* in length. The energy of a monomer in the longer state *b* is ε larger than the energy of a monomer in the shorter state *a*. You may consider the thermodynamic limit $N >> 1$ to simplify the calculations. The following mathematical result may be useful: $\ln N! \approx N\ln N - N$ for $N >> 1$.

a. Determine the partition function for one monomer.
b. At a given temperature *T*, find the average number of monomers in each state, and hence the equilibrium length of the entire molecule.
c. Now, suppose that the molecule is forced to be a fixed length *L*, where $Na < L < Nb$, so that $(L - Na)/(b - a)$ of its monomer are in the stretched length *b* state. Find the internal energy $E(N,L)$ and the entropy $S(N,T,L)$.
d. From question *b*, calculate the Helmholtz free energy $F(N,T,L)$ and finally the force needed to extend the molecule to length *L* at fixed temperature *T*.

Problem 3

An electron is subject to a uniform, time-independent magnetic field directed along the positive *z*-direction, such that its Hamiltonian is given by $H = \omega S_z$, with $\omega > 0$ and S_z denoting the *z*-component of the spin operator. At time $t = 0$, the electron is in an eigenstate of the operator $\vec{S} \cdot \hat{n}$ with eigenvalue $\hbar/2$, where $\hat{n}$ is a unit vector that lies in the *xz* plane and makes an angle $\pi/6$ with the *z*-axis.

Obtain the probability for finding the electron in the $s_x = \hbar/2$ state as function of time.

Problem 4

In a simplified quantum mechanical model for the ammonia molecule (NH_3), the nitrogen atom can be either above or below the plane formed by the three hydrogen atoms (at a certain fixed distance). Both states (labeled by $|1\rangle$ and $|2\rangle$, respectively) have the same energy, which we hereafter set to zero, i.e. $\langle 1|H|1\rangle = \langle 2|H|2\rangle = 0$, where H is the Hamiltonian. The transition matrix elements $\langle 1|H|2\rangle = \langle 2|H|1\rangle = -V$, where V is positive.

a. Find the eigen-energies and eigenstates of the molecule.
b. When the nitrogen atom is above the plane, the ammonia molecule has an electric dipole moment $+\eta$, whereas when it is below the plane, the molecule has a dipole moment $-\eta$. Thus, denoting the electric dipole operator by P, we have $\langle 1|P|1\rangle = -\langle 2|P|2\rangle = \eta$. In the presence of an electric field E perpendicular to the hydrogen plane, the Hamiltonian acquires the additional term $H' = -EP$. Find the new ground state energy and the corresponding eigenstate of the molecule.
c. Compute the mean-value of the electric dipole moment of the molecule $\langle P\rangle$ in its ground state and obtain the polarizability of the molecule. Assume a small electric field $|\eta|E \ll V$.

Problem 5

A point electric charge q is placed at the distance r from the center of a hollow thin spherical metal shell of radius R with $r > R$. The sphere has total electric charge Q.

a. Find the force acting on the charge q.
b. How much energy is required to move the point charge q to infinity?
c. Find the force acting on the charge q if the sphere is grounded (instead of having fixed charge Q), so that its potential is maintained at the same value as at infinity. Any effect of the grounding wire can be neglected.

Hint: prove that the electric field outside the sphere is equivalent to that of the system, where the sphere is replaced by two 'image' point charges, one at the center of the sphere and one elsewhere on the line between the center and the charge q. You will have to find the position of this second image charge and the values of both image charges.