Echolocation in air: biological systems, technical challenges, and transducer design

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Abstract: Echolocation systems used by bats may be used by engineers wishing to emulate the performance of these biological systems. Comparisons are made between biosonar systems based on water and air, the signal structures used by animals echolocating in air, and the limits on resolution. The current thinking in how biological systems operate are discussed, as are the engineering challenges of replicating the performance levels demonstrated by echolocating animals.

Keywords: biosonar, echolocation, ultrasound, biomimetics

1 BIOSONAR

Since the discovery that bats echolocate to orientate and find their prey [**1**, **2**], engineers have sought to emulate the impressive performance that bats appear to be capable of. Such a biomimetic approach, the mimicry of a biological system by an engineering one, could improve resolution or target characterization in any field that uses sound to probe environments remotely, or probe material properties by direct contact through a coupling medium.

The process of echolocation can be considered the active or intentional production of sound which reflects from objects in the environment and conveys information via the echo back to the producer. Such a definition is necessary since the passive production of sound provides cues about environmental spaces via reflections and reverberation, a phenomenon that humans exploit as well as many other animals [**3**]. Animals may deliberately and actively produce unstructured high frequency signals for the same purpose. Rats use ultrasonic signals to gauge approximate distances when jumping [**4**], and the mole-rat *Spalax ehrenbergi* use low frequency self-generated seismic waves to detect obstacles when digging underground burrows [**5**].

The use of more tightly structured signals to provide a quantitative estimate of range appears to have evolved at least five times: once in the oilbird [**6**]; at least once in cave swiftlets [**7**]; once in odontocete cetaceans [**8**]; and at least twice in bats, in the megachiropteran genus *Rousettus* [**9**] and in the microchiroptera [**10**]. Of these latter five convergent systems, all have evolved to allow the animal to orientate and navigate when vision cannot provide the necessary informational bandwidth. In oilbirds, swiftlets, and *Rousettus*, this is because of their roosts being deep in caves where there is very little light. In toothed whales, this may be either due to light absorption when hunting in deep water, or low visibility caused by sediment. In microchiropteran bats, which forage at night, echolocation is used to navigate and detect food, which may be invertebrate, vertebrate, fruit, or nectar.

The temporal and spatial resolution of the clicks used by both oilbirds and swiftlets is largely unknown, but appears to be relatively poor compared with bats and cetaceans. Obstacle avoidance experiments show reasonable performance by the megachiropteran bat *Rousettus* [**9**], but it is the microchiropteran bats and odotocete cetaceans such as dolphins, which show the greatest capabilities to identify the location and characteristics of remote targets. Consequently, it is these groups which have been the focus for engineers looking to develop sonar systems capable of mimicking the performance of biological systems, primarily bats for those working with terrestrial robotic systems, and dolphins for those working with underwater acoustics, materials testing, and medical ultrasonics.

It is important at this point to make the distinction between a biomimetic approach versus a biologically inspired one. Bats are limited by energetics and the biological properties of materials they are made from. This limits the energy of the acoustic output and hence range. Their nervous systems are largely analogue systems, slow, hugely parallel, and with high noise levels with a limited ability to phase lock onto returning echoes which constrains any signal processing algorithm. Bats have also evolved over millions of years to produce echolocation calls optimized to detect biological targets rather than targets that engineers consider important. Therefore, while engineers may learn a great deal about alternative techniques for working with sonar in air, mimicking a bat echolocation system using a biomimetic approach may not provide the optimal solution to any given engineering problem. However, a biologically inspired approach, knowing that bats are capable of resolution far in excess of what is currently technologically possible, should open up new ways of approaching sonar problems with applications in robotic guidance, medical imaging, underwater mapping, and geological surveying. Such inspiration may arise from understanding the subtleties of signal structure, signal production, and reception, or signal processing.

2 PROPERTIES OF ULTRASOUND IN AIR

In air, the choice of call frequency will largely dictate the range at which a target is detected. This is the result of two issues; first that the amount of acoustic energy reflected from a target (termed the target strength) is dependent on the size of the target relative to the wavelength of the ensonifying signal, and second, that the level of absorption of sound by the air is frequency dependent.

In order to model the effects of these parameters, a common form of the sonar equation can be used [**11**]

$$\mathrm{SNL} = \mathrm{SL} - 2\mathrm{TL} + \mathrm{TS} - (\mathrm{NL} - \mathrm{DI}) \qquad (1)$$

where SNL is the signal to noise ratio of the returning echo, SL is the source level, 2TL are the two way transmission losses, TS is the target strength, NL is the noise level, and DI is the directivity index. All parameters are in decibels. The convention in air is to reference the sound pressure level to 20 μPa, whereas in water, this reference value is 1 μPa.

Source levels for bats recorded in the laboratory are of the order of 106 dB peSPL at 10 cm [**12, 13**] or possibly even higher in the field, over 120 dB peSPL at 10 cm [**14–16**], where dB peSPL is the peak-equivalent SPL and is the r.m.s. sound pressure of a sine wave that matches the peak–peak amplitude of the bat's call [**17**]. Bat echolocation intensities are traditionally referenced to 10 cm, while those of cetaceans tend to be referenced to 1 m. Source levels in different media are also referred to different reference levels, that of 2×10^{-5} Nm^{-2} in air (or 20 μPa) where Nm^{-2} is equivalent to the SI unit of the Pascal (Pa) and 1 μbar in water where 1 μbar is equivalent to 0.1 Pa. Intensity at 1 m is more compatible with other measured parameters in the sonar equation and is easily converted by subtracting 20 dB from the source level at 10 cm.

A bat generally wishes to detect small targets, such as insects, requiring a short wavelength and hence a high frequency. However, high frequencies are attenuated more severely by the atmosphere, so a higher target strength because of a higher frequency may be offset by higher attenuation. The total energy lost during transmission is because of spherical spreading from the bat and from the target, and excess attenuation caused by absorption by air. Atmospheric attenuation is affected by both temperature and relative humidity, and to a lesser extent by pressure. At 10 kHz, 20 °C, and 50 per cent relative humidity, conditions usually found in temperate regions in the summer, atmospheric attenuation is of the order of 0.1 dBm^{-1} at 10 kHz, rising to 3.3 dBm^{-1} at 100 kHz, and 8.3 dBm^{-1} at 200 kHz (Fig. 1) [**18**]. The higher temperature and relative humidity found in tropical regions has an even greater effect on attenuation above 40 kHz. These levels of attenuation are especially troublesome for bats since the path length of the call is twice the distance to the target [**19, 20**]. Hence for a temperate region bat at 1 m from a target using a call of 200 kHz, the signal will be attenuated by 16.6 dB over the spherical spreading losses. Aside from losses

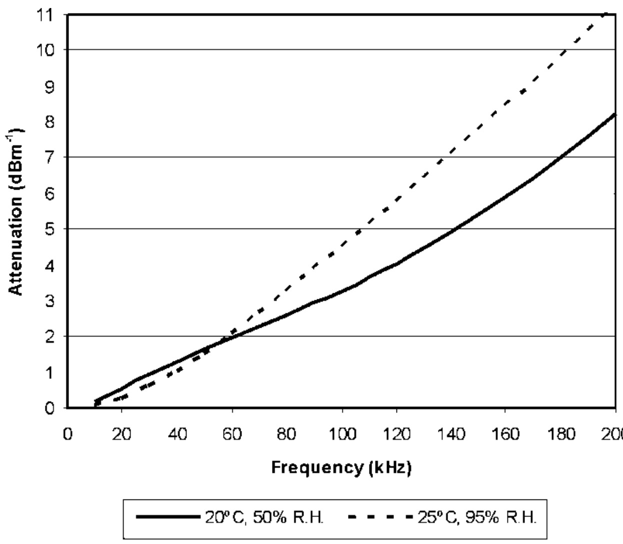

Fig. 1 Atmospheric attenuation as a function of frequency for temperate regions of 20 °C and 50 per cent relative humidity and tropical regions of 25 °C and 95 per cent relative humidity

because of spherical spreading and attenuation, there may also be significant energy loss on reflection from the target. Target strength is represented as the loss in sound energy between the incident sound pulse and the reflected sound pulse. The target strength is referenced to a standard distance (usually 1 m). For example, if the incident sound intensity at a target is 80 dB SPL, and the intensity of the echo recorded at 1 m from the target is 40 dB SPL, then the target strength is −40 dB. Note that this also takes into account the spherical spreading loss at this range from the target. As a general rule of thumb, objects are acoustically large and return significant echo energy if they fall into the simple scattering region and satisfy the following condition [**11**]

$$a > \frac{5\lambda}{2\pi} \tag{2}$$

where a is the object radius and λ is the wavelength of sound. This range is illustrated in Fig. 2 and approximates to the condition where the object is larger than the wavelength of sound impinging on it. When the object is smaller than the wavelength, or for spheres when the circumference is smaller than the wavelength, Rayleigh scattering operates [**21**]. Between the regions of Rayleigh scattering and simple scattering, Mie scattering produces an oscillation in target strength with frequency [**22**]. The actual target strength will depend on the size and geometry of the target and equations and nomograms exist for the calculation of target strength of ideal targets such as spheres and discs [**11**], but this parameter really needs to be measured for biologically realistic targets. Typical measured target strengths for moths with wingspans of 2 cm are of the order of −50 dB standardized to 1 m [**23**].

The final parameters in the sonar equation are the noise levels and the directivity index. Noise can come from many sources such as from other bats, ultrasonically singing insects, wind noise in the ear of the bat as it flies forward, or spontaneous neurone discharge in the bat's cochlea. Some of this noise can be moderated by the directivity index which is a function that improves the overall signal to noise ratio by limiting the directivity of the ear, and hence limiting the direction over which the ear can receive noise. It can be defined as the decibel reduction in the overall noise level which occurs as a result of the narrowing of the angle of view of the ear. Both of these parameters are difficult to estimate in non-idealized situations but quantitative measurements of directivity for the bat pinna do exist [**24**].

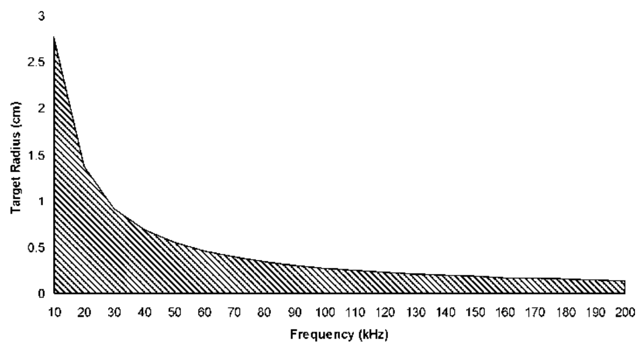

Fig. 2 Range of frequencies and target radii over which the object becomes acoustically large (region above the hatched area) and reflects significant amounts of echo energy

Incorporating the factors above into the sonar equation with real data provides the following example: a bat at 2 m from a moth target with a target strength at −50 dB, using a call at 50 kHz (with an atmospheric attenuation component of 1.7 dBm^{-1}) and calling at 90 dB peSPL at 1 m is likely to receive an echo of

$$= 90 - \left(40 \times \log_{10}\frac{2}{1}\right) - (4 \times 1.7) - 50$$
$$= 21.2 \text{ dB SPL}$$

All values in the above equation have been normalized to a distance of 1 m.

The value of 21.2 dB SPL excludes any noise terms. At 4 m the value would be 2.3 dB SPL. It is assumed that the threshold of hearing is at 0 dB SPL (as it is for humans), then it can be seen that even with very high source levels, the range of echolocation is very limited.

In terms of the useful range of frequencies in air, attenuation generally limits bat echolocation calls at the upper end to frequencies lower than 200 kHz, while the lower end is limited by low target strengths from small targets to about 10 kHz. Total range is limited by the physiological mechanism of producing high volume sounds.

3 ECHOLOCATION CALL STRUCTURES

Echolocation calls are produced in the larynx in microchiropteran bats, by clicking the tongue in *Rousettus* and through movement of air across the phonic lips in the toothed whales [**25**]. The frequency structure of all three systems is broadly classed as ultrasonic, that is above 18 kHz, the upper limit of hearing in humans. Such a high frequency ensures a short wavelength ensuring echo returns from small objects. In aquatic systems, the frequency must be higher to compensate for the higher speed of sound in water (1500 ms^{-1} compared with 344 ms^{-1}), which results in a longer wavelength for any given frequency.

The function of echolocation, to estimate the range of a target and provide information on the target in the returning echo, requires a signal structure which can be accurately localized in time and is broadband. An idealized function which fulfils these requirements is analogous to a Dirac delta function, which is a function on the real line which is zero everywhere except at the origin where it is infinite

$$\delta(x) = \begin{cases} \infty, x = 0 \\ \hline 0, x \neq 0 \end{cases} \tag{3}$$

and which is constrained to satisfy the condition

$$\int_{-\infty}^{\infty} \delta(x)\mathrm{d}x = 1 \tag{4}$$

The Fourier transform of such a function leads to a flat spectrum over an infinitely wide frequency range. The echo return from such a function can therefore be localized accurately since the time course of the signal is so restricted, and will contain spatial information from the target because of the interaction of the signal with the target in a frequency-dependent manner. Such signals are clearly not realistic in a real world situation because of limitations in producing such short duration signals with sufficiently high intensity. A very wideband signal may not be desirable either because some energy may simply diffract around the target at low frequencies or be absorbed by the medium at high frequencies. The animal may also have a physiologically limited hearing range, meaning that energy returned at frequencies outside the range of the animals hearing are wasted. These factors combine to target energy at frequencies that are useful. Dolphins appear to use such a compromise in their sonar signals, which are clicks lasting a few hundred microseconds with typical frequencies between 100 and 150 kHz. Output levels are of the order of 120–220 dB (referenced to 1 μPa). Although signals have only a few cycles, the amplitude modulated envelope shows an approximation to a Gaussian curve. Such signals show close relationships to Gabor functions, which provide the minimum bandwidth for any given signal duration [**8**]. Such an arrangement allows a wideband signal to return frequency-dependant information from targets while maintaining a restricted duration for localization in time. An almost identical system is used by *Rousettus aegyptiacus* in air [**26**], but not by other bats, which use a variety of frequency and amplitude modulated sine waves, often with harmonics [**10**] (Fig. 3). One possible reason for this difference in echolocation behaviour between water and air is the match between acoustic impedance for the two media. The acoustic impedance can be defined as

$$Z = \rho \cdot c \tag{5}$$

where Z is the acoustic impedance, ρ is the density of the medium in k gm^{-3}, and c is the velocity of sound in the medium in ms^{-1}. Z has the units Pa sm^{-1}.

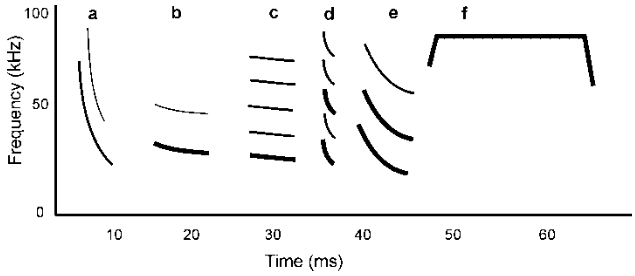

Fig. 3 Schematic sonograms of call types produced by echolocating bats after [**10**] (a) broadband with limited harmonics, (b) narrowband with limited harmonics, (c) narrowband multi-harmonic, (d) short duration broadband multi-harmonic, (e) long duration broadband multi-harmonic, and (f) constant frequency

The difference between the acoustic impedances of two media at an interface provides an estimate of the efficiency of energy transfer between the two media, more closely matched impedances provide greater transfer efficiency.

The fraction of energy reflected at a boundary between two media, termed the reflection coefficient, can be approximated by

$$R = \left(\frac{(Z_2/Z_1) - \sqrt{1 - [n-1]\tan^2 \alpha_i}}{(Z_2/Z_1) + \sqrt{1 - [n-1]\tan^2 \alpha_i}} \right)^2 \tag{6}$$

where Z_1 is the acoustic impedance towards the source and Z_2 is the acoustic impedance towards the load of the two media, $n = (c_2/c_2)^2$ and α_i is the angle of incidence of the sound wave to the boundary.

For an angle of incidence of zero, this simplifies to

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2 \tag{7}$$

As the proportion of energy reflected from the boundary and transmitted through it must sum to one, the proportion of energy transmitted is simply the reflection coefficient subtracted from one. The acoustic impedance for air at 20 °C is 413 Pa sm^{-1} and for sea water 1.54×10^6 Pa sm^{-1}. The reflection coefficient for an air–water boundary for a sound wave normal to that boundary is therefore 0.999, meaning that 99.9 per cent of acoustic energy is reflected at the boundary and only 0.1 per cent of energy is transmitted. The density of the acoustic structures of dolphins is much more closely matched to water than those of bats is to air allowing greater transfer efficiency in aquatic environments than air. In fact, dolphins exploit the material properties of tissues to grade the acoustic impendence of the melon, the large fat-filled structure on the front of the skull, to both match that of seawater, and to act as an acoustic lens to focus acoustic energy into a tight beam in front of the animal [**26**].

The energy content of cetacean echolocation signals is of the order of 164 dB (referenced to 1 $\mu Pa^2 s$) [**27**], equating to 8×10^{-7} Jm^{-2} when corrected for the density of the medium. This compares with a similar structure from *Rousettus*, which contains approximately 3.5×10^{-8} Jm^{-2} [**8**] suggests that echolocation calls from bats and dolphins contain similar amounts of energy, although this appears highly species specific with wide variation, mostly as regards signal energy in odontocetes [**28**]. A further feature of the aquatic environment is that because of the high density of water, any given particle displacement will generate a higher pressure and higher sound energy density. Therefore, in an aerial environment, it is difficult to produce signals of high amplitude, whereas in an aquatic environment, large amounts of energy can be transmitted using more modest amplitudes. This means that even if bats could afford the metabolic energy to transmit high intensity short duration impulsive signals such as dolphins, the high amplitudes necessary would be physiologically difficult. Because the energy transmitted in a sound pulse is a function of both the amplitude and the duration, it is possible to propagate more energy if the duration of the signal is extended. Microchiropteran bats appear to use this approach. The disadvantage is that the signal is now less localized in time, making estimates of time of arrival of the echo, and hence target range, more difficult.

Call structure in echolocating bats is matched to the task the bat wishes to perform and shows strong correlations with wing morphology and flight style [**29**]. Within the microchiroptera, call structures can be broadly divided into long duration constant frequency (CF) calls, primarily used by Rhinolophoid bats, and frequency modulated (FM) calls of shorter duration and wider bandwidth used by all other species. Bats using CF calls generally produce echolocation pulses between 20 and 50 ms long with a very narrow bandwidth in the middle part of the call, but with rising FM components at the start of the call, and falling FM components at the end of the call. The CF portion of the call can be very high in frequency, and includes that produced by *Cleotis percivali* at 212 kHz, the highest known frequency used by bats [**30**]. Although higher frequencies increase the detectability of small targets such as insects, their range is constrained by atmospheric attenuation. An external factor that may have driven CF bats to use higher frequency echolocation calls is that moths have evolved auditory systems to alert them to the presence of echolocating bats. By using higher echolocation call frequencies, bats may be able to bypass the moths hearing [**31**, **32**] by using frequencies to which the moths are less sensitive. Such confounding effects can blur the link between call design and task from an engineering perspective, since other biological effects may take the call structure away from what is deemed 'optimal'.

Bats are generally intolerant of pulse-echo overlap, where the echo from the start of the call returns while the call is still being produced, as would happen in close proximity to a target. This is because of various mechanisms bats employ to avoid self-deafening or forward masking. For a 5-ms signal, pulse-echo overlap occurs at a range of 0.86 m from the target. For a 50-ms signal, as used by many CF bats, this rises to 8.6 m. Bats using CF calls get around the pulse-echo limit by having a very narrow band of high sensitivity in the cochlea, termed the acoustic fovea. By calling just below this region, the bat is effectively deaf to its own emissions. However, as the bat flies towards a target, the returning echo is Doppler shifted up in frequency back into the region of the acoustic fovea making the echo audible. The long duration of the CF call can also encode both amplitude and frequency modulations from movement of the prey's wings, allowing the bat to determine prey type.

Bats using FM signals show a very wide range of signal types, from shallow long duration FM, to short multi-harmonic signals. The call shape is generally matched to the bats' foraging style. Bats which fly fast and high tend to produce long duration low frequency echolocation calls which are attenuated less by the atmosphere [**33**]. This is necessary since the bats require a long detection range as they fly faster. Conversely, bats which fly close to vegetation tend to produce short duration steep FM signals which reduce the zone of pulse-echo overlap. The calls themselves are variable within a species depending on the task. Many species are able to modify bandwidth and duration as they approach the target and the call structure changes as the bat moves from search phase to approach phase and the final terminal buzz [**34**]. In this last stage, the repetition rate of the signal can rise to as much as 200 Hz.

Although the broad scale differences in call structures between species are reasonably well understood, the reasons for the fine scale differences are less clear. Two similar species, such as *Myotis mystacinus* and *Myotis nattereri* may forage in the same area and both use a call of 2.5 ms duration, yet one will have an end frequency of 30 kHz, and the other will end at 20 kHz [**35**].

Sweep patterns are also unusual since the optimal pattern should be a hyperbolic frequency sweep to give Doppler tolerance [**36**]. Some bats appear to use optimal call designs to give a depth of focus at which ranging errors are minimized, this depth of focus changing as the bats range to the target changes [**37**]. However, not all bats use this pattern, and the pattern changes with task [**38**] suggesting no single underlying reason for modulation pattern. The relationship between frequency modulation pattern and Doppler tolerance appears complex, and is also reliant on the type of receiver model used [**39**]. Clearly, there is still much work to be done on the fine scale signal structure that bats use in relation to the task it has to perform.

4 RESOLUTION OF BIOSONAR IN AIR

Echolocation in bats has two purposes. The first is to locate targets, both in azimuth, elevation, and range. The second is to resolve features of the target, such as size, shape, and movement. Bats resolve azimuth and elevation in the same manner that other mammals do, using interaural intensity differences, interaural timing differences, and head-related transfer functions, facilitated by a complex pinna shape [**40**]. The directional nature of the sound beam means that bats need only be receptive to a narrow range of target locations since echoes are only likely to be received from the general area in front of the bat [**41**]. Most work has been conducted on the accuracy with which bats can deduce range from timing delays between sound emission and reception of the echo, and on how bats may infer target structure from features encoded within the returning echo.

A matched filter is the optimal design for detecting a signal is noise, such as a low amplitude echo from a target, and also the optimal design for determining the time delay of an echo relative to an emission [**42**]. Results are ambiguous in that a matched filter system does not appear to operate for target detection [**43**] but is used for target ranging [**44**]. The concept of a matched filter relies on convolving a template of the outgoing signal with the returning echo. This implies phase sensitivity at high frequencies which does not appear to be the case in some studies [**45**] but does in others [**46**]. However, alternative models exist which circumvent this need for phase sensitivity and yield results comparable with a coherent receiver [**47, 48**].

At its simplest, the detection system of the bat can be thought of as a simple pulse-echo system. However, timing accuracy is crucial for accurate target location. An error of 1 ms translates to a target distance error of 17 cm. Since the typical time-course of a neurone action potential is of the order of 1 ms, any finer time resolution appears to be impossible. However, some experimental evidence suggests that bats may be accurate to time resolutions of the order of a few hundreds of nanoseconds, equivalent to submillimetre range estimations [**49**]. Temporal resolution of this magnitude, coupled with the ability to identify signals in noise suggest a form of cross-correlation receiver model, whereby the returning echo is cross-correlated with a template of the original outgoing signal. An FM signal produces a much sharper spike in the resulting plot than a CF signal, suggesting a reason why bats use FM signals. However, a cross-correlation receiver requires the bats to be sensitive to phase at very high frequencies. Phase sensitivity is lost in humans at frequencies about 1.5 kHz, and there is no currently known mechanism by which phase at such high frequencies can be encoded.

One model which has been proposed to deal with these inconsistencies is the spectrogram correlation and transformation receiver (SCAT) model [**47**]. This model uses a series of modular blocks to process incoming echoes. The first of these processing units is the cochlear block, where the incoming echo signal is bandpass filtered into 81 separate 3 kHz bands. The outputs from this filterbank are then low-pass filtered to recover the envelope. This is followed by a temporal block in which the envelopes from each filter are passed into a series of parallel delay-lines. Neurones, tuned to specific delays in increments of 1 μs look for coincidences across the delay lines. The final block, the transformation block, is used to reconstruct target 'shape' from multiple overlapping echoes derived from surface features of the target. Although models such as SCAT are useful approaches, what remains is evidence that bats can discriminate echo delay to an accuracy of 10–15 ns with no known physiological mechanism for doing so [**50**].

Aside from consideration of target range, bats may also be able to extract features from the target. Any three-dimensional target will return echoes with multiple wave-fronts because of the spatial extent and structure of the target. Bats could reconstruct the target shape through time-domain based analysis of echo delay, though this does not appear to be the case [**49**], though indirect use of time-delays may be used for shape reconstruction [**51**]. However, multiple overlapping wave-fronts also result in characteristic constructive and destructive interference resulting in impulse responses of targets that may be used in target classification [**52**]. Bats using CF calls can also decode amplitude modulated and FM shifts in the returning echo to assess prey type [**53**], but the extent to which bats using FM calls can do this is unknown.

It is clear that the returning echo conveys a great deal of information on target location and characteristics. In addition, there is a wealth of information on the neural pathways and properties of the auditory system [**54, 55**]. The paradox remains that range estimation in bat echolocation is that bats appear to be able to time differences in subsequent echoes (and hence range estimation) with a greater accuracy than currently possible using any know physiological mechanism. Although signal processing models exist which approximate to known neural pathways, the mismatch between performance and known mechanisms of auditory processing remains a challenge.

5 INSPIRATION FROM BATS

Many engineers have been inspired by the way that bats can exploit airborne sonar to develop better systems for sensing via echolocation [**56–58**]. Such an approach can be divided into a number of areas where an understanding of bat biology can be useful. An understanding of the way that signal structure is matched to target can provide insight into the best signal structures to resolve specific target characteristics and the way that returning echoes are acquired and binaurally processed can inform receiver technology [**59**]. The way that those signals are decomposed and processed by the bat's cochlea reveals how information on relevant parameters can be preserved during encoding [**60**], while understanding of how the bat's brain deals with the processing of complex signals for feature extraction can drive new signal processing techniques [**47**].

A biomimetic approach, by definition, would be limited to ultrasound in air, yet many of the potential benefits listed above are applicable to other frequency ranges or media. One limitation of airborne ultrasound has been the availability and performance of transmitting and receiving transducers. The simplest approach of using airborne ultrasound for target ranging does not make use of the information available on target structure. The potential of airborne sonar to encode information about complex targets or surfaces [**61**] opens up many more possibilities than the simple obstacle detector it is often used for. Prior to any new approaches in signal processing, however, the outstanding problem in biomimetic sonar is the reliable production of repeatable, accurate, and high power acoustic signals, which are similar to those produced by bats. Only when it is possible to produce signals which are similar to bats will it be possible to understand what information is contained within the echoes of such signals.

6 AIRBORNE ULTRASONIC TRANSDUCERS

The function of an ultrasonic transducer is to take mechanical motion and transfer that motion to the air for a transmitter, and to take vibration of the air and to convert that to mechanical motion for a receiver. The engineering considerations of both are largely similar and transceivers, which act as both transmitter and receiver exist. Because of the high frequency vibration required to impart ultrasonic vibration to the air, lower mass transducers with small active areas are most efficient meaning that most conventional loudspeaker and microphone technology will not function in the ultrasound region.

The first ultrasonic detector to be used in conjunction with bats was a Rochelle salt crystal in conjunction with an amplifying horn used by Donald Griffin to establish that bats emitted echolocation signals in the ultrasonic range [**1**, **2**]. Coupled with the process of heterodyning, where an internal oscillator produces a sine wave which is mixed with the incoming ultrasonic signal to create sum and difference frequencies, the same technique is still used in commercial 'bat detectors' today. The piezo-electric effect of the crystal produces an opposite charge on two faces of the crystal when it vibrates and changes shape as sound impinges upon it. These small changes in charge can then be amplified. Single crystal piezo materials such as quartz are less sensitive by about two orders of magnitude than ceramic materials such as lead zinconate titanate (PZT). The properties of a piezo ultrasonic transducer can be manipulated both through the material properties and the shape and size of the transducer. Ultrasonic loudspeakers can deliver high output levels, around 110 dB SPL at 10 cm, and are also sensitive when used as microphones where commercially available transducers typically operate at −60 dB re 1 V/Pa. However, they suffer from resonance where maximal output/sensitivity is only obtained over a very narrow range of frequencies, typically a few kilohertz. An alternative design uses piezoelectric polymers, such as polyvinylidene difluoride (PVDF) which can be curved to create simple air-coupled transducers where the resonant frequency is simply a function of the radius of curvature [**62**]. However, since it is difficult to engineer a design with more than one radius of curvature, transducers of this design also suffer from a narrow bandwidth.

Condenser microphones suitable for ultrasound fall into three types; electret, solid dielectric, and air dielectric. In electret microphones, a permanent polarization charge is applied to a material such as a polymer. The electret material is usually a thin membrane held close to a backplate. When sound impinges on the membrane, it vibrates and causes a change in the capacitance between it and the backplate inducing a voltage change in phase with the sound. The low current generated requires a high input impedance amplifier to be placed close to the microphone, usually inside the microphone capsule. With solid dielectric microphones and loudspeakers, an insulating material is coated on one side with a conducting material and held against a perforated conducting backplate forming a capacitor [**63**]. A high polarization charge of a few hundred volts is applied across the capacitor and, in the case of a loudspeaker, a voltage swing applied. For a microphone a high impedance amplifier is placed close to the capacitor. The net effect is that the perforated backplate, and the elastic properties of the insulator produce a large number of small pockets which can vibrate in parallel. The advantages of such a transducer are high output levels for a loudspeaker and high sensitivity for a microphone and they have been shown to be very effective as sensors in robotics [**64**, **65**]. The frequency response of these transducers can be controlled by membrane properties, tension and on the geometry of the backplate perforations [**66**], with surprisingly linear frequency responses possible [**67**]. One such transducer is the widely available polaroid transducer, originally developed for ranging for focus distance in cameras but finding many other applications [**65**]. The disadvantage is that sensitivity can vary from day to day, and that, as microphones, they are very susceptible to humidity which causes breakdown in the polarization charge, a problem that bats do not face in the high humidity of the tropics.

Air dielectric microphones replace the solid insulator with air, such that a thin metal membrane is stretched over a conductor with a thin layer of air between the two. A polarization charge is applied across this capacitor. The advantage is that these microphones are capable of highly repeatable sound level measurements and are very stable across a wide range of temperatures. They also can have very linear frequency responses from a few hertz to over 100 kHz. This makes them suitable for measurements of sound intensity. The disadvantages are that they are delicate, and for frequencies over 40 kHz, the protective grid must be removed. They are also very expensive, and as the capacitance is very low, around 6 pF, a very high input impedance of 10 GΩ is required in the amplifier resulting in high levels of thermal noise. To use for ultrasound, the output has to be filtered to remove low frequency ambient sound down to the infrasonic range but the noise floor may still remain around the 35 dB SPL level making the detection of low amplitude signals difficult.

Aside from output levels and frequency response, the most critical parameter in designing sonar devices is the directionality of the microphone or loudspeaker. At its most basic, this is a function of the transducer size and geometry and the wavelength of sound produced. For the most simple model, that of a planar circular piston in an infinite baffle, the main beam angle can be approximated by

$$\alpha = \sin^{-1} \frac{c}{df} \tag{8}$$

where α is the main beam angle with respect to the angle of propagation, c is the speed of sound in ms^{-1}, d is the diameter of the transducer in meters, and f is the frequency of sound in hertz. The full beam angle is therefore 2α. Thus, a 25-mm diameter transducer operating at 40 kHz produces a main beam angle of 40°. This equation is valid for situations where the wavelength is smaller than the diameter of the transducer.

The actual structure of the beam angles is much more complex than this in reality. Where the wavelength of sound is much smaller than the diameter of the transducer, side-lobes are produced, radiating energy off-axis. In addition, energy falls away with angle and there is no hard cut-off in beam angle. The relative intensity in decibels with angle can be approximated using the following formula derived from equations in [**68**]

$$D = 20\log_{10}\left[\frac{|J_1(ka\sin(\theta))|}{|ka\sin\theta|}\right] \tag{9}$$

where the angular wavenumber $k = (2\pi/\lambda)$, a is the radius of the transducer, λ is the wavelength, θ is the angle in radians off the main axis of the transducer, and J_1 is a first order Bessel function.

In practise, beam angle calculations are approximations to ideal situations but where the transducer is measured in isolation, theory is a good match for the real directional properties [**69**]. Where the transducer sits within a complex object such as a housing, the beam angles calculated will only approximate to the real beam angles, and often the only way to establish angular spread is to measure it. Two idealized radiation patterns are shown in Fig. 4 for a low frequency and high frequency pulse. The high frequency case, where the wavelength of sound is much smaller than the transducer, produces a much more directional beam, but with pronounced side-lobes.

None of the transducer types described here has the flexibility to emulate exactly the range of signal types produced by bats, and each has a compromise in terms of resonant frequency, robustness, or cost. The fact that bats produce signals through vibration of a

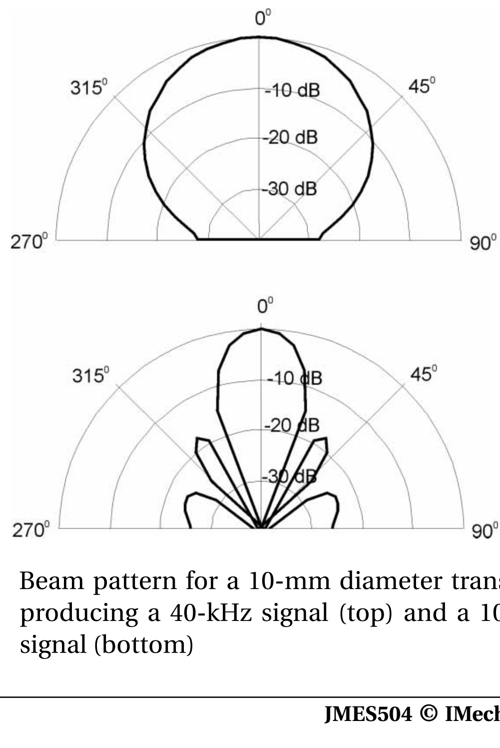

Fig. 4 Beam pattern for a 10-mm diameter transducer producing a 40-kHz signal (top) and a 100-kHz signal (bottom)

membrane in a stream of high pressure air may in fact be appropriate inspiration for new transducer types, which are mechanical rather than electro-mechanical as at present. Such a device may have the flexibility and output levels necessary to fully exploit the potential of ultrasonic sonar in air.

7 APPLICATIONS OF AIRBORNE ULTRASONICS

Airborne ultrasound has largely been relegated to the role of obstacle detection in the past. This is due in part to the lack of commercially available wideband transducers to cover the range 20–200 kHz, but also to the implicit assumption that spatial resolution using the relatively long wavelengths of ultrasound (100 kHz has a wavelength of 3.4 mm) limits what can be achieved in terms of remote sensing small detail. Bats, however, appear to be able to circumvent these considerations and form complex and detailed images of their environment. One of the stumbling blocks is in trying to understand how these acoustic images appear to bats. In medical ultrasonics, ultrasound images are converted to visual representations [**70**]. This is almost certainly not the way that bats view their world. Aside from the signal processing considerations of finding a digital solution to the analogue methods used by the bats auditory system, a greater issue may be in finding better ways to represent echolocation information in a non-visual way [**71**]. Although it is clear that bats are capable of extraordinary feats of detection and classification, an overall view of how this operates is still lacking despite a wealth of information on the individual components of the system, such as call structure, the physiology of peripheral hearing and the central nervous system. Only when these areas of expertize begin to join up will a holistic view of echolocation emerge, and even then, the language in which information about the world is expressed within the bat's brain may still be difficult to interpret.

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APPENDIX

Notation

a	radius of transducer (m)
c	velocity of sound (ms^{-1})
d	transducer diameter (m)
D	relative intensity with angle (dB)
DI	directivity index (dB)
f	frequency (Hz)
J_1	first-order Bessel function
k	angular wavenumber (m^{-1})
NL	noise level (dB)
R	reflection coefficient
SL	source level (dB)
SNL	signal to noise ratio (dB)
TL	transmission losses (dB)
TS	target strength (dB)
Z	acoustic impedance (Pa sm^{-1})
α	main beam angle (rad)
α_i	angle of incidence (rad)
$\delta(x)$	Dirac delta function
θ	angle off the main axis of the transducer (rad)
λ	wavelength (m)
π	pi
ρ	density (kg m^{-3})