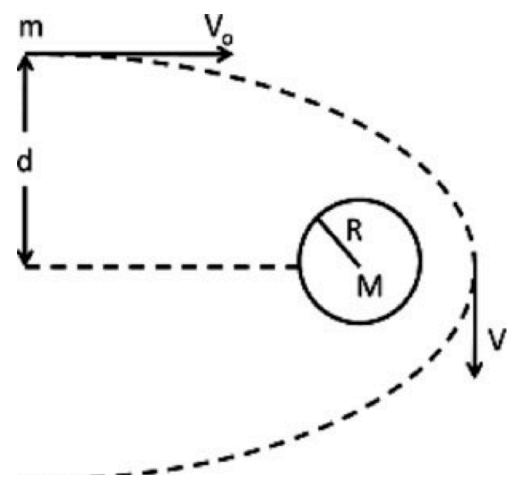
Problem 1

A particle of charge q and mass m is constrained to move along a line between two other equal charges q, fixed at $x = \pm a$. What is the period of small oscillations $(x - x_{eq} \equiv \delta x \ll a)$ about equilibrium?

Problem 2



An asteroid of mass m is moving towards a planet of mass M and radius R, from very far away with initial speed v_o and impact parameter d as shown.

Find an expression for the minimum value d_{min} of the impact parameter such that the asteroid does not hit the planet, in terms of the given quantities and the gravitational constant G.

Problem 3

Consider a simple model for radiation transfer within the Sun in which photons propagate via a random walk process controlled by Thomson scattering from free electrons. The cross-section for Thomson scattering is given in terms of the classical electron radius r_e by

$$\sigma_T = \frac{8\pi}{3} r_e^2 = 6.7 \times 10^{-25}\ cm^2$$

(a) Estimate the average mean free path of the photons assuming the density of the Sun (taken to consist entirely of hydrogen) to be constant at its mean value of ρ = 1.42 g/cm^3.

(b) The radius of the Sun is 2.32 light-seconds: Use your result from part (a) to estimate the time required for a photon to travel from the center of the Sun to the surface.

Problem 4

Consider a binary alloy system $A_x B_{1-x}$ made up of N_A atoms of an element A and $N_B = N - N_A$ atoms of an element B, thereby occupying a total of N sites that are fixed in space, and with $x \equiv N_A/N$. Assume that any type of atom can occupy a given site.

(a) Find the number of possible arrangements $\Omega(N, N_A)$ of A and B in the alloy.

(b) What is the entropy associated with these arrangements (known as the entropy of mixing)?

Problem 5

A light signal, and a neutrino with energy E = 2 MeV and rest mass m_0c^2 = 1 eV are emitted simultaneously in a supernova explosion at a distance of 10^4 light years from Earth.

(a) What is the difference in arrival times at Earth (in s)?

(b) How long does the neutrino's trip take in its rest frame ?

Problem 6

The low-temperature molar specific heat of many materials is found to vary with temperature according to the Debye law

$$c_v = 1.94\times10^3 \left[\frac{T}{\theta}\right]^3 \ \mathrm{J}/(mol \cdot K)$$

where θ is the Debye temperature, taking different values for different materials. For diamond, $\theta = 1860\ K$. (The molar mass of carbon is 12.0 g)

(a) How much heat (in Joules) is required to heat 1.0 g of diamond from 4 K to 300 K at constant volume?

(b) What is the corresponding entropy change of the sample?

Problem 7

The Earth's atmosphere supports a global electric circuit which manifests itself as a fair-weather electric field. While its strength varies according to local conditions, this field (pointing towards the Earth) has an average value on the Earth's surface of about 130 V/m.

(a) What is the average surface charge density of Earth (in $\mathrm{C/m^2}$)?

(b) What is then the total surface charge on earth (in C), including the oceans?

Problem 8

A radio transmitter radiates at a power of 10 kW at a wavelength of 100 m. How many photons does it transmit per second?

Problem 9

A spin one-half particle is prepared in an eigenstate of the operator $\frac{1}{\sqrt{2}}(S_x + S_y)$.

(a) Find the eigenvalues and corresponding normalized eigenvectors for this operator. Hint: it is often easier to work with complex numbers when written in the form $z = |z|\exp(i\varphi)$.

(b) What are the probabilities (for each eigenstate) that a measurement of S_z will yield $-\hbar/2$?

Recall that the spin-1/2 operators are given in terms of the Pauli matrices,

$$\boldsymbol{S} = \hbar\frac{\boldsymbol{\sigma}}{2}, \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Problem 10

The magnetic field of a uniform electromagnetic plane wave in free space is given by $\mathbf{B}(\boldsymbol{r}, t) = \mathbf{B}_0 \exp(-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{r})$ with (in SI units)

$$B_{0x} = 1\ \mu\text{T}, B_{0y} = 2\mu\text{T}$$

$$\boldsymbol{k} = -3\,\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}\ \text{m}^{-1}$$

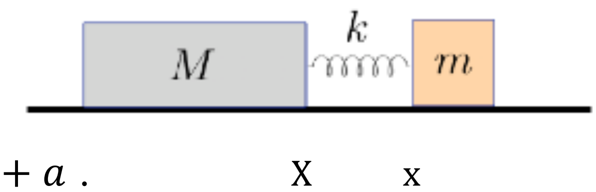
(a) What are the direction of propagation $\widehat{\boldsymbol{k}}$, wavelength λ, and angular frequency ω of the electromagnetic wave?

(b) What is the z component of $\mathbf{B}_0$?

(c) Find the electric field $\mathbf{E}(\boldsymbol{r}, t)$ associated with the given magnetic field.

Problem 1

Two blocks of masses M and m respectively, connected by a spring of spring constant k, are free to move on a frictionless track along the x-axis. The equilibrium length of the (massless) spring is a. For uniformity, let X and x be the coordinates of the right-hand and left-hand edges of the blocks as shown in the figure, and define u via $\mathrm{x} - \mathrm{X} = u + a$.



(a) Write down the Lagrangian for this system, using X and u as generalized coordinates and identify their conjugate momenta.

(b) Derive the equations of motion and identify all conserved quantities.

(c) Find the general solution $u(t)$ and $\mathrm{X}(t)$ to the equations of motion: Obtain $u(t)$ first by eliminating $\ddot{\mathrm{X}}(t)$ to get an equation for $u(t)$ alone, then return to find $\mathrm{X}(t)$. Your solution should include four constants of integration which can be left arbitrary.

Problem 2

A gas of N non-relativistic free electrons at $T = 0$ is constrained to move in <u>two dimensions</u> only, within a square of area A in the x-y plane.

(a) Find the density of single-particle states $g(\epsilon)$ at energy ϵ.

(b) Evaluate the Fermi energy ϵ_F of the gas as a function of the number density of particles $n = N/A$.

(c) Calculate the energy density per unit area $u = E/A$ of the gas as a function of the number density, where E is the total energy.

Problem 3

Block 4 of the nuclear power plant in Chernobyl (Ukraine) had been continuously operated before the accident on April 26th, 1986 with a thermal output power of 1000 MW. About 2% of all fission processes lead to the iodine isotope ^{131}I as a fission product: With a half-life $t_{1/2}$ of 8.04 days, ^{131}I poses a considerable health threat if it is released in the atmosphere as it was over many parts of Europe in the Chernobyl disaster. Assume that an energy of 190 MeV is released in each fission process, and bear in mind the distinction between the half-life $t_{1/2}$ of a radioactive isotope and its lifetime τ.

(a) At what rate r (per second) was ^{131}I produced in the reactor?

(b) Find an expression for the number $N_I(t)$ of ^{131}I atoms present in the reactor as a function of time if none were present initially in terms of r and of the lifetime $\tau = 1/\lambda$. To do so, write and solve the equation for the rate of change $\dot{N}_I(t)$ of this number.

(c) The number of ^{131}I atoms will eventually tend to an equilibrium value. What is this number for the Chernobyl reactor and to what mass (in kg) does this correspond?

(d) How long does it take to reach a number of ^{131}I atoms that is within about 3% of the eventual equilibrium value?

Problem 4

An otherwise free electron is confined to the interior of a hollow spherical cavity of radius R with impenetrable walls.

(a) Find the energy and normalized wave-function of the electron in its ground state (with $\boldsymbol{L}$=0).

(b) What pressure is then exerted on the walls of the cavity by the electron in its ground state?

Recall that the Laplacian in spherical polar coordinates is given by

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} r + angular\ part$$

Problem 5

A thin non-conducting ring of radius R, centered about the origin of coordinates in the x-y plane, is charged with a linear charge density $\lambda(\varphi) = \lambda_o \cos\varphi$ where $\lambda_o > 0$ and φ is the usual azimuthal angle as measured from the x-axis.

(a) Draw a picture of this charge distribution as seen looking down the z-axis, giving a qualitative idea of the density of positive and negative charges and their location. What do you conclude about the direction of the electric field in the plane of the ring?

(b) Find the electric field $\boldsymbol{E}(\boldsymbol{r} = \boldsymbol{0})$ in magnitude <u>and</u> direction at the center of the ring. Use cylindrical coordinates $\boldsymbol{r} = s\,\hat{\boldsymbol{s}} + z\hat{\boldsymbol{z}}$ with $\hat{\boldsymbol{s}} = \cos\varphi\,\hat{\boldsymbol{x}} + \sin\varphi\,\hat{\boldsymbol{y}}$ and $s = \sqrt{x^2 + y^2}$ in setting up the integral for the field.